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Theoretical aspects of internal-external gear pairs with small difference in numbers of teeth and their applications to cycloidal gear mechanisms

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Abstract. The transmission ratio of a cycloidal drive with one internal-external gear pair increases as the difference in numbers of theeth of these gears decrease. For an internal-external gear pair with involute tooth profile, the difference between their theeth numbers cannot be lower than a certain limit because of tooth tip interference. In this paper, a study has been made to examine the possibility of lowering this difference as much as possible, by choosing appropriate value for the distance between the axes of the two gears, along with adjusting the addendum modification coefficients. First, some theoretical aspects and some results on numerical simulation are presented. Then, two case studies concerning the use of these kind of gears for building cycloidal gear mechanisms will be discussed.

Keywords: internal gear, small difference, teeth numbers, cycloidal gear mechanism.

1. Introduction

Internal-external gear pairs with a small difference between the numbers of teeth are frequently encountered in the construction of planetary or differential reducers with high transmission ratios, as well as in the construction of compact gearboxes [1], [2], [3], [4]. They have a number of advantages, when used in such mechanisms, such as: they are very compact, ensuring high transmission ratios at small overall dimensions, both gears rotate in the same direction, etc. This has led to numerous researches in recent years [5]-[15].

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However, in addition to the mentioned advantages, these gear transmissions also have a number of disadvantages such as: the interference phenomenon (involute, trochoid and cutting interference) and a low contact ratio, especially when the difference between the numbers of teeth of the two gears is small. When this difference decreases, the contact ratio is also reduced [9], [10], [11].
To avoid tooth interference, it is necessary to use high addendum modification coefficients. For this reason, both the design and the machining of these gears become difficult [15].
Some theoretical aspects regarding internal cylindrical gears with a small difference between the numbers of teeth, a series of numerical results, as well as two examples of the use of these gears in the construction of cycloidal gear mechanisms, will be presented in this paper.
The paper is organized as follows: first some theoretical aspects of internal-external gear pairs with small difference in numbers of teeth are presented; then two case studies concerning the use of these kind of gears for building cycloidal gear mechanisms are discussed in section 3; conclusions are given in section 4.

2. Some theoretical aspects of internal-external gear pairs with small difference in numbers of teeth

2.1. General aspects

By appropriately selecting the distance between the axes of the two gears, as well as the addendum modification coefficients, it is possible to reduce the difference between the number of teeth on the internal and external gears to one (Fig. 1), and even zero. In this way, mechanisms formed with such gear pairs can achieve high transmission ratios, with small dimensions, using simple and compact design solutions [2]-[6], [14].

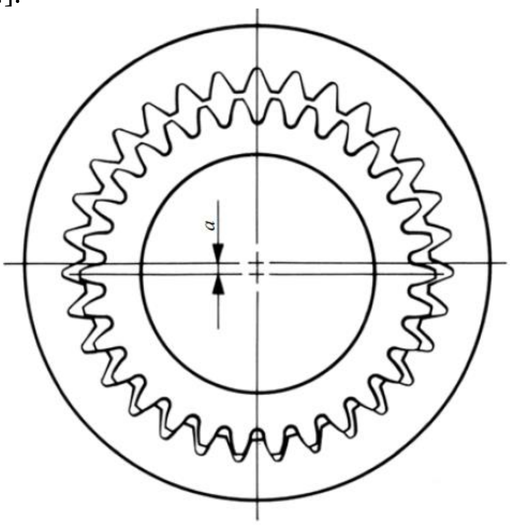

Fig. 1. Internal-external gear pair with small difference in number of teeth ($z_2 - z_1 = 1$) [16].

Despite these significant advantages, internal-external gear-pairs with a small difference between the numbers of teeth also have some drawbacks. Among these, the following can be mentioned: the occurrence of interference phenomena and a reduced contact ratio. If the value of the contact ratio falls below 1, there will be a brief loss of contact between the teeth, leading to improper gear pair operation. If this contact ratio is close to 1, assembly errors, as well as wear, can also cause a loss of teeth contact. Therefore, it is recommended that the value of the contact ratio should be at least 1, 2. As for the interference, next phenomena could appear: involute interference, trochoid interference and trimming interference.
A study was conducted using an original algorithm to investigate the limits of these types of gear pairs concerning: the minimum number of teeth on the internal gear; the minimum difference between the number of teeth on the internal and external gears; the distance between the axes of the two gears; the values of addendum modification coefficients to avoid negative phenomena in the operation of the gear pair; the influence of certain geometric parameters on gear interference phenomena. In the following subsections, only a selection of the numerical results obtained will be presented.

