Technical Sciences
Academy of Romania
www.jesi.astr.ro

Journal of Engineering Sciences and Innovation
Volume 8, Issue 3 / 2023, pp. 227-238
http://doi.org/10.56958/jesi.2023.8.3.227

A. Mechanical Engineering

Received ***27 June 2023*** *Accepted* ***29 September 2023***
Received in revised form ***4 September 2023***

Stress state in perforated discs

COSTICĂ ATANASIU[1,2]*, ȘTEFAN SOROHAN[1]

[1]*Strength of Materials Department, Politehnica University of Bucharest,*
[2]*The Academy of Technical Sciences in Romania*

Abstract. The paper investigates the stress state in perforated discs by circular holes. Circular discs are subjected to two types of loadings: diametrical compression by concentrated forces and radial compression produced by an equivalent uniformly distributed load on the contour of the discs. The discs of radius 150 mm are perforated with two, four or 96 holes, of different radii between 1 mm and 50 mm or maximum possible. The stress distribution in the discs is studied according to the value of the radii of the holes and the distance between them. The stress concentration factors and their variation for the geometrical parameters of the concentrators are determined. The finite element method was used, which allows the modeling of the structure so that it is as close as possible to the real situation of the geometry and the way of application of the load.

Keywords: finite element method, perforated disc, photoelasticity, stress state.

1. Introduction

The non-perforated circular disc, subjected to diametrical compression by concentrated forces F (N/mm) represents a problem of the Theory of elasticity solved by Hertz [1]. By using the stress function, the expressions of the stresses in the disc were obtained, based on which their distribution along the horizontal and vertical diameters was drawn (Fig. 1).

*Correspondence address: atanasiucostica@yahoo.com

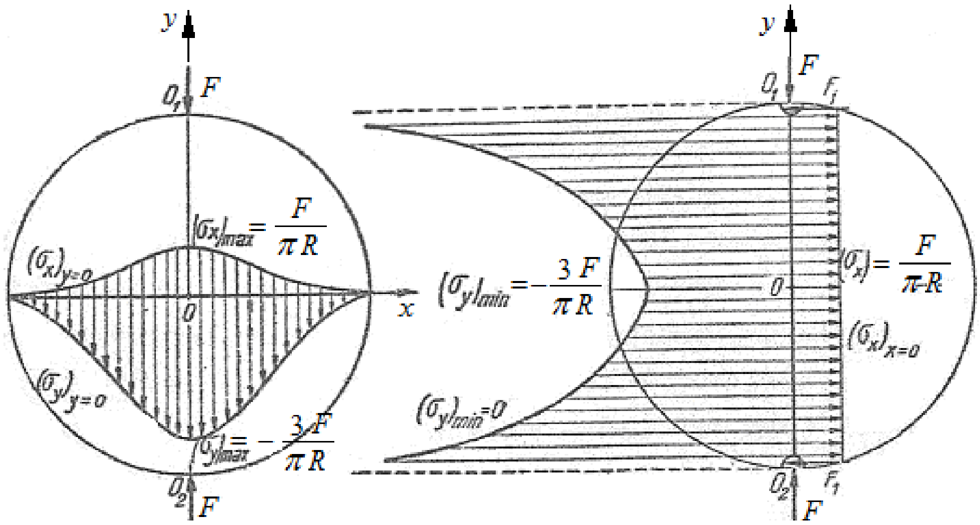


Fig. 1. Stresses distribution in the non-perforated disc.

The expressions of the normal stress in the center of the disc are given on Fig.1. In the center of the disc, of radius *R*, the normal stresses (if the disc thickness *t* = 1, or *F* is the distributed load along the thickness of the disc) are:

$$\sigma_x = \frac{F}{\pi R} \qquad \sigma_y = -\frac{3F}{\pi R}. \qquad (1)$$

It is found that on the vertical diameter which is an axis of symmetry, (except for the concentration area near the applied concentrated force) the normal stress is constant and tensile and the tangential stress is zero. The normal stress in the contact zone is very high and the disc material normally enters the flow zone. Experimentally, Frocht [2] obtained the isochromatic curves for the disk under diametrical compression. In Fig. 2, the isochromic curves for the complete disk, resulting from the photoelasticity study in circularly polarized light, are presented.

In the paper [2], the photoelasticity method in oblique incidences is used to determine the stress concentration factor in a disc perforated by four holes and subjected to diametrical compression [3]. The photoelastic model is examined in the photoelasticity installation [4] and the field of isochromatic lines is recorded (Fig. 3). By performing the integration on the contour of the holes and the disc, which are free of loads, the stress distribution is determined. The stress concentration factor is equal to the ratio of the maximum stresses in the perforated disc to the non-perforated disc.

