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# Analysis of the carrier cable of gravity transport installations subject to the action of the wind

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**Abstract.** The paper presents the calculation of the carrier cable of the gravity transport installations taking into account not only the gravity loads, but also the loads produced by the action of the wind.
It is considered that the action of the wind acts according to some direction, in any case that it has a lateral component that removes the cable from the equilibrium position located in the vertical plane and places it in a plane inclined to it. Scientific studies do not deal with this case of static calculation, although the normative prescriptions provide for the consideration of wind action. The obtained theoretical results are detailed so that they can be used directly by the design engineer.

**Keywords:** carrier cable, zipline, wind action.

## 1. Introduction and formulation of the problem

Cable gravity transport is a broad class of transport frequently used for recreation in mountain tourist areas and amusement parks. They are attractive to young people, who are looking for extreme thrills. These are offered by the traveling speeds, as well as by the long lengths of the route. Openings that exceed 500 *m* are now no longer a rarity.
In their case, on the one hand, the effect on the cable carrying the weight of the payload, which is limited to masses of 110-150 kg, becomes small in relation to the effect of its own weight, and on the other hand, the effect of the wind, as well as the risk of depositing of hoarfrost in the winter become significant.

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The works related to cable cranes and funiculars, from which we quote [1] and [2], present the exact theory of the chain, as well as the practical engineering theory, slightly approximate.
According to this theory, the curve representing the trajectory of the mobile load is a parabola and that in the presence of a concentrated load the cable takes the form of two arcs of a parabola whose junction is found at the point of application of the concentrated load (Fig.1). The loads are considered exclusively gravitational in the two previous biographical reviews: the weight of the cable (considered uniformly distributed on the rope that joins the ends of the carrier cable) and the weight of the mobile load. The papers [3] and [4] dedicated to the study of the wires, presents the particular case of the influence of the wind acting exclusively transversely, in the chain theory and in the absence of a concentrated force, the aim being the determination of the lateral displacement of the wires under these conditions.
The work [5] focuses on determining the speed of moving the mobile load to gravitational cable transport installations, but the issue of wind action was not resolved satisfactorily.
The present work clarifies the effect of wind action. This, theoretically, can act in any direction and sense, and its effect is manifested both on the carrier cable and on the mobile load. The action of the wind on the carrier cable is considered to be uniformly distributed on the cable chord, as in the case of its own weight. The loads equivalent to the action of the wind: on the cable $\bar{q}_v$ and on the mobile load $\bar{Q}_v$ are combined with the gravitational ones, resulting in loads that are no longer vertical. As a result, the equilibrium shape of the cable is no longer in a vertical plane. Thus, heavy string theory is no longer directly applicable. The object of this paper is to solve this problem.

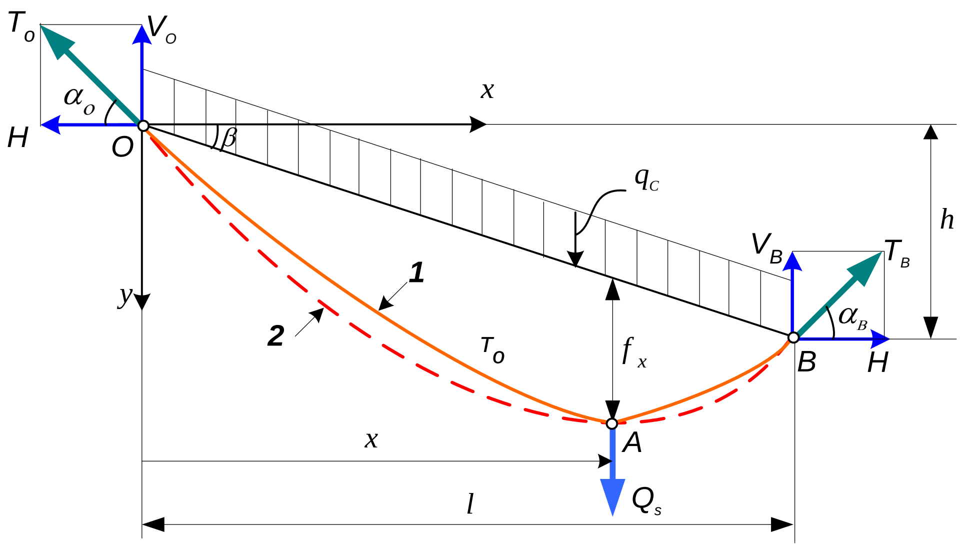


Fig. 1. Scheme for the calculation of the heavy wire loaded with gravity loads:
***1**- the shape of the wire under load; **2** - the path of the mobile load.*

