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Functional principles of the envelope characterized by the variable thermal resistance of the wall as a result of parietodynamic exhaust effect (PDE)

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Abstract. The paper presents the opaque envelope system in the endowment of inhabited buildings characterized by the use of the heat of the air discharged from the inhabited spaces, circulated through the space adjacent to the living environment / outdoor environment in order to reduce the need for heat / sensitive cold of the inhabited spaces (PDE). It is presented mathematical modeling of the processes of heat transfer and flow of indoor / outdoor air circulated through parallel vertical channels. The dimensions of vertical channel opening and air flows are based on the solutions of the property transfer equations associated with Model 1 (forced laminar flow) and Model 2 (globally representative model of both laminar flow and turbulent flow). Model 2 includes both laminar flow (space heating in cold season and cooling spaces in summer season hours with sun) and turbulent flow (cooling living spaces in summer season, hours of night – free cooling). A case study and the interpretation of the results obtained regarding the energy performance of the living space in relation to similar results associated with the conventional envelope is presented.

Key words: laminar air flow regime, undeveloped / developed laminar flow, forced laminar and turbulence flow, energy efficiency heating / cooling spaces.

1. Principles of the PDE system operation

Usually, the polluted air in the living space is discharged from the occupied spaces either through the movable locking elements (doors, windows), through the movable joints or by opening them, or through the mechanical ventilation/air conditioning installations of the living spaces. The exhaust air is characterized by a significant thermodynamic potential in both cold and hot seasons. This potential becomes representative of modern buildings characterized by high energy

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performance, respectively low heat / cold consumption to achieve thermal and physiological comfort in occupied spaces. The other component that participates in the energy performance of a building is the heat flow dissipated outward or penetrated through opaque and transparent construction elements with envelope function. Practically the conventional technical solution used is thermal insulation of opaque elements and replacing, in the case of existing buildings, the “classic” thermal transparent elements with thermal insulation elements, respectively, to furnish new buildings with such closure elements. Compared to the above, the solution for making opaque exterior walls with a parietodynamic effect when discharging the polluted air [1] is meant to ensure, in the cold season, a significant heat recovery on the route of the evacuated air to the outside. Through the intelligent use of the envelope characterized by the usual conventional thermal resistance, the energy effect leads to the generation of a superior thermal resistance of the opaque locking element.

Symbols

ach – air changes per hour [1/h]

α_{cv} – convection heat transfer coefficient [W/m^{2}K]

k – thermal conductivity of air [W/mK]

ν – kinematic air viscosity [m^2/s]

a – thermal diffusivity of air [m^2/s]

R_i – thermal resistance of the boundary element adjacent to the living environment [m^2 K/W]

R_e – the thermal resistance of the border element Adjacent to the outside environment [m^2 K/W]

θ_{Pi} – the temperature of the inner border of the air channel [°C]

θ_{Pe} – the temperature of the outer boundary of the canal air [°C]

$\overline{\theta}(x)$ – average air temperature on the flow section at the rate x [°C]

$\tilde{x}$ – the axial rate at which the boundaries are adiabatic in relation to the air current [m]

δ_t – thickness of the thermal boundary layer [m]

H – the height of the channel air [m]

l – horizontal air channel length [m]

Δ – opening of the air channel [m]

y – opening air channel share [m]

x – axial air flow rate [m]

D_h – hydraulic diameter of air channel [–]

ε – thermal emissivity of the walls bordering the air channel [–]

V – air volume flow [m^3/s]

$\dot{V}$ – air mass flow [kg/s]

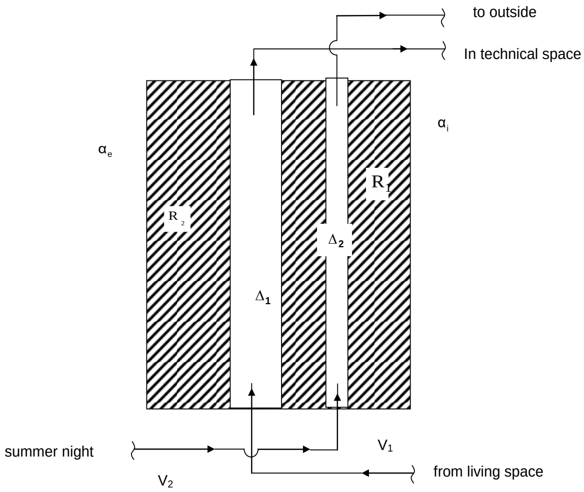
U – average air speed [m/s]
W_c (x) – air velocity at quota x in the non-stabilized rolling flow area [m/s]
λ_n – eigenvalues [–]
Nu – Nusselt dimensionless number [–]
Pr – Prandtl dimensionless number [–]
Re – Reynolds dimensionless number [–]
Pe – Peclet dimensionless number [–]
Gr – Grashof dimensional number [–]
Ra – Rayleigh dimensional number [–]

Basically, a significant reduction in the heat flow dissipated through the transmission is achieved. In addition, in the cold season, the exhaust air still contains enough energy potential to ensure the pre-heating of the fresh air. However, an effect inferior to that of using the heat recovery directly on the circuit of the air discharged from the room (current procedure) is achieved. On the two functions (transmission to the outside and heat recovery from the exhaust air), an energy performance superior to the classical solution (externally insulated wall and heat recovery from the exhaust air as an independent function) is obtained. In the night hours, in the summer season, the only current normal solution for controlling the microclimate is the ventilation of the living spaces. This procedure is partially effective, but the combination with significant cooling of the envelope increases the cooling effect of the building. The system presented offers this possibility by directing the outside air through the channels provided in the envelope, exclusively for the transmission cooling of the building. A system with controlled variable thermal resistance is thus made. The details of the functional principle as well as the mathematical modeling of heat transfer processes along with the sizing of the system are presented below. The functional scheme of the enclosure-wall assembly is shown in Fig. 1.1. The practical solution of the operation of the PDE system involves a structuring of the air routes so that each function is characterized by maximum efficiency, as follows:

– on winter days and nights, the management of thermal flows involves the operation of the air channel route with the Δ_1 opening. According to the scheme in Fig. 1.1, the aim is to minimize the heat flow dissipated through the transmission and preheating the fresh air. Minimization of the heat flow leads to the constructive optimization of the system.