2.2. Minimum number of teeth of the internal gear

Setting aside the manufacturing process, the following will present the influences of various parameters, such as the addendum modification coefficient, x_{n2}, the tooth angle, β, and the reference pressure angle, α_0, on the limits of the minimum number of teeth, $z_{2\min}$, for the internal toothed gear [17].
As we know, for the entire inner tooth of the internal gear to have an involute profile, it is necessary for the diameter of the pitch circle of this gear, d_{a2}, to be greater than or equal to the base circle diameter, d_{b2}. This implies that:

$$m_t \cdot z_2 - 2 \cdot \left(h_{an}^* - x_{n2}\right) \cdot m_t \cdot \cos\beta \geq m_t \cdot z_2 \cdot \cos\alpha_t \, , \tag{1}$$

which means that

$$z_2 \geq \frac{2 \cdot \left(h_{an}^* - x_{n2}\right) \cdot \cos\beta}{1 - \cos\alpha_t} \, , \tag{2}$$

where: m_t is the transversal module; h_{an}^* is the addendum coefficient.
This implies that equation (2) can be utilized to calculate the minimum number of teeth that can be chosen for the internal gear. In the case of a gear with straight teeth ($\beta = 0$), an unmodified tooth profile $x_{n2} = 0$, and a standardized tool ($h_{an}^* = 1$; $\alpha_0 = 20^{\text{o}}$), the minimum number of teeth will be:

$$z_{2\min} \geq \frac{2 \cdot (1 - 0) \cdot \cos 0^{\text{o}}}{1 - \cos 20^{\text{o}}} = 33 \, . \tag{3}$$

This number of teeth, $z_{2\min}$, can be smaller when the tool is non-standardized (the values of the parameters α_0, β and/or x_{n2} are deviating from the above mentioned values). Thus, in Fig. 2, the variation of the minimum number of teeth, $z_{2\min}$, is presented for $x_{n2} = 0\mathrm{K}\ 0.8$. Upon careful examination of these surfaces, it can be observed that when one of the parameters (α_0, β, x_{n2}) increases, the minimum number of teeth for the internal gear decreases, and it may even fall below 10. The most significant influence on the variation of the minimum number of teeth is attributed to the tooth profile modification coefficient, x_{n2}. Following this is the influence of the reference pressure angle α_0, and finally, the value of the tooth angle, β.

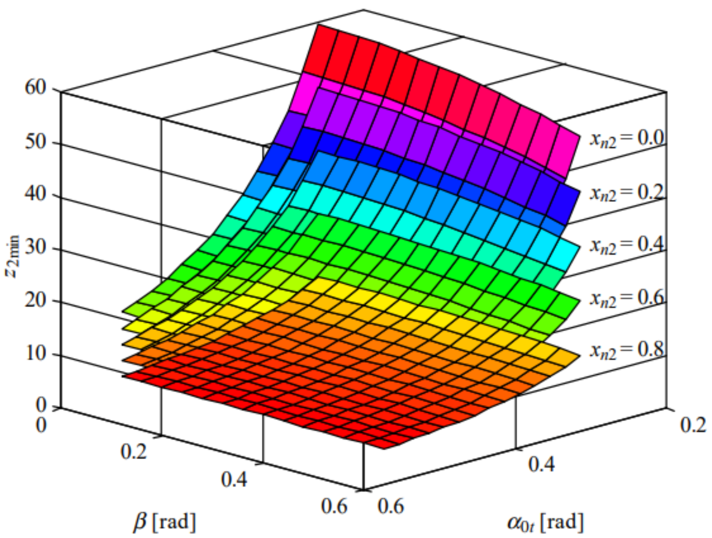

Fig. 2. The influence of the parameters α_{0t}, β and x_{n2} on the minimum number of teeth [17].

2.3. Some aspects on the internal-external gear pair

Based on the above-mentioned algorithm, discussions were conducted on pairs of internal-external gears with $z_2 - z_1 = 1$, having the number of teeth of the external gear, $z_1 = 60$, the reference pressure angle, $\alpha_0 = 15^{\mathrm{o}}\mathrm{K}\ 25^{\mathrm{o}}$, and the tooth angle, $\beta = 0^{\mathrm{o}}\mathrm{K}\ 15^{\mathrm{o}}$. Additionally, an examination was conducted on the impact of the difference in the number of teeth, $z_2 - z_1 = 1\mathrm{K}\ 4$.

The investigation yielded outcomes related to the permissible range of the center distance between the internal and external gears, represented as a_w, as well as findings concerning the boundaries of the addendum modification coefficients, x_{n1}

and x_{n2}. The results are presented in Fig. 3 to 6. These findings can prove highly valuable for individuals seeking to compute the geometric parameters of a gear pair. Further results will be unveiled in forthcoming research.

