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Analysis of the tire vertical stiffness influence on the vehicle safety

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Abstract. The research outlines findings regarding the effect of tire vertical stiffness on the safety of the vehicle. With the intention of analyzing the impact of different frequency vibrations from mechanical parts on the dynamics of the vehicle in motion, various tire sizes, inflation pressures and tire vertical stiffness values were examined to achieve this aim. In this regard, through the MATLAB software, the input data served as the basis for the implementation of the mathematical model for the entire vibrating vehicle. This modeling process resulted in the generation of comparative results that were graphically analyzed to determine the influences of the tire vertical stiffness on the safety of the considered vehicle.

Keywords: vehicle safety, tire, mathematical modelling, vibrations, inflation pressure.

1. Introduction

1.1. The present paper's scope and objectives

This study aims to assess the response of suspended mass to road surface irregularities due to vehicle movement, taking into consideration different tire dimensions, inflation pressures, and vertical stiffness. The primary goals are the development of a dynamic model to analyze the tire influence on vehicle dynamics and determining optimization trends for the vehicle's dynamic behavior based on the tires utilized.

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1.2. The necessity of the proposed topic

Vehicle safety is essential to minimize the chances of traffic accidents, injuries, and fatalities, making it a critical aspect of modern transportation systems. The World Health Organization approximates that traffic accidents cause 1.19 million deaths annually, making them a significant global factor of death [4, 5].
According to data from the National Transportation Safety Board [1], tire failures cause over 33000 accidents on an annual basis in the United States, more than 2000 of which involves blowouts.
Tire blowouts can trigger vehicle crashes, leading to property damage, higher insurance costs, injuries, and even fatalities. Accidents related to tires can arise from several factors, including blowouts, worn tread, underinflation, or improper tire matching. Proper tire maintenance and selection of the right tires for the specific vehicle and driving conditions is equally important, being crucial in preventing accidents [1, 9].
Vehicle safety has become a key focus in today's road transport systems. Increasing attention is being given to active safety measures in the automotive industry, with safety standards continually rising to reduce accidents. Tires are fundamental to active safety, as traffic regulations alone cannot ensure safe driving, particularly when an average driver unknowingly pushes a vehicle to its physical limits [7, 23, 28].

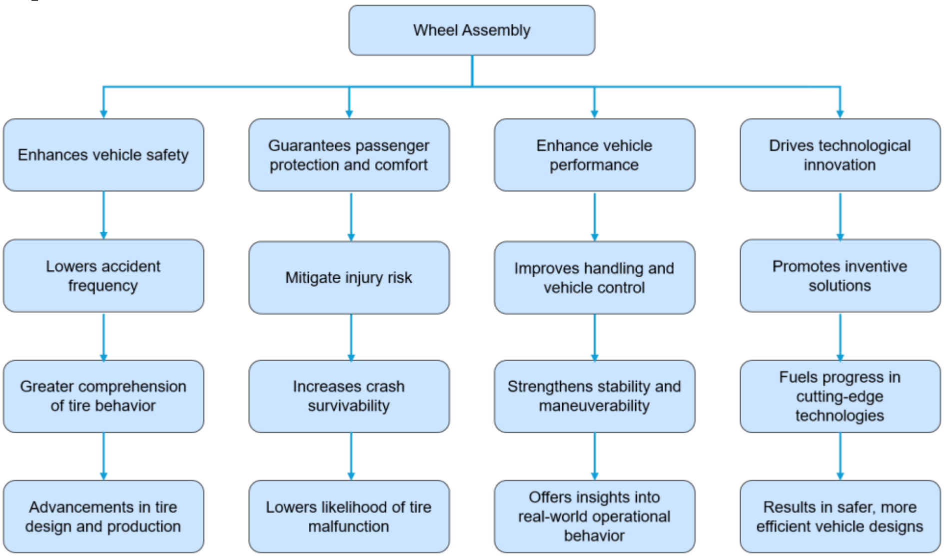

Fig. 1. Necessity of studying wheel assembly impact on the vehicle safety [11, 12, 13, 15, 16, 18, 24].

For a variety of reasons, examining the influence of wheel assembly on vehicle safety is essential. Proper maintenance of the wheel assembly enhances stability and maneuverability, leading to reduced accident rates and greater overall safety (Fig. 1). Additionally, ensuring the wheel assembly operates correctly is important for passenger comfort and safety, as it helps minimize injury severity and improves crash survivability.

Improving vehicle performance leads to better handling and control. This research advances technological progress by encouraging innovation and the development of new technologies. It also ensures full compliance with safety standards and legal regulations, while addressing contemporary challenges effectively [8, 13, 15, 17, 18, 24].