- the optimal values specific to the cold season, referring to the air route taken from the inside and circulated through the channel characterized by the Δ_1 opening, are also maintained in the sunny hours of the warm season in which the space is used;

– on the nights of the summer season, heat recovery involves operation, according to the scheme in Fig. 1.1, by using the air channel with the Δ_2 opening. The aim is to maximize the heat flow dissipated through the transmission through the opaque wall. The air is discharged outside without any further processing.



(Δ_1) air transport channel in winter and summer, sunny hours

(Δ_2) outdoor air transport channel in summer, night hours

Fig. 1.1. The structural and functional scheme of the PDE envelope.

Note: The channels (Δ_1) and (Δ_2) are used complementary (Δ_1 in the heating season and in the summer season when solar gains are significant and Δ_2 in the summer season in the night hours).
The volumic flow rate circulated (evacuated) in the cold season is lower than the flow rate of outdoor air circulated during the summer season in the night hours: $\dot{V}_1 < \dot{V}_2$. The flaps are positioned based on the programmed instructions and their ability to respond to the weather conditions, as determined by the programmed parameters (summer hours night $\theta_{i_0} > \theta_e + 2^o C)$. The structural characteristics ensuring the energy efficiency of the PDE system are as follows:
• on winter days and in summer sunny hours: achieving an equivalent thermal resistance, with a value higher than the conventional thermal resistance resulting from the succession of layers of material that make up the opaque envelope of the PDE system, $R_{ECH} > R_{CONV}$;
• during the night hours of the warm season: achieving a thermal resistance $R_{N.SUM.} < R_{CONV}$, with a value lower than the conventional thermal resistance, which allows cooling of the occupied space.
The functional optimization of the PDE system is intended to determine the appropriate R_i, R_e values of the thermal resistances of the plates adjacent to the air routes. It also aims to determine the thermal characteristics of the air spaces included in the PDE materialized by the convection coefficients α_{CV} and by the thermal emissivities ε_1, ε_2 of the airflow channel borders. The PDE physical model was tested between 2008 – 2010, both by numerical modeling and dynamic simulation and by empirical validation [2].

2. Mathematical modeling of the processes of non-isothermal laminar flow of air through the exhaust channel and heat transfer characteristic of the PDE system (Model 1 - flow of air in laminar regime between vertical plates)

Minimizing the heat transfer coefficients of the mentioned processes leads to an efficient solution for applying the PDE system. In the case of radiation heat transfer between the planes bordering the air flow path characterized by temperatures θ_{P1} (adjacent to the living space) and θ_{P2} (adjacent to the external environment) the intensity is reduced by reducing the average emissivity $\overline{\varepsilon}$ achieved by minimizing the emissivity of the two flat surfaces.

In the case of convection heat transfer from the exhaust air to the surfaces that configure the exhaust air route, geometric solutions (referring to the opening of the air outlet channel) and the choice of the exhaust air flow regime (by forced circulation) are required. Minimization of heat flow is achieved by minimizing the heat transfer coefficient by convection α_{CV} (average value on the height of the flow path). The coefficient minimization is achieved by maintaining the stabilized laminar air flow regime as close as possible to the H height of the flow path. Basically, this condition involves minimizing the path of the flow stabilization and implicitly stabilizing the temperature profile and air velocity on the thickness of the channel designated by the two flat surfaces. The value of the Nu number for the bounded path of infinite vertical plates in the air flow laminar regime is 7.54 [3]. The definition of the Nusselt number shows that the coefficient α_{CV} value varies inversely with the hydraulic diameter D_h of the air flow section. The optimum size of the distance between the planar plates is the maximum allowable distance under the conditions of maintaining the laminar air flow regime. The analysis is based on a phenomenological benchmark represented by the possible physical limitation of the value of the α_{CV} coefficient. In order to establish this limitation, the values of the α_{CV} characteristic coefficient of both the vertical plane plate in a natural convection regime (without spatial limitation in the direction perpendicular to the surface of the plane plate) and the system consisting of two infinite planar plates, parallel with opening at the lower and upper parts. From the point of view of the average speed on the flow section, it is determined by the volume flow of exhaust air V, imposed by the ventilation rate of the living space, and by the air flow section imposed by the Δ distance between the flat surfaces. The flow section is customized by opening the flow path (the distance Δ between the two parallel vertical planar plates) and by the horizontal dimension of the flow path, l. The surface of the flow section $\Delta \cdot l$ and the average air speed $U = \frac{V}{\Delta \cdot l}$ are defined. The thickness of boundary thermal layer for natural convection has a higher value than the thickness of the boundary thermal layer characteristic of forced convection $\delta_{t,0} > \delta_t$ in a stabilized flow laminar regime. From the above it follows that the distance between the two vertical plane surfaces, Δ must be determined by reference to the thickness $\delta_{t,0}$ of the thermal boundary layer proper to the flow of air on the surface of a vertical plane plate that is part of the assembly of two infinite planar plates, correspondence with the outside environment. This solution is achievable by placing the two planes so that $\Delta \geq 2\delta_{t,0}$. The result is an independent flow of air by natural convection to the surface of each vertical plane characterized by its own temperature θ_{P1} and θ_{P2}. The thickness of the thermal boundary layer (natural convection) $\delta_{t,0}$, in the case of air characterized by the number Pr <1 ($Pr \approx 0{,}71$), is around 0,033 m for the temperature range $\theta_{P1} - \overline{\theta}(x) \approx 1°C$ [4]. If the temperature difference is approx. 3 - 5°C, the value is reduced to approx. 0,018 m. The average value of the number Nu in the case of natural convection between two infinite plane plates, located at the distance Δ, shall be determined with the relation [4]:

$$\text{Log (Nu)} = 0{,}0307\,[\text{Log}\,(x)]^3 - 0{,}3129\,[\text{Log}\,(x)]^2 + 1{,}3118\,\text{Log}\,(x) - 1{,}4375 \qquad (2.1)$$

where

Nu–Nusselt number = $\frac{\alpha_{cv} \cdot D_h}{k}$ and x – dimensionless coordinate $\frac{D_h}{H} \cdot Gr_{Dh} \cdot Pr$

Gr_{Dh} – dimensionless number Grasshoff $= 0{,}125 \cdot \frac{g \cdot D_h^3}{\nu^2} \cdot \beta \cdot \left(\theta_P - \overline{\theta}\right)$; $D_h \approx 2\Delta$, H >> Δ