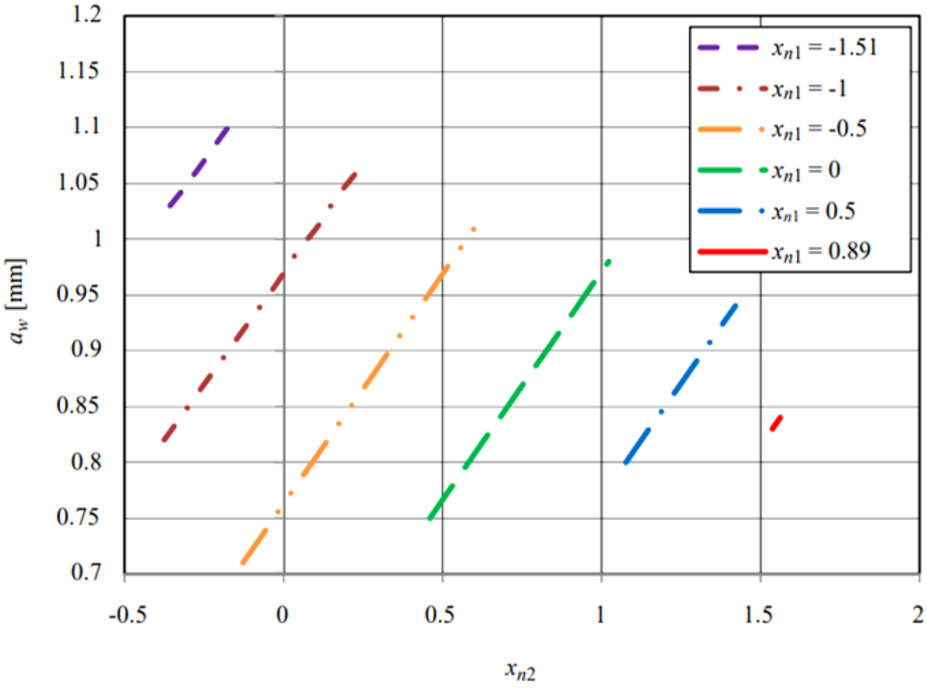

Fig. 3. The center distance, a_w, in relation to the addendum modification coefficient, x_{n2}, for the parameters: $z_1 = 60$, $z_2 = 61$, $\alpha_0 = 20^o$, $\beta = 0^o$, and module $m_n = 1$.

Examining the diagrams illustrated in Figs. 3 to 6, it is evident that the fluctuations in the angles α_0 and β, as well as the variation in the difference $z_2 - z_1$, there is an escalation in the absolute values of $x_{n2\min}$ and $x_{n2\max}$, noting that the impact of the angle β is relatively small compared to the other two parameters under consideration.

By selecting the center distance and the addendum modification coefficients within the specified ranges and applying the provided algorithm, one can determine the geometric parameters of the internal-external gear pair. This approach helps mitigate potential adverse effects that may arise during manufacturing or gear engagement.

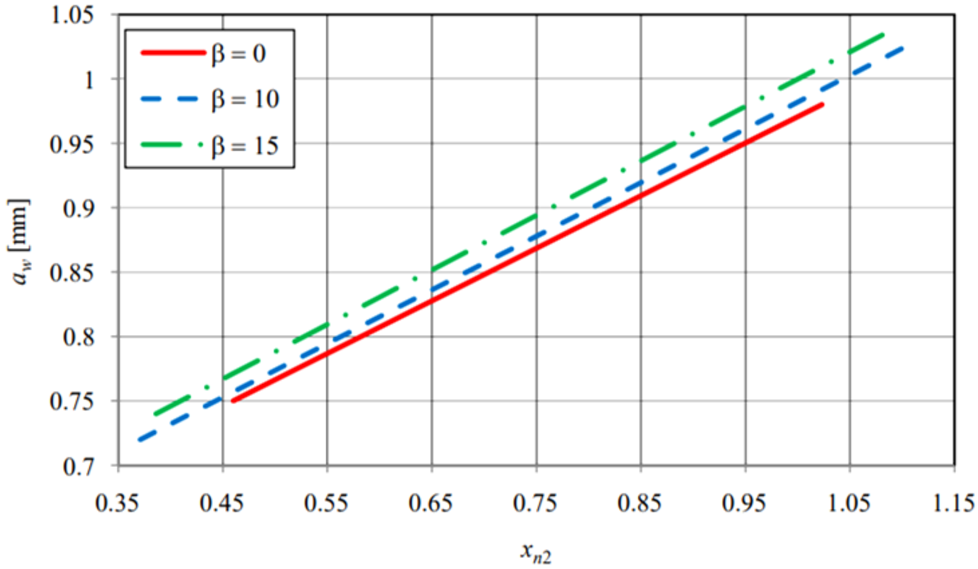

Fig. 4. The center distance, a_w, in relation to the addendum modification coefficient, x_{n2}, for the given specifications: $z_1 = 60$, $z_2 = 61$, $\alpha_0 = 20^o$, $x_{n1} = 0$, and module $m_n = 1$.

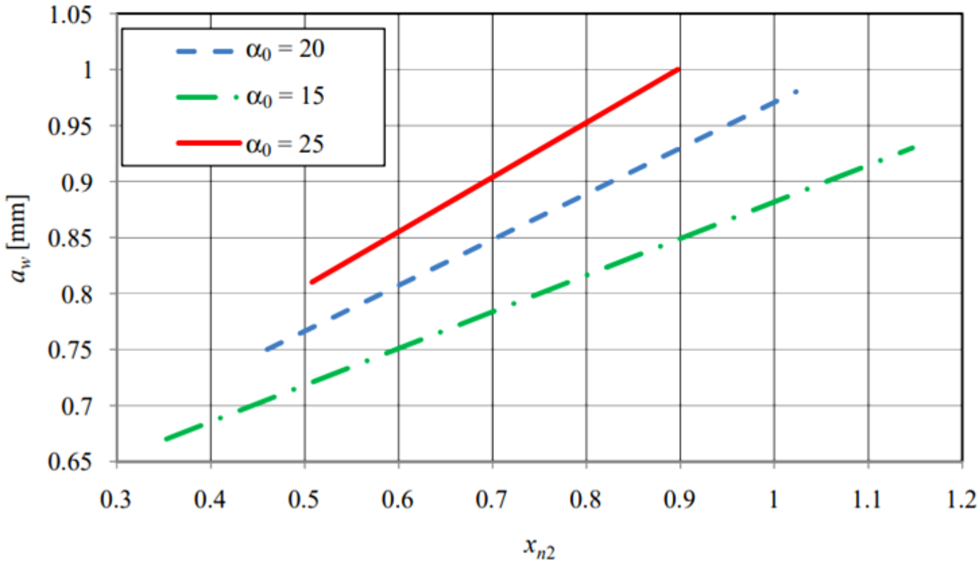

Fig. 5. The center distance, a_w, in relation to the addendum modification coefficient, x_{n2}, for the specified parameters: $z_1 = 60$, $z_2 = 61$, $\beta = 0^o$, $x_{n1} = 0$, and module $m_n = 1$.