## 2. Main ideea

It starts from the observation that, in the case of the heavy wire acted only by gravitational forces, the problem is solved by considering an *xOy* planar reference system: the *Ox* axis being horizontal and *Oy* vertical, which means that the *Ox* axis is perpendicular to the loads, and *Oy* is parallel to these. This observation provides the key to solving the proposed problem. Indeed, in the case of wind action, its lateral component (perpendicular to the *xOy* plane) makes the equilibrium configuration of the wire go out of the vertical plane.
Assuming that this new equilibrium configuration is planar and that it relates to a system $x'Oy'$ in this plane, the axis $Ox'$ being perpendicular to the direction of the resulting gravitational forces and those due to the wind, and the axis $Oy'$ being parallel to this direction, the problem can be treated in a similar way to the case of the heavy wire loaded only with gravitational forces. Thus, all the results found for the heavy thread can be used if the action of the wind is also considered.
However, it must be stated that the assumption that the equilibrium shape of the cable remains flat even in the case of lateral forces is not exactly rigorous because the resultant of the loads on the mobile load (weight plus the force of the wind) is generally not found in the plane of the resultant forces distributed on the wire (own weight plus wind). Thus, locally, in the area of the mobile load, the wire exits this plane. However, the approximation is not rough! Because the wire is longer, heavier, and stiffer, the approximation is almost accurate (in reality, it is a steel cable with several wires). Furthermore, as will be shown below, the approximation can be minimized. However, for out-of-service installations, since no moving load is present, the assumption is correct, accurate.

## 3. Assumptions

All the assumptions used in the study according to the parabola method of the heavily loaded wire with a concentrated gravitational load are preserved, as well as the related simplifying approximations ([1], [2]). The relations and calculation formulas derived in this way are assumed to be known and will be used when necessary. In addition, the hypothesis formulated in point 2 above is considered.

## 4. Identifying the plane where the carrier cable is located

The planar system of *xOy* axes in Figure 1 will be completed with the *Oz* axis perpendicular to the *xOy* plane and is the axis along which the lateral component of the wind action acts. Thus, the components of the forces equivalent to its action on the cable are denoted by: $q_x^v, q_y^v, q_z^v$, and for the mobile load with: $Q_x^v, Q_y^v, Q_z^v$. Considering the weights: $q_c$ of the cable and $Q_s$ the load, the vectorially resultants are

$$\begin{cases} \bar{q} = q_x^v \cdot \bar{i} + (q_c + q_y^v) \cdot \bar{j} + q_z^v \cdot \bar{k} \\ \bar{Q} = Q_x^v \cdot \bar{i} + (Q_s + Q_y^v) \cdot \bar{j} + Q_z^v \end{cases} \tag{1}$$

Where the unit vectors of the coordinate axes are: $\bar{i}, \bar{j}, \bar{k}$.

As mentioned in paragraph 2, the plane determined by the chord *OB* and the resultant $\bar{Q}$ generally does not coincide with the plane determined by the chord and the resultant $\bar{q}$. However, the degree of approximation of the assumption that it is plane, can be diminished if the resultants (1) will be transformed into a single equivalent distributed resultant, $\bar{q}^{ech}$, and the plane in which the equilibrium form of the wire is considered to be found is the plane defined by the chord and this resultant.

For this it is noted that the equation of the wire heavily loaded with a concentrated charge is, [1],

$$y(x) = x \cdot tg\beta + f(x) = x \cdot tg\beta + \left( \frac{q_c}{2\cos\beta} + \frac{Q}{l} \right) \cdot \frac{x(l-x)}{H}$$