1.3. State of the art

Due to their capacity to transfer forces connecting road and vehicle, such as accelerating, braking, and cornering forces, pneumatic tires have become important in the production of road vehicles. These forces have become essential for the stability, safety, and operational efficiency of the vehicle. Because of their flexible structure, they provide better ride comfort and shock absorption, while advances in tire technology have enhanced handling, durability, and fuel efficiency. Pneumatic tires play a major role in current automotive engineering as they optimize the vehicle behavior [13, 20, 25, 27].

In order to produce the forces required to maneuver the vehicle, there is a requirement for tires, representing the vehicle's single point of contact with the road, which is essential to vehicle efficiency and performance [25].

The forces acting on the tire's contact area strongly affect vehicle behavior. Consequently, tire characteristics have a significant impact on this behavior. Vehicle performance, especially in motorsport, is strongly affected by tire behavior, particularly the optimal functioning of the tire compound. Recently, advancements in tire technology have become essential in both motorsport and the automotive industry. The increasing need for accurate simulation of vehicle dynamics has led to more research into vehicle systems analysis and modeling [13, 14, 25, 26].

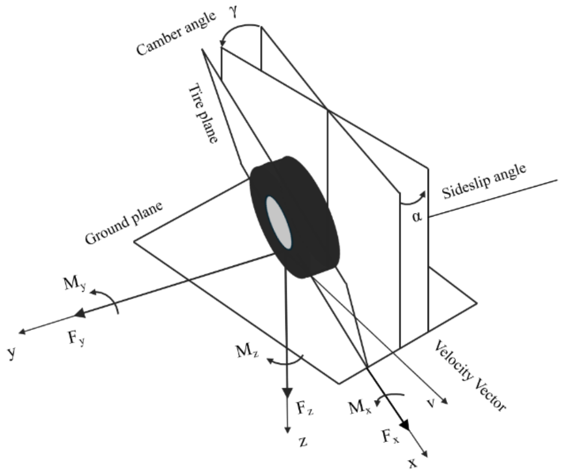

Fig. 2. Tire axis terminology [10, 18, 25, 28].

The longitudinal force F_x (Fig. 2) acts along the x-axis, where a positive value indicates acceleration and a negative value signifies braking, also known as the forward force. The perpendicular force to the ground surface is the normal force F_z, with a positive value indicating an upward direction, commonly referred to as the vertical force. The lateral force F_y, which lies in line with the ground surface and perpendicular to F_x and F_z, has a positive value when it is oriented along the y-axis [18, 25, 28].

The roll moment M_x signifies the x-axis longitudinal moment, where a positive value tends the tire to rotate around the x-axis, being called tilting torque or overturning moment. The lateral moment at the y-axis is known as pitch moment M_y, with a positive value inducing forward rotation of the tire around the y-axis, often referred to as resistance torque due to rolling. The yaw moment M_z describes the moment about the z-axis, with a positive value leading to rotation about this axis, also known as the aligning moment or self-aligning torque [18, 25, 28].

Evaluating tire forces is essential for vehicle safety, performance optimization and development, as it guarantees precise control, stability, and handling while ensuring compliance with regulatory standards and enhancing consumer satisfaction. This process supports data-driven design decisions, particularly in the context of electric vehicles, and enables manufacturers to sustain a competitive edge by delivering high-performing, and reliable vehicles [12, 15, 18, 19, 25, 26].

2. Mathematical modelling for vibration analysis

The objective of this section is to highlight the frequencies generated by different tire configurations while the analyzed vehicle operates under typical road conditions. The entire vibrating vehicle simulation is developed with MATLAB, starting with the vehicle's technical characteristics and the chosen tires' stiffness values (Table 1).

Table. 1. Main parameters of the tested vehicle [2, 3, 21, 22].

Parameter	Value	Unit
Suspension system damping coefficient	1725	[Ns/m]
Tire damping coefficient	5020	[Ns/m]
Stiffness of front spring	28000	[Ns/m]
Stiffness of rear spring	98000	[Ns/m]
Stiffness of tire (conventional, 215/45 R16, 0.2 MPa)	215521	[N/m]
Stiffness of tire (conventional, 215/45 R16, 0.25 MPa)	260949	[N/m]
Stiffness of tire (run-flat, 215/45 R16, 0.2 MPa)	265091	[N/m]
Stiffness of tire (run-flat, 215/45 R16, 0.25 MPa)	320968	[N/m]
Stiffness of tire (conventional, 225/45 R17, 0.20 MPa)	199766	[N/m]
Stiffness of tire (conventional, 225/45 R17, 0.25 MPa)	241763	[N/m]
Stiffness of tire (conventional, 255/40 R17, 0.20 MPa)	263101	[N/m]
Stiffness of tire (conventional, 255/40 R17, 0.25 MPa)	292722	[N/m]
Vehicle mass	1370	[kg]
Unsprung mass	47	[kg]
Longitudinal mass moment	870	[kg·m^2]
Lateral mass moment	1200	[kg·m^2]