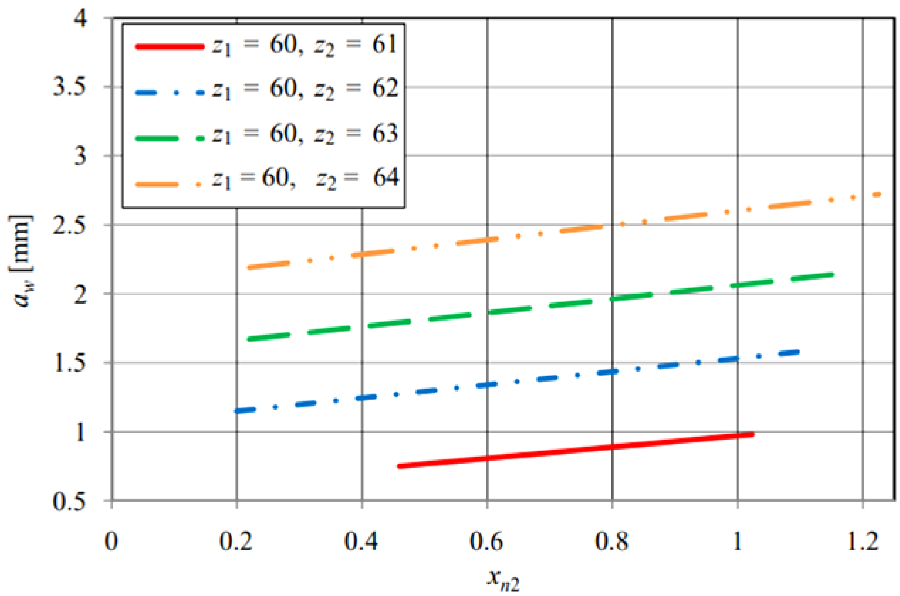

Fig. 6. The center distance, a_w, in correlation with the addendum modification coefficient, x_{n2}, under the conditions: $\alpha_0 = 20^o$, $\beta = 0^o$, $x_{n1} = 0$, and module $m_n = 1$.

3. Applications to cycloidal gear mechanisms

Various research studies and technical solutions exist in literature regarding the development of gear drives with high transmission ratios while maintaining small dimensions. Among them, mention can be made of cycloidal drive and harmonic drive. In addition to the numerous advantages of these mechanical transmissions, there are also several disadvantages, such as the difficulty in achieving the tooth profile (cycloidal drive) or the challenge of obtaining a flexible gear ring (harmonic drive). Starting from these aspects, we will propose and discuss here two new technical solutions of planetary gear mechanisms, using normal internal – external gear pairs (gears with involute tooth profiles).

The small difference in the number of teeth in internal-external gear pairs enables the design of planetary mechanisms featuring a single solar gear. These mechanisms are not only compact but also offer a high transmission ratio and substantial torque within a confined space. A small difference in teeth numbers results in a very small distance between the axes of the gear pair. This small distance allows for the extraction of rotary motion from the planet gear (when the sun gear is stationary) or the prevention of planet gear rotation around its axis (when the sun wheel is mobile). Various types of couplings can be employed for this purpose, facilitating a minimal distance between two parallel axles. Common choices include the Oldham coupling, universal joint, bolted couplings, or some flexible couplings. In extreme cases, when the difference $z_2 - z_1 = 0$, the second gear pair may function as a coupling mechanism.

In the following, we will discuss two new solutions, using some balls (Fig. 7) or double conical rollers (Fig. 12), which may eliminate the rotation of the planet gear around its axis. We will first examine a mechanism that employs certain balls, potentially preventing the rotation of the planetary gear around its axis (see Fig. 7). A technical solution of the planetary gear is shown in Fig. 8.

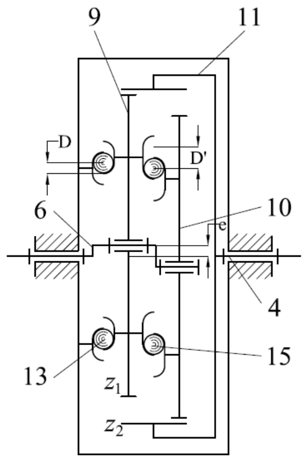

Fig. 7. Kinematics of a first planetary gear mechanism.