Pr = $\frac{\nu}{a} \approx 0{,}71$;

Depending on the specific curves of natural convection shown in the graphs in Fig. 2.1 (based on the relationship (2.1)) and Fig. 2.2, the α_{CV} coefficient value is capped at 1.70 W/m^{2}K. The same graph shows the specific curve of forced convection that is obtained by using the heat transfer model with laminar air flow. It is found that, over the distance between the plates of 0,055 m, the value of the heat transfer coefficient by convection in the forced convection mode becomes less than 1,70 W/m^{2}K (proper to the natural convection between the planar plates). This result attests to a phenomenological impossibility (the heat transmission coefficient associated with forced convection to be lower than the similar one associated with natural convection), therefore the α_{CV} coefficient is capped at 1.70W/m^2 K. Model 1 analysis targets winter days and sunshine hours in summer. The positioning of the air circulation channel in the outer wall structure generates two zones with different thermal characteristics, depending on the thermal resistances of the opaque elements of the envelope located between the air channel and the environments adjacent to the zones, namely the indoor environment (living space) and the natural outdoor environment (Fig. 1.1).

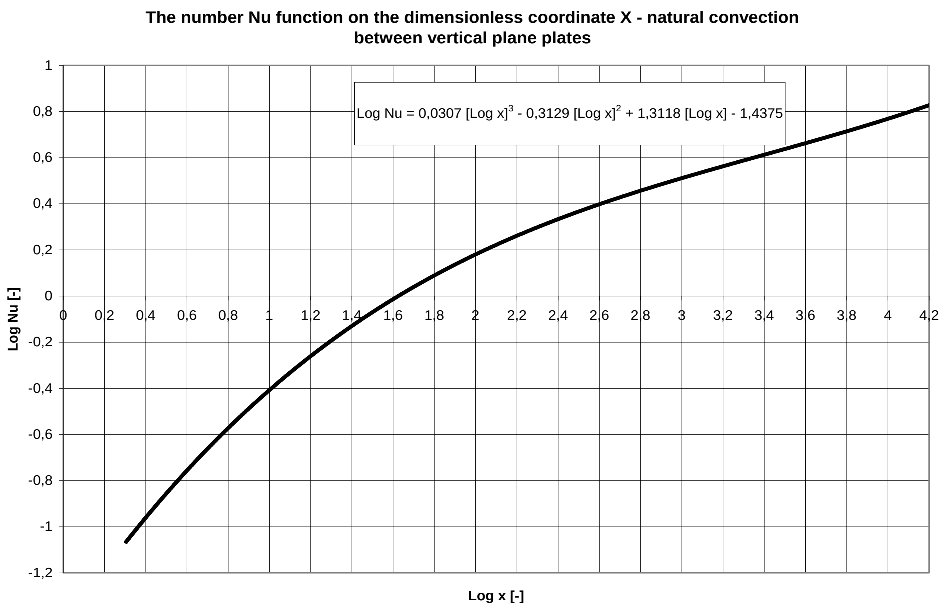


Fig. 2.1. Correlation of Nu values with the values of the dimensionless coordinate x.

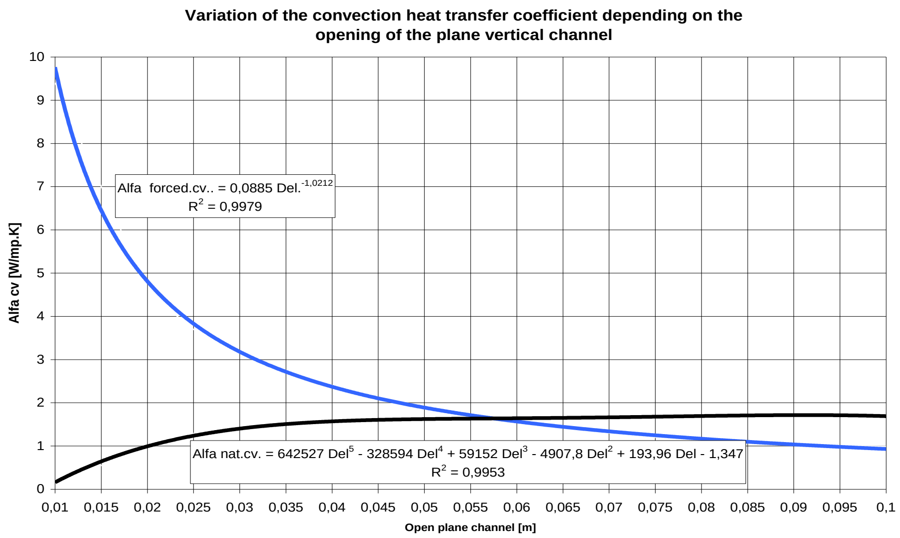


Fig. 2.2. The values of the α_{CV} coefficient specific to natural convection between vertical parallel plates and the similar coefficient specific to forced convection between vertical parallel plates.

Based on all the processes, depending on the increase in the room ventilation rate, there is a reduction in the heat flow related to the transmission to the outside, concerning the similar flow of the conventional building. Also, there is an increase of the heat flow related to the heating of the fresh air compared to the similar values belonging to the conventional building. Therefore, the *recommended beneficiaries of the energy efficiency of the system are residential buildings and similar ones (homes, hotels, etc.).* The conclusion is also maintained for the interval of sunshine hours, during the summer season. The calculation scheme that forms the basis of the mathematical model is presented in Fig. 2.3. The calculation model uses the laminar air flow hypothesis on the channel path adjacent to the living space. Heat transfer is described by the equations specific to the steady state regime. The penetration of air taken from an infinite environment, characterized by temperature θ_{i_0} (occupied space), into the air channel is done with the known mass flow rate, $\dot{V}_m$. In section x = 0, the speed is uniform with the U resulting value:

$$\dot{V}_m = \rho \cdot \Delta \cdot U \cdot l \text{ in which } (\rho = ct.) \tag{2.2}$$

The value $\dot{V}_m$ refers to the dimension l, in the direction perpendicular to the plane of the calculation scheme. Portion (a) (Fig. 2.3) is the region of the unstabilized flow, in relation to the development of the hydrodynamic boundary layer thickness $\delta(x)$ from the value $\delta = 0$ in the input section x = 0, to the value $\delta(x = L) = \frac{\Delta}{2}$ that marks the hydrodynamic stability of the flow. The unstabilized flow is characterized by changing the air velocity profile from the uniform value U in the entry section x = 0 to the parabolic distribution (Hagen – Poisseuile) stabilized in zone (b) (Fig.2.3)