Parameter	Value	Unit
Center of gravity to front axle distance	1.4035	[m]
Center of gravity to rear axle distance	1.157	[m]
Track width in half (front axle)	1.511	[m]
Track width in half (rear axle)	1.559	[m]

The maneuvering, stability, general driving performance, and ride comfort, considered key factors to improve vehicle safety, are greatly influenced by tire frequencies. To be able to determine how a vehicle responds to different road conditions, these frequencies are usually linked to the tire's natural frequencies and modes of vibration [11].

2.1. Development of the entire vehicle vibration analysis model

The entire vehicle model is a generic vibrating vehicle model. The components of the model are represented in Figure 3, which contains: wheels hop (denoted by x_1, x_2, x_3, x_4), independent road inputs y_1, y_2, y_3, y_4, vertical body motion x, body roll φ, with body pitch θ. The vehicle body is treated as a rigid structure, characterized by a mass m, a longitudinal moment of inertia I_x, and the lateral moment of inertia I_y. Each wheel is modeled with mases m_1, m_2, m_3, and m_4, while the stiffness of the front and rear tires is represented by k_{tf} and k_{tr} [18].

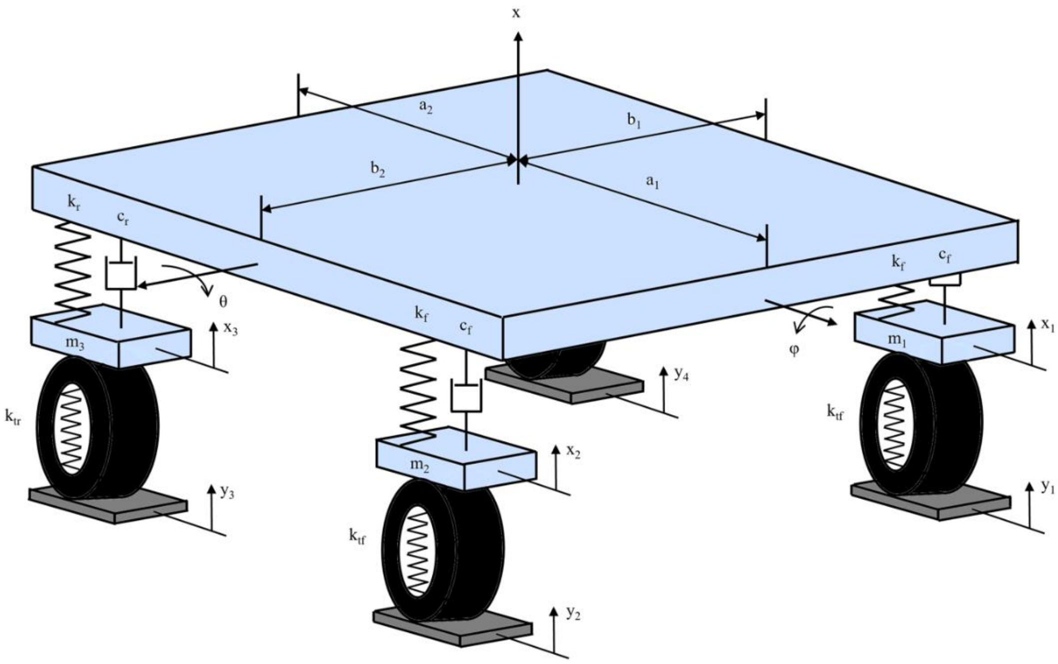

Fig. 3. Entire vehicle vibration analysis model [18].

$$m_1 = m_2 = m_f \tag{2.1}$$

$$m_3 = m_4 = m_r \tag{2.2}$$

Front suspension system is equipped with damping c_f and stiffness k_f, while the rear suspension utilizes damping c_r and stiffness k_r. It is standard practice for the left and right wheel suspensions to be identical in both rigidity and damping [18].

It may be used [18]:

$$w_f \equiv w = b_1 + b_2, \tag{2.3}$$

$$M_R \equiv -k_R \cdot \varphi, \tag{2.4}$$

where M_R is torque delivered by antiroll bar (if exists), proportional to the roll angle φ.