The planetary gear reducer, according to Fig. 8, consists of an outer casing (1), closed at both ends by a front cover (2) and a side cover (3). In the central part of the side cover (3), a driven shaft (4) is mounted, on which an inner flange (5) is attached. In the central part of the front cover (2), a driving shaft (6) is mounted, equipped with two eccentric steps (a) and (b), which function as satellites. The carrier (6) is supported, with its inner end, in a front bore (c) of the shaft (4). On the eccentric steps (a) and (b), two planet gears (9) and (10) are mounted, externally toothed and having a number of teeth z_1, engaging with a ring gear (11), internally toothed, which has a number of teeth z_2, and the relationship between the numbers and is $z_2 - z_1 = 1$. The ring gear (11) is fixed on the inner flange (5), mounted on the driven shaft (4). Between the profiled grooves (d) on the cylindrical parts with flanges (12), mounted on the front cover (1), and the grooves (f) on the front surface of the planet gear (9), balls (13) are mounted, and the relationship between the diameter D of the grooves and the eccentricity e of the driving shaft (6) is $D = 2 \cdot e$. Between the profiled grooves (g) on the cylindrical parts with flanges (14), mounted on the planet gear (10), and the grooves (h) on the lateral surface of the planet gear (9), balls (15) are mounted, and the relationship between the diameter of these grooves D' and the eccentricity e of the driving shaft (6) is $D = 2 \cdot e$. The number of balls (13) and (15) is at least two for each, and the use of four balls is recommended to retain the rotational movement of the planet gears (9) and (10). By rotating the carrier (6), the planet gear (9) performs a circular translational movement relative to the front cover (1), with a circle radius equal to e, and the balls (13) eliminate the possibility of rotation of the planet gear (9) around its own axis. Simultaneously, the planet gear (10) performs a circular translational movement relative to the planet gear (9), with a circle radius equal to double the eccentricity e, and the balls (15) eliminate the possibility of rotation of the planet gear (10) around its own axis.

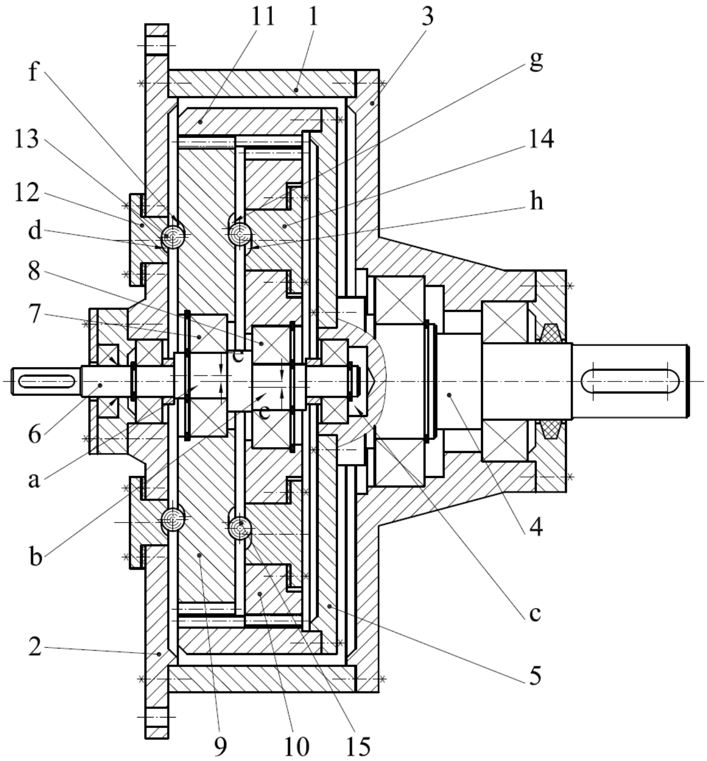

Fig. 8. Technical solution of a first planetary gear mechanism.

In Figs. 9 to 11, details regarding the technical solutions for restraining the planet gears (9) and (10) from rotating around their own axes are presented.
The transmission ratio of the planetary gear reducer, when carrier (6) acts as the driving link, is given by the expression:

$$i_{6-4}^{9} = \frac{z_2}{z_2 - z_1}, \tag{4}$$

reaching its maximum value when $z_2 - z_1 = 1$, respectively,

$$i_{6-4}^{9} = z_2. \tag{5}$$

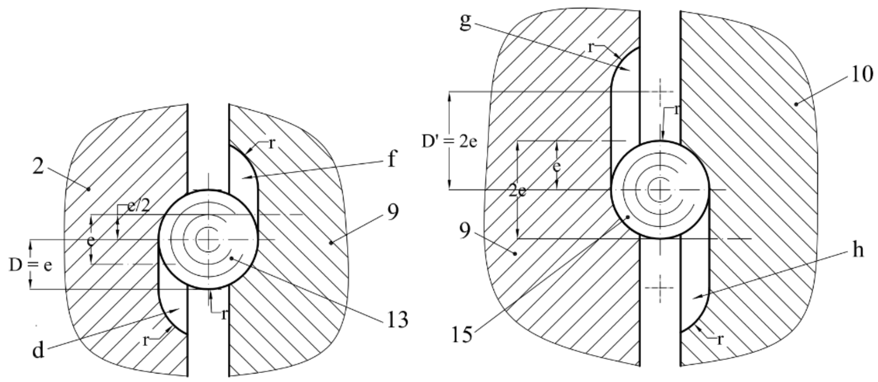

Fig. 9. Details, in axial section, regarding the technical solutions for restraining the planet gears from rotating around their own axes: a) for planet gear (9); b) for planet gear (10).