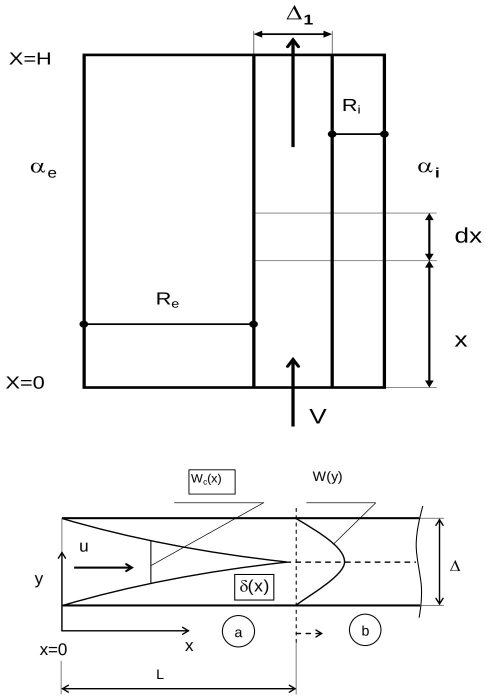


Fig. 2.3. The calculation scheme for model 1.1

The mathematical model, which refers to the laminar flow of air, shall be validated by the value of the dimensionless number Re (Reynolds) by proving that the resulting values belong to the range of the laminar flow:

$$Re_{Dh} \le 2300 \tag{2.3}$$

in which:

$$Re_{D_h} \approx 4 \cdot \frac{U \cdot b}{\nu} \quad b = \frac{\Delta}{2} \tag{2.4}$$

The hydrodynamic regime analysis is performed using the equation of conservation of momentum (amount of motion) (Navier Stokes) and the equation of conservation of air mass flow. Equations are characteristic of both the flow stabilizing area ($x \in [0; L]$) and the stabilized flow area, $x \ge L$.

2.1. Determination of the length in the channel within which the flow is unstabilized (undeveloped) (Model 1.1)

The mathematical model is materialized by the system of the two equations associated with the flow of fluid. The integral equation of the amount of motion has the form:

$$\frac{d}{dx}\int_0^{\delta(x)}[W_c(x) - u(x.y)] \cdot dy + \frac{dW_c(x)}{dx} \cdot \int_0^{\delta(x)} dW_c(x) \cdot dy = \left.\frac{\partial u(x,y)}{\partial y}\right|_{y=0} \tag{2.5}$$

In the area between the two boundary layers the fluid moves forward at a uniform but variable speed in relation to the x value ($W_C(x)$) (Fig.2.3), starting from the U-value characteristic of section x = 0. The equation for conservation of air mass flow has the form:

$$\int_0^{\delta(x)} \rho \cdot u(x,y) \cdot dy + \int_{\delta(x)}^{\frac{\Delta}{2}} \rho \cdot W_c(x) \cdot dy = \rho \cdot U \cdot \frac{\Delta}{2} \tag{2.6}$$

In equation (2.6) the hypothesis $\rho \ne \rho(\theta)$ (fluid is incompressible) was used, a hypothesis justified by the real range of variation in air temperatures. The unknowns of the two integral equations are $\delta(x)$ and $W_C(x)$, for $x \in [0, L]$. Based on the solution [3], [5], [6] of the "spline" form of grade 2, it follows:

$$\frac{u(x,y)}{W_c(x)} = 2 \cdot \frac{y}{\delta(x)} - \left[\frac{y}{\delta(x)}\right]^2 .$$

The integration of the two equations results [3]:

$$\frac{x}{\Delta \cdot Re_\Delta} = \frac{3}{40} \cdot \left[9 \cdot \frac{W_C(x)}{U} - 2 - 7 \cdot \frac{U}{W_C(x)} - 16 \cdot \ln \frac{W_C(x)}{U}\right] \tag{2.7}$$

$$\frac{\delta(x)}{\Delta} = \frac{3}{2} \cdot \left[1 - \frac{U}{W_c(x)}\right] \tag{2.8}$$

The geometrical condition of stabilization of flow is:

$$\delta(x = L) = \frac{\Delta}{2} \tag{2.9}$$

from which it follows:

$$L = 0{,}026 \cdot \Delta \cdot Re_{\Delta} \tag{2.10}$$

length at which the laminar flow stabilizes. The variation L by Δ and V [m^3 /h] is shown in the graph in Fig. 2.4. The approximate function of correlating the kinematic viscosity with the air temperature shall be used $\nu \approx 13{,}374 \cdot 10^{-6} \cdot \exp(0{,}0059 \cdot \theta)$. It is found that for $\Delta < 0{,}05$ m, the undeveloped flow portion is very small compared to the channel height, H = 3 m. The graph in Fig. 2.5 shows the number Re_{Dh} variation according to V [m^3/h] and air temperature. The graph in Fig. 2.5 attests to the placement of the flow in the laminar field.

2.2. Analysis of forced convection heat transfer in flat channels, in the area of the developed stabilized laminar flow (Graetz-Nusselt problem, Model 1.2).

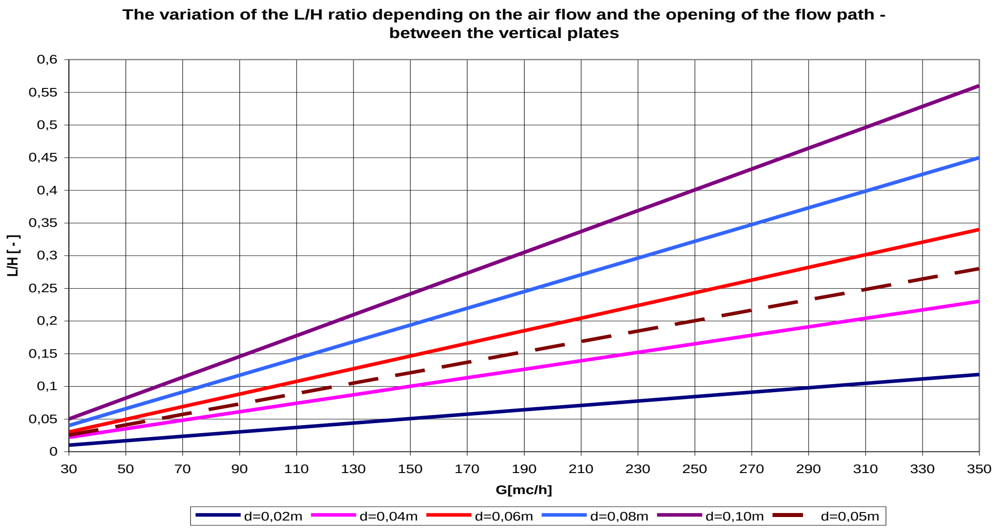


Fig. 2.4. Variation of the air flow stabilization length depending on air volume flow and opening of plane channel (Model 1).