The equations have been developed in MATALB using scripting techniques, utilizing the technical characteristics of the vehicle considered. Consequently, the following matrix of mass [m], matrix of damping [c], and matrix of stiffness [k] needed to be solved [18]:

$$[m] = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_y & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_f & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_r \end{bmatrix}. \tag{2.5}$$

$$[c] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & -c_f & -c_f & -c_r & -c_r \\ c_{21} & c_{22} & c_{23} & -b_1c_f & b_2c_f & b_1c_r & -b_2c_r \\ c_{31} & c_{32} & c_{33} & -a_1c_f & a_1c_f & -a_2c_r & -a_2c_r \\ -c_f & -b_1c_f & a_1c_f & c_f & 0 & 0 & 0 \\ -c_f & -b_2c_f & a_1c_f & 0 & c_f & 0 & 0 \\ -c_r & b_1c_f & -a_2c_r & 0 & 0 & c_r & 0 \\ -c_r & -b_2c_r & -a_2c_r & 0 & 0 & 0 & c_r \end{bmatrix}, \tag{2.6}$$

$$\begin{gathered} c_{11} = 2c_f + 2c_r, \\ c_{21} = c_{12} = b_1c_f - b_2c_f - b_1c_r + b_2c_R, \\ c_{31} = c_{13} = 2a_2c_r - 2a_1c_f, \\ c_{22} = c_R + b_1^2c_f + b_2^2c_f + b_1^2c_r + b_2^2c_r, \\ c_{32} = c_{23} = a_1b_2c_f - a_1b_1c_f - a_2b_1c_r + a_2b_2c_r, \\ c_{33} = 2c_fa_2^2 + 2c_ra_2^2. \end{gathered} \tag{2.7}$$

$$[k]=\begin{bmatrix} k_{11} & k_{12} & k_{13} & -k_f & -k_f & -k_r & -k_r \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & b_1k_r & -b_2k_r \\ k_{31} & k_{32} & k_{33} & a_1k_f & a_1k_f & -a_2k_r & -a_2k_r \\ -k_f & k_{42} & a_1k_f & k_{44} & -\frac{k_R}{w^2} & 0 & 0 \\ -k_f & k_{52} & a_1k_f & -\frac{k_R}{w^2} & k_{55} & 0 & 0 \\ -k_r & b_1k_r & -a_2k_r & 0 & 0 & k_r+k_{t_r} & 0 \\ -k_r & -b_2k_r & -a_2k_r & 0 & 0 & 0 & k_r+k_{t_r} \end{bmatrix}, \tag{2.8}$$

$$\begin{gathered} k_{11}=2k_f+2k_r, \\ k_{21}=k_{12}=b_1k_f-b_2k_f-b_1k_r+b_2k_R, \\ k_{31}=k_{13}=2a_2k_r-2a_1k_f, \end{gathered} \tag{2.9}$$

$$\begin{gathered} k_{22}=k_R+b_1^2k_f+b_2^2k_f+b_1^2k_r+b_2^2k_r, \\ k_{32}=k_{23}=a_1b_2k_f-a_1b_1k_f-a_2b_1k_r+a_2b_2k_r, \\ k_{42}=k_{24}=-b_1k_f-\frac{1}{w}k_R, \\ k_{52}=k_{25}=b_2k_f+\frac{1}{w}k_R, \end{gathered} \tag{2.10}$$

$$\begin{gathered} k_{33}=2k_fa_2^2+2k_ra_2^2, \\ k_{44}=k_f+\ k_{t_f}+\frac{1}{w^2}k_R, \\ k_{55}=k_f+k_{t_f}+\frac{1}{w^2}k_R. \end{gathered} \tag{2.11}$$

Natural frequencies values and the mode shapes are determined by using the matrix formulation $[A]=[m]^{-1}[k]$, paired with eigenvalue and eigenvector techniques [6]. These approaches generate a column vector tied to the eigenvalues of the matrix A. Through eigenvector analysis, the most critical components of the mode shape, labeled u_1 through u_7, include: φ (roll angle about x-axis), x (longitudinal displacement), θ (pitch angle about y-axis), and the vertical displacements at the front and rear wheels (x_1, x_2, x_3, x_4). At a resonance frequency, every mode shape analyzes the relative amplitude of each location in the entire vehicle model [18].