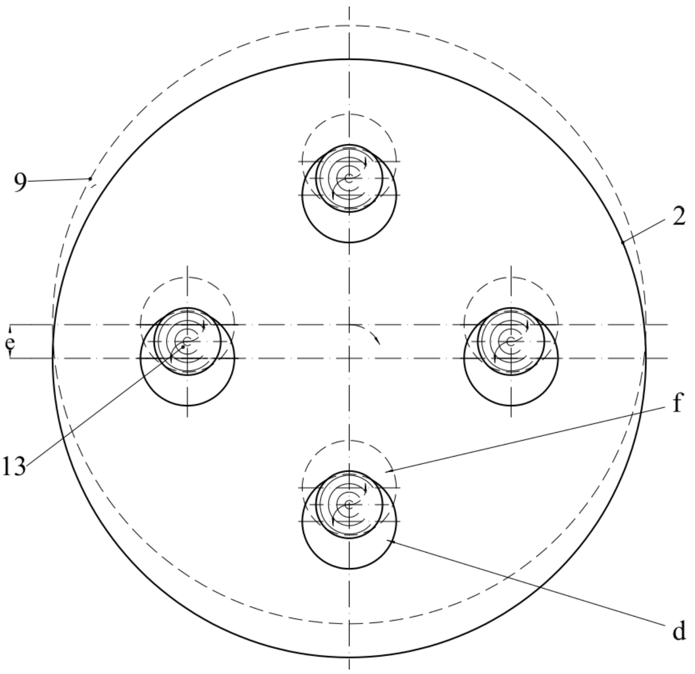

Fig. 10. Details, in frontal view, regarding the technical solution for restraining the planet gear (9) from rotating around its own axis.

When the driving link is the shaft (4), the transmission operates as an amplifier, and the transmission ratio is:

$$i_{4-6}^{9} = \frac{z_2 - z_1}{z_2}, \tag{6}$$

reaching its minimum value when $z_2 - z_1 = 1$, respectively,

$$i_{4-6}^{9} = \frac{1}{z_2}. \tag{7}$$

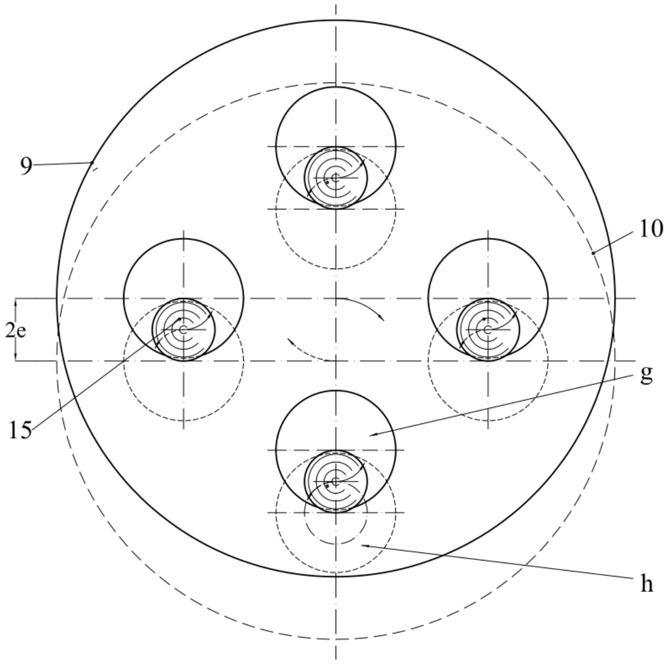

Fig. 11. Details, in frontal view, regarding the technical solution for restraining the planet gear (10) from rotating around its own axis.

We will now discuss the second planetary gear mechanism, which is using certain double conical rollers to prevent the rotation of the planet gears around their own axes (see Fig. 12). A technical solution of this mechanism is shown in Fig. 13.

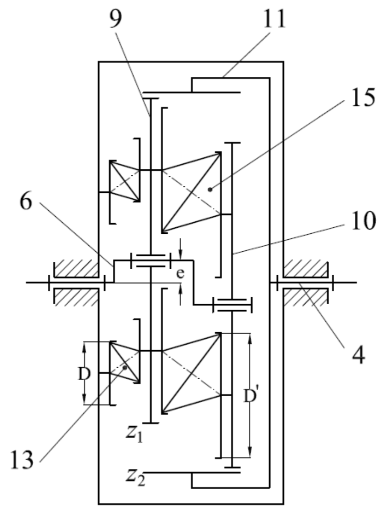

Fig. 12. Kinematics of the second planetary gear mechanism.