Comparing this equation with that of the heavy wire without concentrated charge

$$y(x) = x \cdot tg\beta + \frac{q_c}{2\cos\beta} \cdot \frac{x(l-x)}{H}$$

the equivalent distributed load can be defined $q^{ech}$, the equivalence condition being the equality of the ordinate *y* which is the same thing as the equality of the arrows *f*. The meanings of the notations result from Figure 1. Equating the two expressions, in the second one replacing $q_c$ by $q^{ech}$ is obtained

$$q^{ech} = q_c + \frac{2Q}{l}\cos\beta = q_c + \frac{2Q}{L} \tag{2}$$

Taking (2) into account, the expressions of the components of the equivalent uniformly distributed load, on the three axes, are obtained:

$$\begin{cases} q_x^{ech} = q_x^v + 2Q_x^v / L \\ q_y^{ech} = (q_c + q_y^v) + 2(Q_s + Q_y^v) / L \\ q_z^{ech} = q_z^v + 2Q_z^v / L \end{cases} \tag{3}$$

Thus the equivalent uniformly distributed force is

$$\begin{cases} \bar{q}^{ech} = q_x^{ech} \cdot \bar{i} + q_y^{ech} \cdot \bar{j} + q_z^{ech} \cdot \bar{k} \\ q^{ech} = \left| \bar{q}^{ech} \right| = \sqrt{\left(q_x^{ech}\right)^2 + \left(q_y^{ech}\right)^2 + \left(q_z^{ech}\right)^2} \end{cases}, \tag{4}$$

and the plane in which the carrier cable can be considered to be located (plane *P*, according with Figure 2) is the plane determined by this vector and a vector collinear with the chord *OB*, whose expression is

$$\begin{cases} \overline{OB} = l \cdot \overline{i} + l \cdot tg\beta \cdot \overline{j} \\ \left|\overline{OB}\right| = \sqrt{l^2 + \left(l \cdot tg\beta\right)^2} = L \end{cases} \tag{5}$$

The plane *P* containing the chord *OB* is specified if the dihedral angle $\rho$ it makes with the vertical plane containing the same chord is known, the plane marked *V* in Figure 2. To establish this angle it is first necessary to establish
the angle $\theta$ that the vector $\overline{q}^{ech}$ makes with the vector $\overline{OB}$. This angle can be established by expressing the scalar product of these vectors in two ways

$$\overline{q}^{ech} \cdot \overline{OB} = q^{ech} \cdot L \cdot \cos\theta$$

By using (4) and (5) it is easily obtained

$$\cos\theta = \frac{q_x^{ech} \cos\beta + q_y^{ech} \sin\beta}{q^{ech}} \tag{6}$$

From Figure 2 it follows that $\beta' = \pi/2 - \theta$, therefore $\sin\beta' = \cos\theta$.
To establish the calculation expression for the dihedral angle between the planes *P* and *V*, denoted $\rho$ (Fig. 4*b*), first determine the *r* direction of the unit verctor normal to the plane *P*, denoted by $\overline{u}$, with the help of the vector product $\overline{q}^{ech} \times \overline{OB}$

$$\overline{u} = \frac{\overline{q}^{ech} \times \overline{OB}}{q^{ech} \cdot L \cdot \sin\theta}$$

and then, with the help of the scalar product of this vector with the unit vector of the normal to plane *V*, which is $\overline{k}$, it results $\rho$.