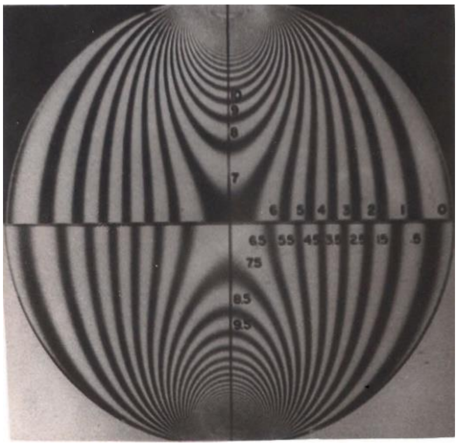


Fig. 2. Isochromatic fringes for non-perforated disc.

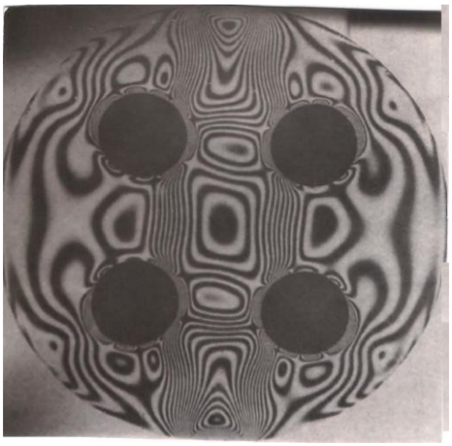


Fig. 3. Isochromatic fringes for the perforated disc.

2. Stress distribution in the perforated disc subjected to diametrical compression

Perforated circular discs represent sub-assemblies frequently found in the construction of motor vehicles, agricultural machines, machinery in the process industry, etc.

The research carried out refers to circular discs with radius R = 150 mm and constant thickness t = 10 mm, perforated by with two, four or 96 circular holes with radii r in the range 1 to maximum 50 mm (1, 2, 5, 10, 20, 30, 40, 50 mm), arranged as in Fig. 4. The two and four holes respectively (Fig. 4, b,c) are arranged on a circle with radius R_1 = 75 mm. The centers of the 96 holes (Fig. 4, d) are arranged in a grid of squares with a side of a = 24 mm. The discs are made of plexiglas with the elastic characteristics: the longitudinal modulus of elasticity (the Young's modulus) E = 3400 MPa and the transverse contraction coefficient (Poisson's ratio) ν = 0.3. Perforated discs subjected to diametrical compression present both geometric and loading symmetry.

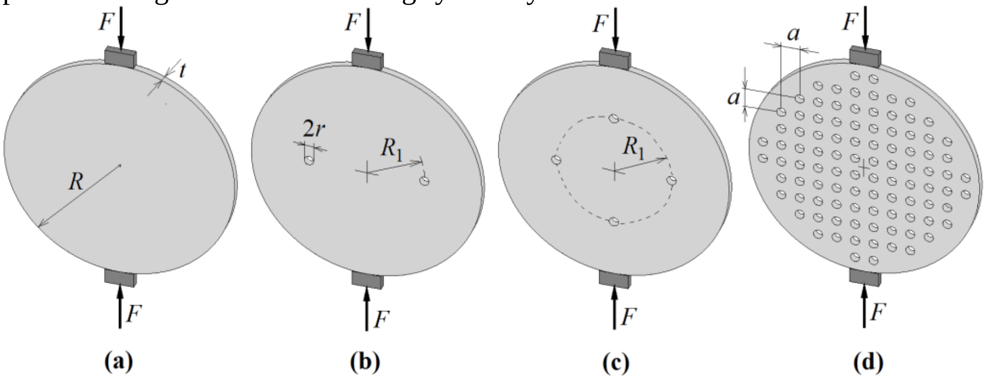


Fig. 4. Full and three perforated discs subjected to compression.

The perforated disk has reduced stiffness and the stresses have increased values compared to the non-perforated disk. The existence of holes, material discontinuities, also introduces a stress concentration effect. The stress concentration factor in the perforated disc k_σ is defined here by the ratio between the maximum absolute stress in the concentrator $\sigma_{\max}$ and the maximum absolute stress σ in the center of the non-perforated disc.

The plexiglas discs were subjected to diametrical compression through forces applied by means of plexiglas plates of the same thickness and with a width B = R/4= 37.5 mm and a height H = B/2 = 18.75 mm, so that one material does not penetrate the other. The applied compressive force was F = 1000 N, resulting in a pressure of 0,266 N/mm applied on the top of the rectangular plates. In this case the normal stresses in the center of the non-perforated disc, resulting from the application of the relations established by the Theory of Elasticity [1], [2] are σ_x = 0,212 N/mm and $_y$ =-0,637 N/mm. So the maximum absolute stress in the center of the non-perforated disc is σ = - 0,637 N/mm. In the case of the disc to be studied with a thickness of 10 mm, the stresses expressed in MPa are: σ_x = 2.12 MPa and σ_y = -6.37 MPa.

