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A note on circuit elements

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Abstract. The notion of circuit element is commonly used in electrical and electronic engineering and not only. However, the significance of this notion is sometimes ambiguous mainly since several new concepts have been introduced under the syntagm of circuit elements. In this communication we discuss some aspects related to concepts of generalized circuit elements and separable circuit elements and their synthesis by means of mutators. Relationships to classical nonlinear circuit elements as well as with dynamical nonlinear state-space systems are also highlighted. Also, we discuss the definition of mem-elements, i.e. memristive, mem-capacitive and mem-inductive and explain the reason they are characterized by pinched hysteresis loops, insisting on the need of making a clear distinction between circuit elements and circuit devices.

1. Introduction

The simplest answer to the question: *what are circuit elements*? is: *Circuit elements are resistors, capacitors and inductors and perhaps some others like independent sources etc*. Often these elements are characterized by the syntagm *ideal*. Even though there are situation when this simple answer is reasonably useful, often the answer should be more involved.

A fundamental aspect is that of making a clear separation between physical elements (devices – resistors, capacitors and inductors) and models. The point of view almost unanimously adopted is that of models. For sure, modelling needs often more refined methods that are essentially related to algebraic – differential equations. In any physical device modeling, two basic techniques are used, the physical and black-box ones.

As shown in [1] the physical approach consists of device physics analysis and partitioning, physical equation formulation, simplification and optimization and, if necessary, nonlinear network synthesis. On the other hand, the black-box approach

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∗Correspondence address: lgoras@etti.tuiasi.ro consists of experimental observations i.e., data acquisition, mathematical modeling, model validation, and, again, if necessary, nonlinear network synthesis.

Each approach is strongly dependent on aspects like, class of input signals envisaged to be applied, practical consideration regarding precision, knowledge of physical laws governing the device behavior, and often it is also an art beside being a science. The last step i.e. nonlinear network synthesis is more and more necessary when combined simulations using specialized software tools are expected. A significant simplification in making classifications refer to the set of lumped nonlinear circuit elements that can be classified into algebraic and dynamic, even though the universe of such elements is remarkably large. Coming back to nonlinear synthesis, as a last part of modelling, it can be considered in reverse order, i.e., it has been considered useful to *invent* special elements synthesizable using common existing elements.

2. Classical circuit elements: lumped/distributed, linear/nonlinear

Lumped circuit elements are those used when for a physical device to be modeled the smallest wavelength associated to the highest relevant frequency in the signals components is much larger than the physical device dimension. Lumped refers to the physical dimensions but also to the model – it means that the model is described by algebraic and ordinary differential equations. We reiterate the fact that the model of a system strongly depends on the class of signals the system is supposed to process. Distributed models are described by partial derivative equations and correspond to devices with dimensions comparable to the smallest wavelength of the signals of interest.

In the most general classical sense, the three types of **lumped** (algebraic) n-port **nonlinear** circuit elements are: resistive, capacitive and inductive.

Resistive circuit elements are described by an algebraic relation between the (vector) port variables current **i**, voltages **u,** and time $f(\mathbf{u},\mathbf{i},t)=0$. This definition covers not only low frequency models for various devices but also voltage and current autonomous sources as well as any type of linear or nonlinear controlled sources.

Capacitive elements are (in general) multiports characterized by port variables voltage **u**, electric charge **q**, and time $f(\mathbf{u},\mathbf{q},t)=0$. An interesting observation is that the concept covers small signal nonreciprocal capacitive multiports encountered, among others, in the models of CMOS devices.

Similarly, inductive elements are (in general) multiports characterized by port variables magnetic flux $\boldsymbol{\varphi}$, voltage **u** and time $f(\boldsymbol{\varphi},\mathbf{i},t)=0$.

It can be observed that, formally, it is possible to define another circuit element, the fourth, named by Chua [2], memristor – “the missing circuit element” described by an algebraic nonlinear relationship between magnetic flux $\boldsymbol{\varphi}$ electric charge, **q** and time of the form $f(\boldsymbol{\varphi},\mathbf{q},t)=0$.

Even though the memristor has been defined as the *missing element*, in the seminal paper [2] a schematic for synthesizing a memristor using OA’s transistors, resistors and capacitors is given. Later, in 2008, researchers from Hewlett Packard built a device from TiO2 the results being published in the May 2008 issue of Nature, as well as an article in the December 2008 issue of the IEEE Spectrum. In these papers it is stated that *Chua's missing memristor was finally discovered by a team of researchers at Hewlett-Packard* - this memristor proved to be used as a nonvolatile memory device and as a possible replacement for the traditionally used flash memories and DRAMS, and research continues nowadays as well.