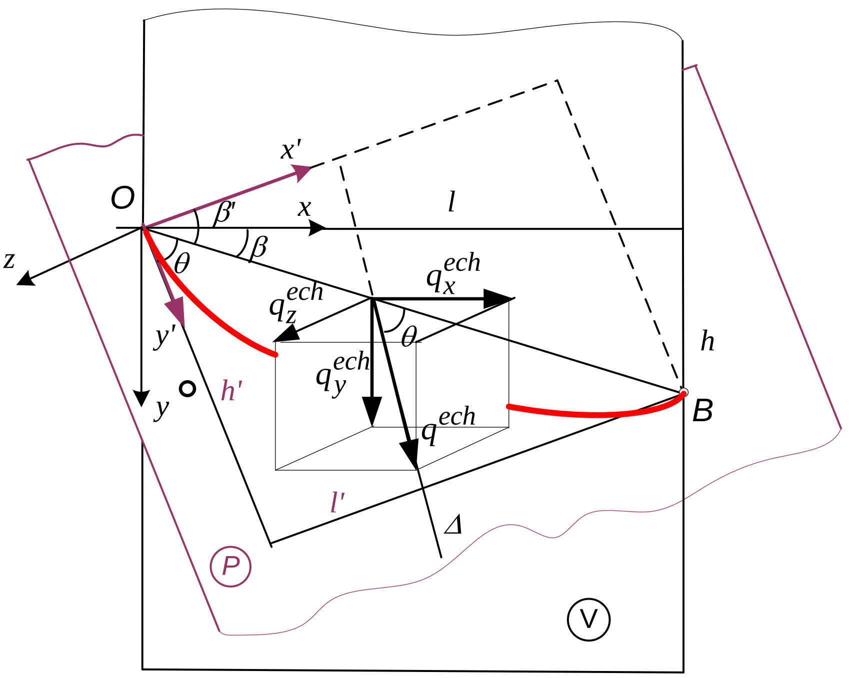


Fig. 2. The plane in which the equilibrium configuration of the wire is established, driven by gravity loads and those determined by the action of the wind.

After performing the products of the denominator and the numerator of the above expression, the result is finally obtained

$$\cos\rho = \frac{\left(q_y^{ech} - mq_x^{ech}\right)}{\sqrt{\left(q_y^{ech} - mq_x^{ech}\right)^2 + \left(1+m^2\right)\cdot\left(q_z^{ech}\right)^2}} \tag{7}$$

where $m = tg\beta$.

There are two particular cases of interest:

- The first is that of the exclusively longitudinal wind action; in this case it will be taken $q_z^{ech} = 0$, and the thread will remain in the vertical plane. This case is studied to determine the speeds of the mobile load: maximum (when the wind acts in the direction of movement) and minimum (when the wind opposes the movement; in this situation in all the relations above $q_x^{ech}$ it must be replaced by -$q_x^{ech}$).
- The second case corresponds to the lateral action of the wind; in this case $q_x^v = q_y^v = 0$. This case is taken into account when checking the resistance of the cable out-of-service, in the event of the hoarfrost being deposited ($q_y^{ech} = q_c + q_{ch}$ and $Q_s = Q_x^v = Q_y^v = 0$).

**5. Carrier cable calculation**

It is considered the carrier cable in the reference system $x'Oy'$ located in the plane $P$ and having the axis $Ox'$ perpendicular to $\overline{q}^{ech}$. When calculating the carrier cable, all the results obtained can be used in the study of the heavy wire can be used, actuated only by gravitational forces, replacing them in formulas with $q^{ech}$ , respectively with $Q^{ech}$, and instead of dimensional elements $l, h, \beta$ using $l', h', \beta'$ ($l' = L \cdot \cos(\beta')$, $h' = L \cdot \sin(\beta')$ ).
By interpreting the results obtained and examining Figure 2 it can be seen that the action of the wind has the following effects:
**a)** The components along the *Ox* axis modify the calculation opening $(l \rightarrow l')$, as well as the slope of the chord *OB* $(\beta \rightarrow \beta')$; if the components $q_x^v$ and $Q_x^v$ act in the positive direction of the *Ox* axis, their effect consists in reducing the calculation opening and increasing the slope, and if it is in the opposite direction, they increase the opening and reduce the slope;

**b)** The components along the *Oz* axis remove the cable from the vertical plane; the angle $\rho$ made by the *P* plane is all the greater as the components $q_z^v$ and $Q_z^v$ are larger;

**c)** The components $q_y^v$ and $Q_y^v$ cumulate with the respective gravitational loads; increase them if they are in the sense of the *Oy* axis and reduce them otherwise.
All these components of wind action, of course, change the tension in the cable, but what is equally important, they are also likely to change the speed with which the mobile load travels along the zipline (cable) route. The direct and major effect on the speed of the mobile load is the component $Q_x^v$.

However, two problems remain to meet the requirements of the zipline design calculation.
-The first, simpler one, is to establish the components of the reactions at the attachment points at the ends of the cable along the axes of the *Oxyz* trihedral, depending on the components of the tension in the cable $H'$ and $V_O'$, respectively, $V_A'$ located in the *P* plane.
-The second question is to determine how cable tensions can be deduced under conditions other than those considered of his choice, given that this choice is made in the most unfavorable conditions, they also involve the action of the wind. For example, the parameters of the cable (tension and arrow) are of interest during installation, when it is assumed that the wind is not present.