In the stabilized flow area the speed profile does not vary along the flow, so that at any x > L elevation:

$$u(y) = \frac{3}{2} \cdot U \cdot \left[1 - \left(2 \cdot \frac{y}{\Delta}\right)^2\right] \tag{2.11}$$

It is noted that for $y = 0$ (channel shaft):

$$u(y = 0) = \frac{3}{2} \cdot U \tag{2.12}$$

The relationship (2.11) represents the profile of Hagen – Poiseuille characteristic of the stabilized laminar flow. Being known the variation in speed in the channel section, the analysis equations are that of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.13}$$

and that of heat transfer (Kirchhoff – Fourier: K – F)

$$u\frac{\partial \theta(x,y)}{\partial x}+v\frac{\partial \theta(x,y)}{\partial y}=a\left[\frac{\partial^2 \theta(x,y)}{\partial x^2}+\frac{\partial^2 \theta(x,y)}{\partial y^2}\right] \tag{2.14}$$

By applying the *scale analysis* to the equations (2.13) and (2.14) (the derivative in relation to the x direction, proportional to the H height of the exhaust airflow path, has a value lower than the derivative in relation to component y, proportional to the thickness of the net thermal limit layer δ_t lower than the H dimension), and taking into account that the component v of the speed is zero, the equation (K-F) of the boundary layer to which the boundary conditions are associated is obtained.

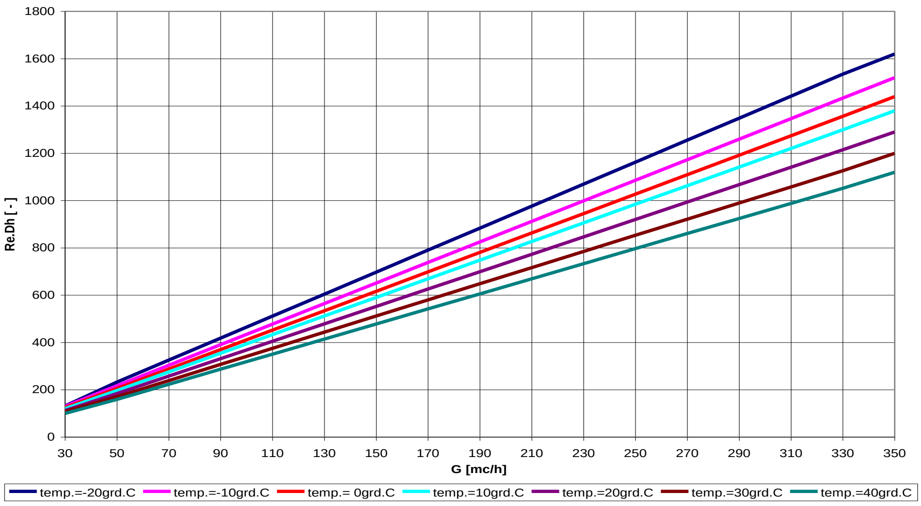


Fig. 2.5. The number Re variation depending on the volume flow rate of air and of the air temperature – $\Delta = 0{,}05$ m

$$0{,}375\cdot\left(1-y^2\right)\cdot Pe_\Delta\cdot\left(\frac{\Delta}{H}\right)\cdot\frac{\partial\Theta(\dot{x},\dot{y})}{\partial\dot{x}}=\frac{\partial\Theta(\dot{x},\dot{y})}{\partial\dot{y}} \tag{2.15}$$

$$\Theta(\dot{x}=0,y)=1 \tag{2.16}$$

$$\Theta(\dot{x},\dot{y}=1)=\Theta(\dot{x},\dot{y}=-1)=0 \tag{2.17}$$

$$\left.\frac{\partial\Theta(\dot{x},\dot{y})}{\partial\dot{y}}\right|_{\dot{y}=0}=0 \tag{2.18}$$

where: $\dot{x}=\dfrac{x}{\left(\dfrac{\Delta}{2}\right)\cdot Pe_{D_h}}$; $\dot{y}=\dfrac{y}{\left(\dfrac{\Delta}{2}\right)}$; $Pe_{\Delta}=Re_{\Delta}\cdot Pr$ și $Pe_{D_h}=Re_{D_h}\cdot Pr$; $D_h \approx 2\Delta$

Equation (2.15) admits solution with variables separated by the form:

$$\Theta(\dot{x},\dot{y})=X(\dot{x})\cdot Y(\dot{y})$$

The synthesis paper [7] presents solutions for a wide variety of fluid types and boundary conditions (combined). The solution, in the general form of equation (2.15), has the form:

$$\Theta(\dot{x},\dot{y})=\sum_{n-0}^{\infty}K_n\cdot Y_n(\dot{y})\cdot\exp\left(-\dot{\beta}\cdot\lambda_n^2\cdot x^2\right) \tag{2.19}$$

in which $Y_n(\dot{y})$, the solutions of the second order differential equation, are:

$$Y''(\dot{y})+\dot{\beta}\cdot\lambda_n^2\cdot\left(1-\dot{y}^2\right)\cdot Y(\dot{y})=0 \tag{2.20}$$

and in case of parallel plates, $\dot{\beta}=\dfrac{8}{3}\approx 2,67.$ Eigenvalues (resulting from the application of boundary conditions) can be determined with the semi-empirical relationship

$$\lambda_n\approx 4\cdot n+\frac{5}{3}\qquad n\in[0,1,2......\infty)$$

The solution specific to the flow through a flat channel, to which the boundary conditions of case I (Dirichlet) are associated, at the level of the surfaces that delimit the flow channel, has the form:

$$\Theta(\dot{x},\dot{y})=\frac{\theta(\dot{x},\dot{y})-\theta_P}{\theta_i-\theta_P}=\sum_{n=0}^{\infty}K_n\cdot Y_n(\dot{y})\cdot\exp(-2,67\cdot\lambda_n^2\cdot\dot{x})\,. \tag{2.21}$$