Let us point out a certain ambiguity between physical realities and models involved in the above discussion. In fact, the device produced by HP does not represent a model but a physical reality described by a set of algebraic-differential equations of the form

$$\begin{gathered} u = R(w)i \\ \frac{dw}{dt} = f(i) \end{gathered} \qquad (1)$$

where, integrating in time corresponds to a relationship between charge and flux even though there is no physical magnetic flux involved. In fact, the initial memristor was introduced as a *model* which fortunately happened to describe, in certain limits of precision, various device with interesting applications potential.

3. Higher order elements

Coming back to the above discussion let us observe that, the main idea involved in the definition of lumped nonlinear elements is that of algebraic relationships between u and i for resistive elements, $s^{-1}u$ and i for inductive elements, u and $s^{-1}i$ for capacitive elements and $s^{-1}u$ and $s^{-1}i$ for memristive elements where $s^{-1}(x)=\int_{-\infty}^{t} xdt$ where x is either voltage or charge. The general form of this set of circuit elements is $f(s^{-k}u, s^{-i}i, t)=0$ where k=i=0 for resistive elements, k=1, i=0 for inductive elements, k=0, i=1 for capacitive elements and k=i=1 for memristive elements. This observation led to the generalization proposed by Chua and Szeto [3], [4] i.e., a class of generalized circuit elements i.e., higher order elements, characterized by algebraic relationships of the form $f(s^{-k}u, s^{-i}i, t)=0$ for (i,k) €{Z, Z}. In these relations, the significance of s^k is

$$s^k = \begin{cases} \frac{d^k(.)}{dt^k} & k > 0 \\ \int_0^t dt_1 \int_0^{t_1} \dots \int_0^{t_{|k|-1}} (.)dt_{|k|} & k < 0 \end{cases} \qquad (2)$$

The mechanism allowing the synthesis/conversion of higher order elements is through generalized mutators which are two ports described by the transmission matrices

$$T_c = \begin{bmatrix} as^m & 0 \\ 0 & ds^n \end{bmatrix} \quad \text{or} \quad T_i = \begin{bmatrix} 0 & bs^m \\ cs^n & 0 \end{bmatrix} \qquad (3)$$

with a,b,c,d €**R**-{0} and m,n €**Z** and subscripts c and i denoting conversion type and inversion type respectively.

Two common examples of (linear) higher order elements, generally not recognized so, are the frequency dependent negative resistor, FDNR described by the symbolic relationship $u=ks^{-2}i$ and the frequency dependent negative conductor, FDNC described by $i=ks^{-2}u$ which, for s=jω behave as linear resistors/conductors with frequency dependent values and applications in analog filter design.

4. Separable circuit elements

A more general class of higher order elements has been defined when it has been observed that s^k defined in (2) is nothing but the symbol of a particular case of a linear integro-differential operator. Thus, a broader class of higher order circuit elements, the so-called separable dynamic elements [5], [6] has been introduced. These elements, contain as particular cases all non-linear elements, already defined and contain elements described by an algebraic relation between the result of a linear operator symbolically represented by A(s) acting on the voltage u and the result of another operator, D(s) acting on the current i, f(A(s)u, D(s)i)=0 with various particular cases regarding time dependence or not or type of control (u- controlled and/or i-controlled) associated to the shape of f.

For example, a current controlled time invariant separable element starting from zero initial conditions can be described by the equation $u = A^{-1}(s)f(D(s)i)$ and the schematic looks like an algebraic nonlinearity sandwiched between to linear dynamic operators. Moreover, in [6] a simple architecture (generalized mutator, Fig.1) for converting a higher order element to another circuit element has been proposed.

$$T_c = \begin{bmatrix} A(s) & 0 \\ 0 & D(s) \end{bmatrix} \quad \text{or} \quad T_i = \begin{bmatrix} 0 & B(s) \\ C(s) & 0 \end{bmatrix}. \tag{4}$$

In particular, the architecture allows the synthesis of a separable dynamic element from a memoryless nonlinear resistor and a linear two port operator having on the principal (secondary) and zero in rest. When port 2 is loaded with a nonlinear resistor $f(u_2, -i_2)=0$, the input port will be described by the relations

$$f(A^{-1}(s)u, (D^{-1}(s)i)=0$$

or (5)

$$f(C^{-1}(s)i, (B^{-1}(s)u)=0$$

which correspond to a separable nonlinear element. A generalized mutator can be synthesized with four controlled sources as shown in the figure below for a type I mutator [6] where the linear transfer functions are voltage gains.