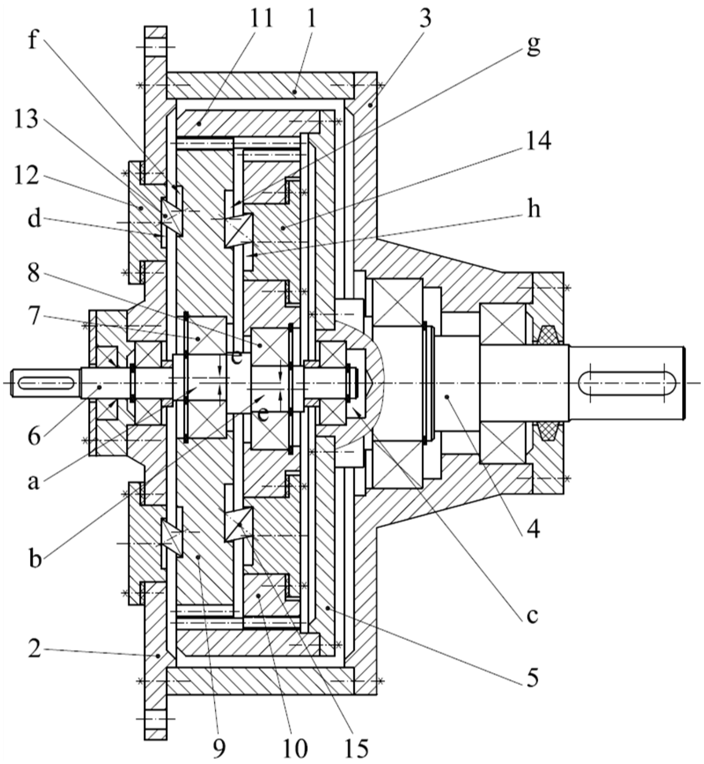

Fig. 13. Technical solution of the second planetary gear mechanism.

The main difference between the first (Fig. 8) and the second (Fig. 13) planetary gear reducers consists in using double conical rollers instead of balls to avoid the rotation of the planet gears around their own axis. For this second solution, between the circular grooves (d) on the cylindrical parts with flanges (12), mounted on the front cover (1), and the circular grooves (f) on the front surface of the planet gear (9), double conical rollers (13) are installed. Also, between the circular grooves (g) on the cylindrical parts with flanges (14), mounted on the planet gear (10), and the circular grooves (h) on the lateral surface of the planet gear (9), rollers with double conicity (15) are installed. The number of rollers (13) and (15) is at least two for each, and the use of two sets of four rollers each is recommended to retain the rotational movement of the planet gears (9) and (10).

By rotating the carrier (6), the planet gear (9) performs a circular translational movement relative to the front cover (1), with a circle radius equal to the eccentricity e, and the rollers (13), performing a nutation movement relative to the central axes of the grooves (d) and (f), eliminate the possibility of rotation of the satellite (9) around its own axis. Simultaneously, the planet gear (10) performs a circular translational movement relative to the planet gear (9), with a circle radius equal to double the eccentricity e, and the rollers (15), performing a nutation movement relative to the central axes of the grooves (g) and (h), eliminate the possibility of rotation of the planet gear (10) around its own axis.

In Figs. 14 to 16, details regarding the technical solutions for restraining the planet gears (9) and (10) from rotating around their own axes are presented.

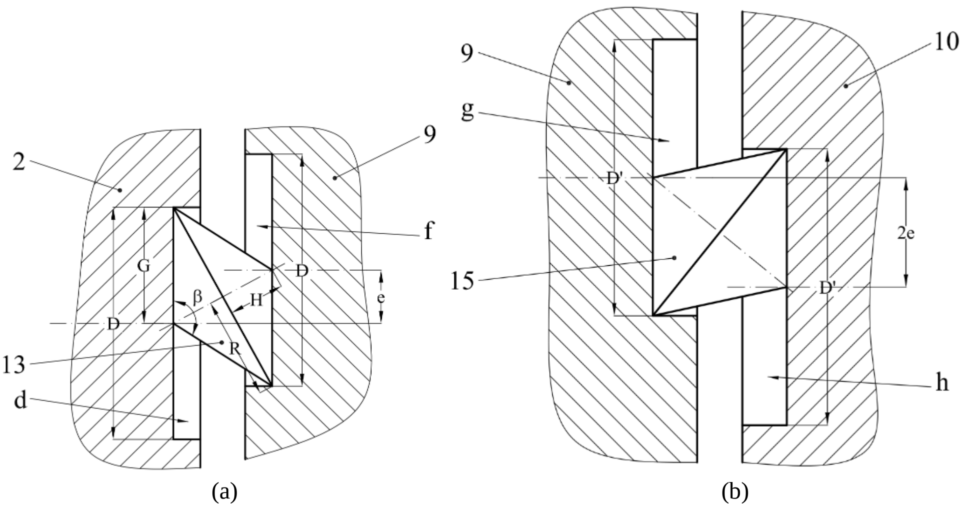

Fig. 14. Details, in axial section, regarding the technical solutions for restraining the planet gears from rotating around their own axes: a) for planet gear (9); b) for planet gear (10).