The analytical solution (2.21) has been used as a reference solution in determining the average air temperature values as a function of the x (along the air flow path) on the flow section of the exhaust air through the PDE system. From the expression (2.21) results the expression of the average air temperature as a variable function on the exhaust route:

$$\overline{\theta}(\dot{x})=\theta_P+2\cdot Nu_{\Delta}^{-1}\cdot\sum_{n=0}^{\infty}[-K_n\cdot Y_n'(\dot{y}=1)]\cdot\exp\left(-2,67\cdot\lambda_n^2\cdot\dot{x}\right)\cdot\left(\theta_i-\theta_P\right) \tag{2.22}$$

Note:

$$G_n=-K_n\cdot Y_n'(\dot{y}=1) \tag{2.23}$$

Table 2.1 shows the eigenvalues λ_n and the coefficients G_n (eigenfunctions) [8]. The two useful expressions of air temperature result:

$$\overline{\theta}(\dot{x}_H)=\theta_P+0,26911\cdot\sum_{n=0}^{\infty}G_n\cdot\exp\left(-2,67\cdot\lambda_n^2\cdot\dot{x}_H\right)\cdot\left(\theta_i-\theta_P\right) \tag{2.24}$$

Where

$$\dot{x}_H = \frac{H}{\left(\frac{\Delta}{2}\right)\cdot Pe_{D_h}} \tag{2.25}$$

and:

$$\overline{\theta} = \theta_P + 0{,}0504\cdot Pe_{D_H}\,\frac{\Delta}{H}\cdot\sum_{n=0}^{\infty}\frac{G_n}{\lambda_n^2}\cdot\left[1-\exp\left(-\frac{5{,}34}{Pe_{D_h}}\cdot\lambda_n^2\cdot\frac{H}{\Delta}\right)\right]\cdot\left(\theta_i-\theta_P\right) \tag{2.26}$$

Table 2.1

Eigenvalues order number	**Paralel plan tube**	
n	λ_n	G_n
0	1,67	0.683
1	5,67	0.454
2	9,67	0.38
3	13.67	0.338
4	17.67	0.311
5	21.67	0.291
6	25.669	0.274
7	29.668	0.262
8	33.668	0.251
9	37.668	0.242
10	41.668	0.23

The air current temperature values useful for analyzing the energy performance of the PDE system are the temperature value at the rate x = H (the rate at which the air leaves the exhaust route to the outside), $\overline{\theta}\left(\dot{x}_H\right)$, and the mean air temperature value over the length of the evacuated air outlet route, $\overline{\theta}$. In both cases the mean symbol refers to the average temperature on the air flow section.

3. Global thermal balance specific to the air circulation route (Model 2)

The overall thermal balance of the air circulated through the channel with flat surfaces is described by the elementary equation:

$$\frac{\mathrm{d}\,\overline{\theta}_G(x)}{\mathrm{d}\,x} + B\cdot\overline{\theta}_G(x) = B\cdot\overline{\theta}_P \tag{2.27}$$

where:

$$B = 2\cdot\frac{\alpha_{CV}\cdot\sum_m L_m}{V\cdot\rho_a\cdot c_{pa}} \tag{2.28}$$

$$\theta_P = 0{,}50\cdot\left(\theta_{P.1}+\theta_{P.2}\right) \tag{2.29}$$

with the specification that:

$$\theta_{P.1}\neq\theta_{P.1}(x);\theta_{P.2}\neq\theta_{P.2}(x)\ \text{şi}\ \theta_{P.1}\neq\theta_{P.2} \tag{2.30}$$

Equation (2.27) is assigned the boundary condition:

$$\theta_G(x=0)=\theta_i \tag{2.31.1}$$

for winter and summer sunny hours

$$\theta_G(x=0)=\theta_e \tag{2.31.2}$$

for summer night.

Integration results in the solution (based on the boundary condition (2.31.1)):

$$\theta_G(x)=\left(\theta_i-\overline{\theta}_P\right)\cdot\exp(-B\cdot x)+\theta_P \tag{2.32}$$

(in case of operation in summer season, night hours, the boundary condition is adopted (2.31.2)), leading to the expression of the average air temperature on the height of the exhaust path, H:

$$\overline{\theta}_G=\theta_P+\left(\theta_{i,e}-\theta_P\right)\cdot\frac{1-\exp(-B\cdot H)}{B\cdot H} \tag{2.33}$$

Coefficient B is written in condensed form:

$$B=\Delta^{-1}\cdot\frac{Nu_\Delta}{Pe_\Delta}=\Delta^{-1}\cdot\frac{Nu_{D_h}}{Pe_{D_h}}$$

(2.34)

and the x dimension in the form:

$$x=0{,}50\cdot\Delta\cdot Pe_{D_h}\cdot\dot{x}$$

from which the dimensionless coordinate is determined with the relation:

$$\dot{x}=\frac{x}{\left(\frac{\Delta}{2}\right)\cdot Pe_{D_h}} \tag{2.35}$$

The exponent $B\cdot x=0{,}50\cdot Nu_{Dh}\cdot\dot{x}$.