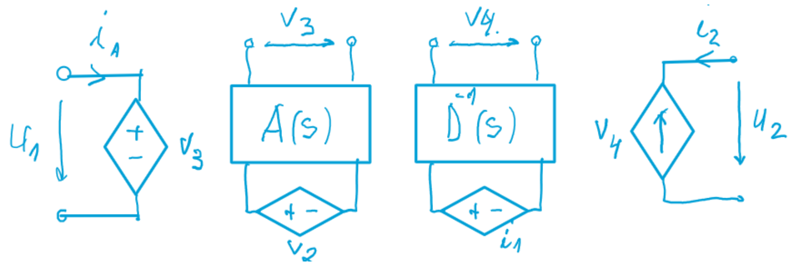

Fig. 1 Schematic of a generalized mutator.

5. Mem-elements

Coming back to memristors, in what follows we discuss the philosophy behind the memristor and that of the similar concepts of memcapacitive and meminductive elements.

To discuss the peculiarity of these circuit elements, let us first review the general state- space model of a dynamical nonlinear system which is of the form

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{g}(\boldsymbol{x},\boldsymbol{u},t) \\ \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{t}) \end{aligned} \qquad (6)$$

where **u, x**, **y** are vectors representing input, state and output respectively, t is time and **f** and **g** are vector-valued functions. In general, in the above equations, the physical dimensions of the input, output and state variable are not constrained. On the other hand, in [7] and [8] general definition of mem-elements (memristor, memcapacitor and meminductors) are defined and analyzed. According to [9], the general form of a **u**-controlled n-port mem-element is

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{g}(\boldsymbol{x},\boldsymbol{u},t)\boldsymbol{u} \\ \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{t}) \end{aligned} \qquad (7)$$

where **y** is the complementary variable associated to **u** (current-voltage, flux-current, charge-voltage) both having the same dimension, n. Thus, the nature of the n-ports we refer to, is related to the physical significance of **u** and **y** as shown in the table below.

- voltage controlled memristor (voltage input current output)	$\boldsymbol{i} = \boldsymbol{g}(\boldsymbol{x},\boldsymbol{u},t)\boldsymbol{u}$ (8) $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{t})$
- current controlled memristor (current input voltage output)	$\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{x},\boldsymbol{i},t)\boldsymbol{i}$ (9) $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{i},\boldsymbol{t})$
- voltage controlled mem-capacitor (voltage input charge output)	$\boldsymbol{q} = \boldsymbol{g}(\boldsymbol{x},\boldsymbol{u},t)\boldsymbol{u}$ (10) $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{t})$
- charge controlled mem-capacitor (charge input voltage output)	$\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{x},\boldsymbol{q},t)\boldsymbol{q}$ (11) $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{q},\boldsymbol{t})$
- current controlled mem-inductor (current input flux output)	$\boldsymbol{\varphi} = \boldsymbol{g}(\boldsymbol{x},\boldsymbol{i},t)\boldsymbol{i}$ (12) $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{i},\boldsymbol{t})$
- flux controlled mem-inductor (flux input current output)	$\boldsymbol{i} = \boldsymbol{g}(\boldsymbol{x},\boldsymbol{\varphi},t)\boldsymbol{\varphi}$ (13) $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{\varphi},\boldsymbol{t})$

Let us observe that there are two fundamental differences between the general model of a nonlinear dynamic system and the above definitions of mem-elements. The first one is the physical nature of the port variables and the second one is that the input variable appears *multiplied* by a function in the algebraic input-output constitutive relations. This last property explains the pinched off hysteretic characteristic in all cases as shown in Fig.2 where neither the physical significance of the input and output signal nor the output and state equations were specified.

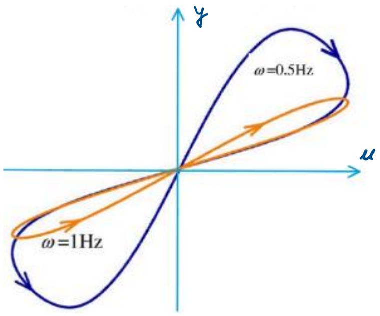

Fig. 2. Pinched off characteristics for two frequencies harmonic inputs.

6. Concluding remarks

By means of the general equations above, three classes of mem-elements have been defined which, „due to their versatility (including analog functionalities), the combined operation of these memory devices in electronic circuits is still largely unexplored" [8], [9]. This opinion should be however considered from two points of view, one corresponding to the hope that new devices will be discovered and they will behave such that the model correspond more or less precisely to one of these models and, the other one, associated to a synthesis point of view, consisting of architectures built with such elements that could be used as signal processing tools.

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