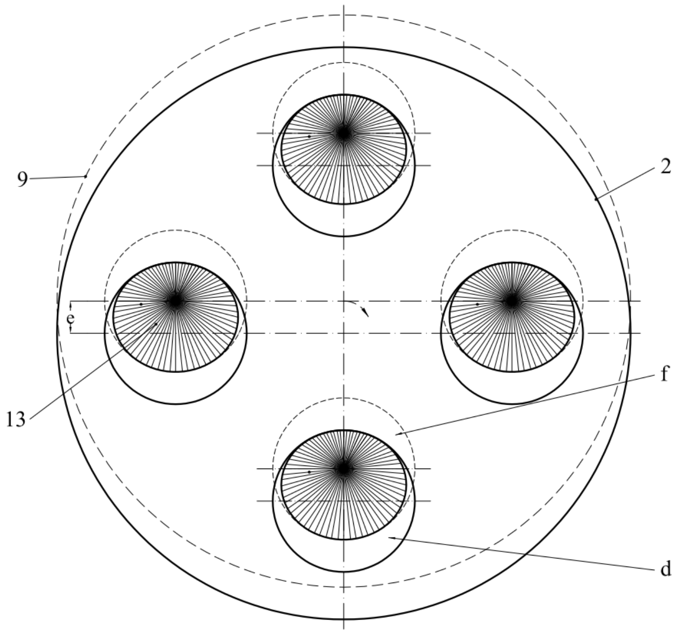

Fig. 15. Details, in frontal view, regarding the technical solution for restraining the planet gear (9) from rotating around its own axis.

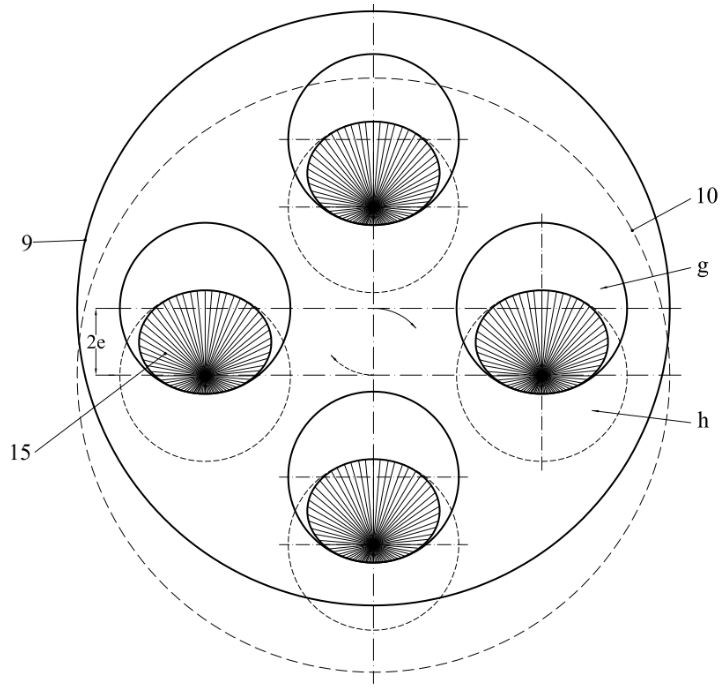

Fig. 16. Details, in frontal view, regarding the technical solution for restraining the planet gear (10) from rotating around its own axis.

The transmission ratio of the second planetary gear reducer, when carrier (6) acts as the driving link, is given by the expression:

$$i_{6-4}^{9} = \frac{z_2}{z_2 - z_1}, \tag{8}$$

reaching its maximum value when $z_2 - z_1 = 1$, respectively,

$$i_{6-4}^{9} = z_2. \tag{9}$$

When the driving link is the shaft (4), the transmission operates as an amplifier, and the transmission ratio is:

$$i_{4-6}^{9} = \frac{z_2 - z_1}{z_2}, \tag{10}$$

reaching its minimum value when $z_2 - z_1 = 1$, respectively,

$$i_{4-6}^{9} = \frac{1}{z_2}. \tag{11}$$

If is considered as the distance between the axes of the circular grooves (d) and (f), a distance equal to the eccentricity e of the carrier (6), the following interdependence relationships between this distance and the dimensions of the roller (13) can be determined:

$$e = 2H\cos\frac{\beta}{2} = 2R\frac{\cos\frac{\beta}{2}}{\mathrm{tg}\frac{\beta}{2}}, \tag{12}$$

or

$$H = \frac{e}{2\cos\frac{\beta}{2}}, \tag{13}$$

or

$$R = e\frac{\mathrm{tg}\frac{\beta}{2}}{2\cos\frac{\beta}{2}}, \tag{14}$$

and

$$\beta = 2\arccos\left(\frac{e}{2H}\right), \tag{15}$$

respectively.

The angle of the roller's conicity must be at least 100° so that they can perform the necessary nutation movement for the proper operation of the reducer.

The diameter D of the circular grooves (d) and (f) can be calculated using the relation:

$$D = 2G = \frac{2H}{\cos\frac{\beta}{2}} = \frac{e}{\cos^2\frac{\beta}{2}}. \tag{16}$$

For the rollers (15), the previous relationships need to be rewritten, considering that the distance between the axes of the circular grooves (g) and (h) is equal to $2e$.

5. Conclusions

By carefully choosing the distance between the axes of the two gears of an internal-external gear pair, along with adjusting the addendum modification coefficients, it becomes feasible to minimize the difference in the numbers of teeth to one or even zero. These geared mechanisms are frequently encountered in the construction of planetary or differential reducers with high transmission ratios, with compact construction and reduced overall dimensions. In this paper, some theoretical aspects of internal-external gear pairs with small difference in numbers of teeth and some results on their numerical simulation have been presented. These results could be helpful for those who want to design cycloidal gear mechanisms using such kind of gears, avoiding the occurrence of interference phenomena and a reduced contact ratio. Also, two examples of cycloidal gear mechanisms with internal-external gear pairs having a small difference in numbers of teeth have been discussed.

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