Taking into account the above, the new expressions of useful temperatures result:

$$\theta_G(\dot{x})=\left(\theta_{i,e}-\theta_P\right)\cdot\exp\left(-0{,}50\cdot Nu_{Dh}\cdot\dot{x}\right)+\theta_P \tag{2.36}$$

respectively:

$$\overline{\theta}_G=\left(\theta_{i,e}-\theta_P\right)\cdot\frac{1-\exp\left(-\frac{Nu_{Dh}}{Pe_{Dh}}\cdot\frac{H}{\Delta}\right)}{\frac{Nu_{Dh}}{Pe_{Dh}}\cdot\frac{H}{\Delta}}+\theta_P \tag{2.37}$$

Both the thermal balance equation (2.27) and the solution (2.37) are representative of the *transient and turbulent flow and heat transfer regimes* under Dirichlet boundary conditions, characteristic of the PDE *system during the summer season, night hours*. The flow laminar thermal balance model based on the Kirchhoff-Fourier equation (K-F) and the Graetz problem solution is the appropriate model for the analysis of the heat transfer characteristic of the PDE system in the cold season and in the summer season in sunny hours. The dimensionless solutions derived from the global calculation model with the relationships shall be used:

$$\theta_G(\dot{x}) - \theta_P = (\theta_{i,e} - \theta_P)\cdot \exp(-0{,}50\cdot Nu_{Dh}\cdot \dot{x}) \tag{2.38}$$

where:

$$\dot{x}_H = \frac{H}{\left(\frac{\Delta}{2}\right)\cdot Pe_{D_h}}$$

$$\frac{\overline{\theta}_G - \theta_P}{\theta_{i,e} - \theta_P} = \frac{1-\exp\left(-\frac{Nu_{Dh}}{Pe_{Dh}}\cdot\frac{H}{\Delta}\right)}{\frac{Nu_{Dh}}{Pe_{Dh}}\cdot\frac{H}{\Delta}} \tag{2.39}$$

Similar solutions specific to the laminar regime (cold season and summer regime, sunshine hours) are [9]:

$$\overline{\Theta}(\dot{x}_H) = \frac{\overline{\theta}(\dot{x}_H) - \theta_P}{\theta_i - \theta_P} = 0{,}26911\cdot\sum_{n=0}^{\infty} G_n\cdot\exp\left(-2{,}67\cdot\lambda_n^2\cdot\dot{x}_H\right) \tag{2.40}$$

$$\overline{\Theta} = \frac{\overline{\theta} - \theta_P}{\theta_i - \theta_P} = 0{,}0504\cdot Pe_{D_H}\cdot\frac{\Delta}{H}\cdot\sum_{n=0}^{\infty}\frac{G_n}{\lambda_n^2}\cdot\left[1-\exp\left(-\frac{5{,}34}{Pe_{D_h}}\cdot\lambda_n^2\cdot\frac{H}{\Delta}\right)\right] \tag{2.41}$$

Two general calculation relationships may be used as follows:

$$\overline{\theta}(\dot{x}_H) = (\theta_{i,e} - \theta_P)\cdot C(\dot{x}_H) + \theta_P \tag{2.42}$$

and

$$\overline{\theta} = (\theta_{i,e} - \theta_P)\cdot\overline{C} + \theta_P \tag{2.43}$$

where:

$$C(\dot{x}_H) = \begin{cases} 0{,}26011\cdot\sum_{n=0}^{\infty} G_n\cdot\exp\left(-2{,}67\cdot\lambda_n^2\cdot\dot{x}_H\right) & \text{if } Re_{D_h} \le 2300 \\ \exp\left(-0{,}50\cdot Nu_{D_h}\cdot\dot{x}_H\right) & \text{if } Re_{D_h} > 2300 \end{cases} \tag{2.44}$$

$$\overline{C} = \begin{cases} 0{,}0504\cdot Pe_{D_h}\cdot\frac{\Delta}{H}\cdot\sum_{n=0}^{\infty}\frac{G_n}{\lambda_n^2}\cdot\left[1-\exp\left(-\frac{5{,}34}{Pe_{D_h}}\cdot\lambda_n^{2\cdot}\cdot\frac{H}{\Delta}\right)\right] & \text{if } Re_{D_h} \le 2300 \\ \dfrac{1-\exp\left(-\frac{Nu_{D_h}}{Pe_{D_h}}\cdot\frac{H}{\Delta}\right)}{\frac{Nu_{D_h}}{Pe_{D_h}}\cdot\frac{H}{\Delta}} & \text{if } Re_{D_h} > 2300 \end{cases} \tag{2.45}$$

4. Energy performance of the PDE type envelope – case study

The calculation model is written for steady-state heat flow. In relation to this specification, the climatic parameters included in the calculation relationships are average values characteristic of representative time intervals (minimum 12 hours).

The heat flow discharged from the living room to the space through which the polluted air is circulated is the result of the following processes (Fig. 2.3):

1. Transmission through the opaque envelope element from the inner medium to the surface of the flat wall adjacent to the polluted air outlet:

$$Q_1 = \frac{A_{PE}}{R_1 + \alpha_i^{-1}} \cdot (\theta_i - \theta_{P1}) \qquad (3.1)$$

2. Convection heat transfer to the surface of the border between the air channel and the living space and to the surface of the border between the air channel and the external environment:

$$Q_{cv.1} = A_{PE} \cdot \alpha_{cv} \cdot (\theta_{P1} - \bar{\theta}); \qquad Q_{cv.2} = A_{PE} \cdot \alpha_{cv} \cdot (\bar{\theta} - \theta_{P2}) \; ; \qquad (3.2)$$

in which:

$$\bar{\theta} = (\theta_i - \theta_P) \cdot \bar{C} + \theta_P \text{ si } \theta_P = 0{,}50 \cdot (\theta_{P1} + \theta_{P2}) \qquad (3.3)$$

The convection heat transfer coefficient between the air that is circulated in the opaque envelope space and the side walls of the air flow path shall be determined by using one of the relations [3] and [10]:

$\alpha_{CV} = \frac{\lambda}{D_h} \mathrm{Nu}_{Dh}$; $\mathrm{Re}_{Dh} \leq 2100$, Nu = 7,54 [3]

$\alpha_{CV} = 0{,}116 \cdot \frac{\lambda}{D_h} \cdot \left(\mathrm{Re}_{D_h}^{\frac{2}{3}} - 125 \right) \cdot \mathrm{Pr}^{\frac{1}{3}} \cdot \left(\frac{\eta_f}{\eta_P} \right)^{0,14} \cdot \left[1 + \left(\frac{D_h}{H} \right)^{\frac{2}{3}} \right]$; $2100 < \mathrm{Re}_{Dh} < 10^4$ [10]

f – calculated at the average fluid temperature;
P – calculated at wall temperature.
The value shall be determined with the relation (3.3) to one of the expressions of the coefficient $\bar{C}$, according to the value of Re_{Dh} (2.45).