## 6. Reactions in the anchor points of the carrier cable ends related to the natural reference system *Oxyz*

The designer of the structures (to which the ends of the carrier cable are anchored) needs the loads transmitted by this cable according to the directions of the natural *Oxyz* reference system. Therefore, it is necessary that the reactions calculated by referring to the reference system *x'Oy'* located in the plane *P*: $H', V'_O, V'_B$ should be transferred to the natural reference system *Oxyz*. For this, the diagrams in Figures 3 and 4 are used and the components of the reactions on the axes of the natural system are noted with: $X_O, Y_O, Z_O$, respectively $X_B, Y_B, Z_B$. These components are considered positive if they are in the opposite direction to the coordinate axes.
With the notations in Figure 3, each of the forces $V'_O$ and $V'_B$, denoted generically by $V'$ $V'$, give in the *Oxyz* system the components:

$$\begin{cases} X^{V'} = V'\sin\varphi_y \cos\varphi_x \\ Y^{V'} = V'\cos\varphi_y \\ Z^{V'} = V'\sin\varphi_y \sin\varphi_x \end{cases} \quad (8)$$

The angles $\varphi_x$ and $\varphi_y$ being known (see Fig.3): $\varphi_x = arctg\left(q_z^{ech} / q_x^{ech}\right)$, $\varphi_y = \arccos\left(q_y^{ech} / q^{ech}\right)$.

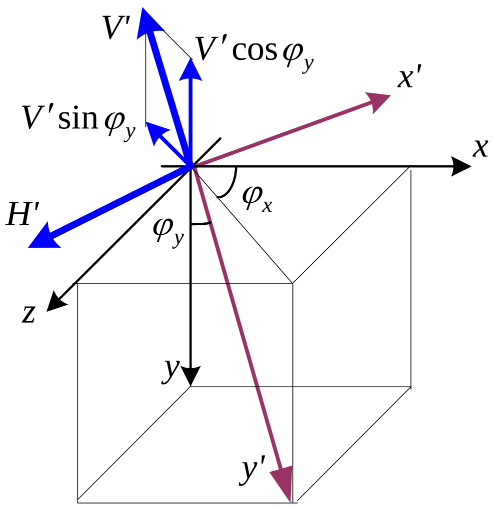


Fig. 3. Decomposition of the reaction $V'$ according to the axes of the Oxyz system.

The reaction $H'$ first decomposes into the components $H'\sin\theta$ located on the chord *OB* and $H'\cos\theta$ located in the plane *P*, perpendicular to the chord, (Fig. 4a).

$$\begin{cases} pr.x = \pm H'\sin\theta\cos\beta \\ pr.y = \pm H'\sin\theta\sin\beta \\ pr.z = 0 \end{cases} \qquad (9a)$$

Using Figure 4b, the component projections $H'\cos\theta$ on the same Oxyz reference system are also easily obtained:

$$\begin{cases} pr.x = \pm H'\cos\theta\cos\rho\sin\beta \\ pr.y = \mp H'\cos\theta\cos\rho\cos\beta \\ pr.z = \mp H'\cos\theta\sin\rho \end{cases} \qquad (9b)$$

In the expressions (9*a*) and (9*b*) the upper sign corresponds to the joint *O*, and the lower one to the joint *B*.
Summarizing the projections on the axes in the expressions (9*a*) and (9*b*) the components of the reactions produced by $H'$:

$$\begin{cases} X^{H'} = \pm H'(\sin\theta\cos\beta + \cos\theta\cos\rho\sin\beta) \\ Y^{H'} = \pm H'(\sin\theta\sin\beta - \cos\theta\cos\rho\cos\beta) \\ Z^{H'} = \mp H'\cos\theta\sin\rho \end{cases} \qquad (9)$$