3. Obtained results

To investigate the impact of tire vertical analysis on vehicle dynamics and to explore various tire configurations, such as conventional versus run-flat tires, different tire sizes (either identical on both axles or narrower on the front with a wider rear tire), or varying inflation pressure for the front and rear axles, 12 distinct scenarios were designed. These scenarios includes configurations like: Scenario 1, which uses conventional tires (215/45 R16) inflated to 0.2 MPa on each axles; Scenario 2, with conventional tires (215/45 R16) with 0.25 MPa inflation pressure on each axles; Scenario 3, featuring run-flat tires (215/45 R16) inflated to 0.2 MPa on each axles; Scenario 4, employing run-flat tires (215/45 R16), 0.25 MPa on each axles; Scenario 5, with conventional tires (215/45 R16) with a inflation pressure of 0.2 MPa on the front and run-flat tires (215/45 R16), 0.2 MPa on the rear axle; and so on in the case of the remaining configurations, each varying in the tire type (conventional or run-flat), size (225/45 R17 for the front and 255/40 R17 in the case of rear axle), and inflation pressure.

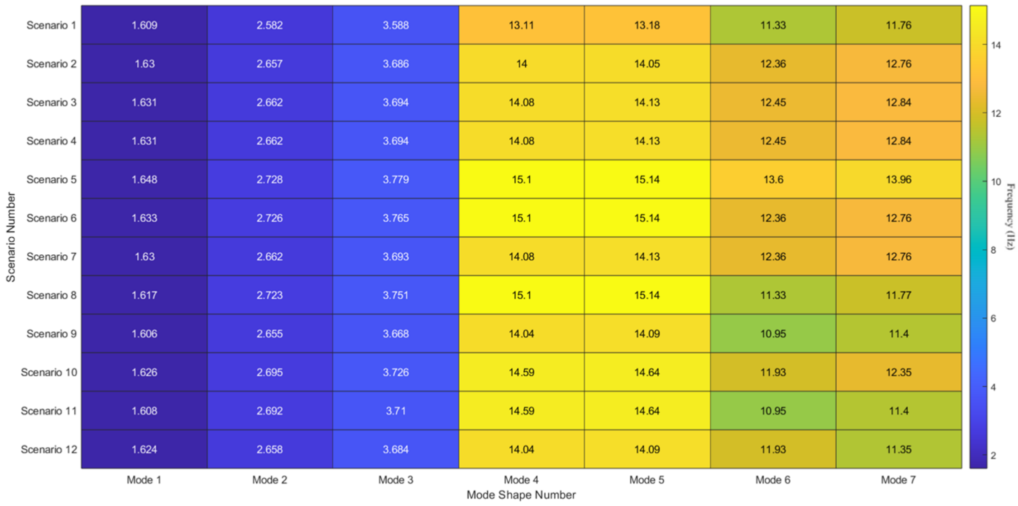

Fig. 4. Heatmap illustrating frequencies for multiple mode shapes and scenarios.

Also, by analyzing the phenomenon evolution of the vehicle behavior, and due to high number of graphical representations, the data was collected and represented in a heatmap (Fig. 4) for a better visualization.

4. Conclusions

The mode shape frequencies tend to rise when the tire pressure is increased from 0.2 MPa to 0.25 MPa, reflecting the stiffer characteristics of the tire, which consequently results in higher oscillation frequencies. This stiffness reduction, resulting from lower tire pressure, enhances tire flexibility, leading to a softer dynamic response. While this may improve riding comfort, it could detract from vehicle control and stability.

A comparative analysis of run-flat tires (as examined in scenarios 3 and 4) with conventional tires under equivalent pressure conditions reveals that higher frequencies in vertical displacement modes are noticed in run-flat tires. This behavior is attributed to the robust sidewall construction of run-flat tires, which are engineered to sustain vehicle load even when deflated. Conversely, conventional tires (considered in scenario 1 and scenario 2) demonstrate lower frequencies in these modes, indicative of a more flexible suspension response.

Scenarios 5 through 8, which explore hybrid configurations of conventional and run-flat tires, reveal frequencies that lie intermediate between those of pure conventional and run-flat setups, suggesting a balance between tire rigidity and flexibility. However, this balance might introduce variability in handling performance and reduce the predictability of vehicle behavior.

In addition, the analysis of the wider tires in scenarios 9 through 12 reveals their significant impact on vehicle dynamics due to their larger contact patches and varying stiffness properties. A larger contact area in which the tire interact with the road enhances the ability of tires to produce grip, which can lead to improved handling, particularly in cornering and high-speed maneuvers. Wider tires are associated with higher mode shape frequencies, which, although enhance vehicle maneuvering, but reduce the ride comfort.

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