3. Radiation heat transfer between flat surfaces bordering the ducting of the polluted indoor air:

$$Q_{R.1} = A_{PE} \cdot \alpha_R \cdot (\theta_{P.1} - \theta_{P.2}) \qquad (3.4)$$

where:

$$\alpha_R = C_0 \cdot \bar{\varepsilon} \cdot (T_{P.1} + T_{P.2}) \cdot (T_{P.1}^2 + T_{P.2}^2) \qquad (3.5)$$

$$\bar{\varepsilon}^{-1} = \frac{1}{\varepsilon_{P1}} + \frac{1}{\varepsilon_{P2}} - 1$$

$C_0 = 5{,}67 \cdot 10^{-8}$ Stefan Boltzmann constant [W/m^2 K^4]

4. Transmission through the opaque outer envelope element, from the polluted air outlet channel, to the surface of the flat wall adjacent to the outside environment:

$$Q_2 = \frac{A_{PE}}{R_2 + \alpha_e^{-1}} \cdot \left(\theta_{P.2} - \theta_e\right) \tag{3.6}$$

The system of algebraic equations is generated (3.7), (3.8). Solving the thermal balance equations (3.7), (3.8) involves the adoption of an iterative method referring to the surface temperature values that bound the width of the air circulation channel.

$$\theta_{P1} = \frac{b_3 c_2 + b_2 c_3}{Num_\theta} \tag{3.7}$$

$$\theta_{P2} = \frac{b_3 c_1 + b_1 c_3}{Num_\theta} \tag{3.8}$$

$$Num_\theta = c_2 b_1 - c_1 b_2;$$

where:

$$b_1 = 0{,}50 \cdot \alpha_{CV} \cdot \left(1 + \bar{C}\right) + \alpha_R + \frac{1}{R_1 + \alpha_i^{-1}}; \; b_2 = 0{,}50 \cdot \alpha_{CV} \; (1 - \bar{C}) + \alpha_R;$$

$$b_3 = \alpha_{cv} \bar{C} \tilde{\theta} + \frac{1}{R_1 + \frac{1}{\alpha_i}} \cdot \theta_i; \; c_1 = 0{,}50 \cdot \alpha_{CV} \; (1 - \bar{C}) + \alpha_R;$$

$$c_2 = 0{,}50 \cdot \alpha_{cv} \left(1 + \bar{C}\right) + \alpha_R + \frac{1}{R_2 + \frac{1}{\alpha_{ei}}}; \; c_3 = \alpha_{cv} \bar{C} \cdot \tilde{\theta} + \frac{1}{R_2 + \frac{1}{\alpha_e}} \cdot \theta_e$$

The temperature $\tilde{\theta}$ has value θ_i in the cold season and in the sunny hours of the summer season, and value θ_e in the nights of the summer season.

Numerical example:

It is considered a room equipped with an opaque envelope integrating the PDE system. Opening of the channel for night ventilation in the warm season (free cooling), Δ_2 = 0.03 m (Fig.1.1), opening of the air outlet from the living space Δ_1 = 0.05 m (Fig.1.1). The height of the air circulation route is H = 3 m, and the horizontal dimension of the envelope is l = 10 m. The volume of the room is 150 m^3. The ventilation rate has the following values: 0.50 ACH for the existing building and 0.30 ACH for the new building. Climatic parameters used in numerical examples refer to values of outdoor temperatures specific to the cold season (2^0C), daylight hours on sunny days of July (38^0C), night hours of the warm season (19^0C). Indoor temperatures are (22^0C) in the cold season and (27^0C) in the summer season. The envelope is characterized by the following thermal resistance values [m^2K/W] in relation to the functions related to the processes of maintaining the comfort microclimate in the inhabited areas (Table 3.1)

Table 3.1

Function	Existing Building			New Building		
System	PDE system		Classic	PDE system		Classic
Envelope position	Inside	Outside	Outside	Inside	Outside	Outside
Space heating	0,646	0,625	0,625	0,646	2,5	3,146
Summer sunny	0,646	0,625	0,625	0,646	2,5	3,146
Summer night	0,238	1,043	0,625	0,238	2,918	3,146

In Tab.3.2 we summarize the performance of the parietodynamic air exhaust system through envelope equipped with PDE systems, in term of the **thermal flow proper to the room envelope** [W]. The physical meaning of the results presented in Tab.3.2

Table 3.2

Function	Existing Building		New Building	
System	PDE system	Classic	PDE system	Classic
Envelope position	Tab.3.1	Outside	Tab.3.1	Outside
Space heating	**500,16**	778,34	220,2	**202,33**
Summer sunny	**-292,54**	-676,82	**-118,08**	-375,03
Summer night	**-611,54**	-394,21	**-603,59**	-272,75

Bold values indicate the *recommended technical solutions*
- *In the case of the* ***existing building****, the values written with bold mean:*
Space heating: Low heat dissipation in the case of PDE compared to the classic equipment (64%)
Summer sunny hours: Low space cooling in the case of PDE compared to the classic equipment (43%)
Summer night hours: Superior exhaust heat flow in the case of PDE compared to the classic equipment (155%)
- *In the case of a* ***new building****, the values written with bold mean:*
Space heating: Low heat dissipation in the classic case compared to PDE equipment (8.8%)
Summer sunny hours: Low space cooling in the case of PDE compared to the classic equipment (31,5%)
Summer night hours: Superior exhaust heat flow in the case of PDE compared to the classic equipment (221,3%)

The main conclusion: Adoption of the PDE solution in *case of renovation of heritage buildings whose architectural aspect is preserved*. Basically, the adoption of PDE envelope leads to superior energy performance of the renovated building in relation to conventional solutions.

5. Conclusions

1. The mathematical model of the processes of air flow through the section of the endowment of the exterior wall system with Parietodynamic effect on the exhaust (PDE) and heat transfer between the interior environment and the external environment can be applied in the study of the energetic configuration of both, an existing building (especially in the case of heritage buildings) as well as a new building (Ch. 4).
2. The properly sized external wall system with Parietodynamic exhaust (PDE) effect (openings and location of air circulation spaces) leads to a sensitive increase in energy performance without the use of thermal insulation materials.
3. The main conclusion that emerges from the numerical analysis, regarding the properties of the PDE system, attests that it can be included in *the integrated design concept* that associates the energy performance with the environmental performance of buildings and the built urban area. The integrated design concept *associates energy performance with the environmental performance* of buildings and urban areas constructed under the synthetic designation of climate sensitive urban design [11], [12].

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