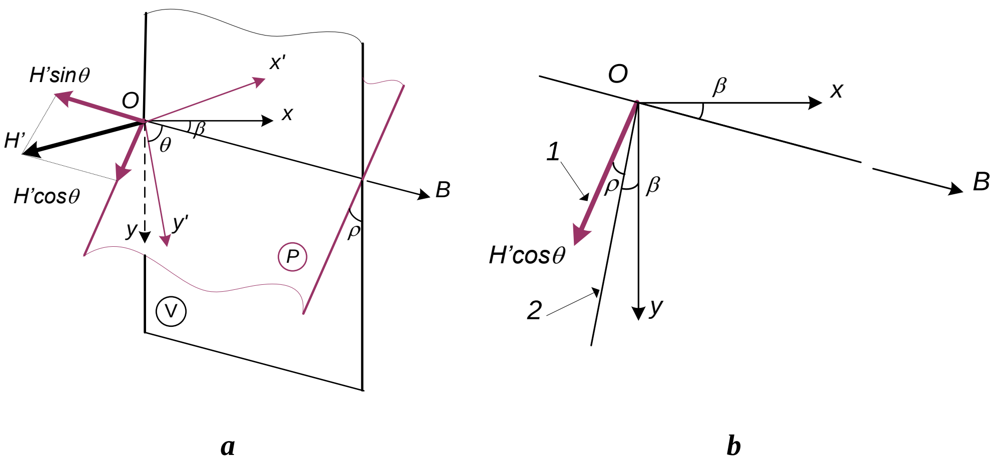


Fig. 4. Decomposition of the reaction $H'$ according to the axes of the *Oxyz* system;
*a) decomposition of the component* $H'\sin\theta$; ***b****) decomposition of the component* $H'\cos\theta$
*b) (**1**- normal to OB in plane P, **2**- normal to OB in plane V).*

Finally, combining (8) with (9), the components along the axes of the *Oxyz* reference system of the total reactions at the anchorage points of the carrier cable are obtained.

## 7. Establishing the horizontal component of the tension in the carrier cable under conditions different from those of the reference situation

Solving the problem of determining the effort in the carrier cable in one of the conditions in which it is assumed to be more stressed and which are, obviously, the conditions when the wind also acts, the problem arises of establishing the tension in the cable and in the conditions in which it is considered that the wind it does not work, for example when mounting. Therefore, assuming the horizontal component of the tension in the cable is known in a situation considered as a reference, in which the equilibrium configuration of the cable is established in a plane inclined to the vertical plane, it becomes necessary to calculate the value of the tension in a situation in which the equilibrium configuration is established in the vertical plane. For this, the same steps are taken as those used to solve the problem when in the two situations the equilibrium configurations are established in the same plane, [1]. In both situations (reference, index *r*) and another one where the parameters are not known (index *x*), the axis of the deformed cable is a flat curve: a parabola - if the moving load is not present, or two arcs of a parabola when it is present (Fig.1). In the following, the second case will be assumed - the general case, the first being deduced by particularization ( $Q_x = 0$ ).

The cumulative lengths of the two arcs of the parabola in the two situations are, [1]:

$$s_x = l + \frac{h^2}{2l} + \frac{G_x^2 l}{24H_x^2} + \frac{x(l-x)}{2lH_x^2} \cdot Q_x\left(Q_x + G_x\right) \tag{10}$$

$$s_r = l' + \frac{h'^2}{2l'} + \frac{G_r^2 l'}{24H_r^2} + \frac{x'(l'-x')}{2l'H_r^2} \cdot Q_r\left(Q_r + G_r\right) \tag{11}$$

Here: $x$ and $x'$ $x'$ are the abscissas of the load $Q_x$ opening positions, respectively $Q_r$ related to the reference systems *Oxyz*, respectively *Ox'y'z'* (Fig. 2). The loads $G_r$ and $Q_r$, since in the reference situation the action of the wind is also assumed, are:

$$G_r = \sqrt{(q_x^v)^2 + (q_c + q_y^v)^2 + (q_z^v)^2}$$
$$Q_r = \sqrt{(Q_x^v)^2 + (Q_s + Q_y^v)^2 + (Q_z^v)^2}\,,$$

and the loads $G_x$ and $Q_x$, which correspond to the situation without wind, are: $G_x = q_c \cdot L$, $Q_x = Q_s$.

The difference in the lengths of the two curves is $\Delta s = s_x - s_r$. This occurs, on the one hand, due to the different elastic elongations of the cable in different states of tension, and on the other hand, due to the different temperatures of the two considered working conditions. Thus:

$$\Delta s = \Delta s_e + \Delta s_\theta = \frac{T_x - T_r}{EA} \cdot L + \varepsilon L\left(\theta_x - \theta_r\right),$$

Here it is noted: $A$ is the cross-sectional area of the cable, $E$ – its modulus of elasticity, and $\varepsilon$ - the coefficient of linear expansion of the steel. In his expression $\Delta s$ it is allowed to consider $H \cong T\cos\beta$, [1], so that

$$\Delta s = \left(\frac{H_x}{\cos\beta} - \frac{H_r}{\cos\beta'}\right)\cdot\frac{L}{EA} + \varepsilon L\Delta\theta \tag{12}$$

By introducing the expressions (10) and (11) in (12), after convenient rearrangement of the terms, the third degree algebraic equation is obtained, the unknown of which is $H_x$

$$H_x^3 - \left[\frac{H_r}{\cos\beta'}\cdot\frac{L}{EA} + \Delta l - \varepsilon L\left(\theta_x - \theta_r\right) - \frac{x'\left(l'-x'\right)}{2l'H_r^2}\cdot Q_r\left(Q_r + G_r\right) - \frac{G_r^2 l'}{24H_r^2}\right]\cdot$$
$$\cdot\frac{EA}{L}\cos\beta\cdot H_x^2 - \left[\frac{x\left(l-x\right)}{2l}\cdot Q_x\left(Q_x + G_x\right) + \frac{G_x^2 l}{24}\right]\cdot\frac{EA}{L}\cos\beta = 0 \tag{13}$$

Here, to simplify the writing, the notation was used

$$\Delta l = \left(l + \frac{h^2}{2l}\right) - \left(l' + \frac{h'^2}{2l'}\right) \tag{14}$$

**7.1. Specifications in the case of typical particular conditions.**

a) If in the reference situation it is considered that the mobile load is found in the middle of the opening $l'$, which is the position of the mobile load $H_r$ for which it takes the maximum value, in (13) will be taken $x' = l'/2$.

b) To determine the horizontal component of the cable tension during mounting, the following should be considered: $Q_x = 0$, $G_x = G_c = mg\cdot L$, $m$ being the mass per linear meter of the cable.

c) For the out-of-service situation at the temperature when it is assumed that hoarfrost is deposited on the cable, it will be considered $G_x = \left(m + m_{ch}\right)g\cdot L$, $m_{ch}$ being the mass per linear meter of the deposited hoarfrost, and $Q_x = 0$.

For instance, if the horizontal component of the cable tension is to be determined during its initial pre-stretching (at mounting), $H_m$, and in addition, if the reference conditions are taken into account $x' = l'/2$, it results:

$$H_m^3 - \left[\frac{H_r}{\cos\beta'}\cdot\frac{L}{EA} + \Delta l - \varepsilon L\left(\theta_m - \theta_r\right) - \frac{l'}{8H_r^2}\cdot Q_r\left(Q_r + G_r\right) - \frac{G_r^2 l'}{24H_r^2}\right]\cdot$$
$$\cdot\frac{EA}{L}\cos\beta\cdot H_m^2 - \frac{EA\cdot G_c^2}{24\cdot L}\cos^2\beta = 0 \tag{15}$$

***Note:*** In fact, the equation (13) is generally valid. The shape of this equation stays unchanged regardless of the reference situation (with or without wind, with or without mobile load, with or without hoarfrost), and regardless of the situation considered when determining the horizontal component of cable tension. It is necessary to introduce only the values that affect the coefficients of the equation that correspond to the two situations.

## 8. Conclusions

While SR EN 12930: 2015, [5] provides for the calculation of cables carrying passenger transport installations considering wind action, the specialized technical literature does not address this issue. We consider that this paper solves all the problems that arise.

The problem shall be resolved, as detailed in paragraph 2, by defining the plane in which the equilibrium configuration of the cable is established under the action of the main (gravitational) loads and the wind.

When the payload is not present, the given solution is theoretically accurate in the case of installations out-of-service, but slightly approximate in the case of installations in the service (see paragraph 4).

The solution found and detailed in this paper makes the influence of wind implicit if the modified opening and level difference ($l'$ instead of *l* and $h'$ instead of *h*) is used in the calculations, see Figure 2 and the comment in paragraph 5. The wind speed no longer appears explicitly in the differential equation that represents the variation of the route speed by the mobile load, whose function is unknown.

The paper is not only limited to specifying the framework in which the approach leads to the theoretical solution of the problem, but also solves the related technical problems that interest the design engineer (paragraphs 6 and 7).

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