The static analysis of the displacements and stresses distribution in the discs was carried out using the finite element method and quadrilateral elements (Plane183) with eight nodes [5], [6]. The symmetry of the discs was respected by imposing the boundary conditions in displacements: along the vertical diameter the displacement u_x = 0 and along the horizontal diameter the displacement u_y = 0. The Ansys R22 code was used in the research. Considering the symmetry of the discs and the small thickness compared to their diameter, a 2D analysis was carried out with finite elements for a quarter discs.

A parametric model was created (using APDL in Ansys classic) for each problem. Finite element discretization was performed for the four types of disks studied: non-perforated disc, discs perforated by 2, 4 and 96 holes (Fig. 4). When defined the non-perforated disc, it was considered that it should be done in such a way that by extracting some areas, the disc perforated through two, four or 96 holes with the expected values of the radii would result. At the same time, it was considered that in the area of the holes with a very small radius and in the area of contact of the disc with the rectangular plate through which the compression force is applied, a fine refinement of the mesh should be done. In the area of transmission of the compression force, contact elements line-to-line was used.

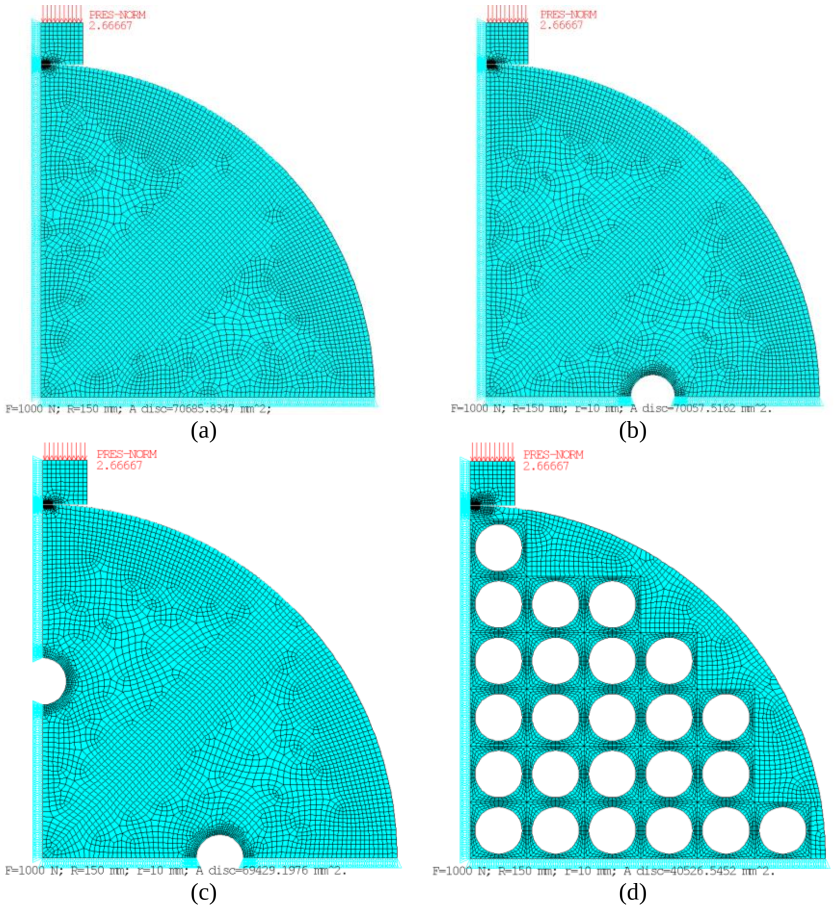


Fig. 5. Quarter models of the analyzed discs - discretization and boundary conditions.

The classic Ansys APDL generates contact elements simpler using "Contact Manager". In Fig. 5, the discretizations of the non-perforated disc and the one perforated by two, four or 96 holes with a particular radius r = 10 mm subjected to compression by the force F = 1000 N distributed over the rectangular plate are presented.

After entering into the Software Ansys the information regarding the geometry of the discs, the values of the elastic characteristics of the material from which they are made, the boundary conditions and the way of applying the compression force, for each case studied it is retained:

- total displacements and of u_x and u_y displacements distribution;
- equivalent stresses and principal stresses distribution in discs;
- the equivalent stresses and the principal stresses distribution in the discs without the area of application of the load;
- the principal stresses values in the center of the discs;
- stress distribution contact.

The verification of the performed modeling was done by calculating the stresses in the center of the non-perforated disc, which were obtained by the finite element method σ_1 = 2.1224 MPa and σ_3 = -6.37 MPa compared to σ_1 = 2.12 MPa and σ_3 = -6.37 MPa, results from the analytical calculation. This confirms the assessment that the schematizations and discretizations used for the applied tasks and the geometry of the discs were very close to the real situation of the studied problem.

3. Stresses state in perforated discs subjected of diametrical compression by concentrated forces

For the contact pressure distribution between the plexiglas block through which the compression force is transmitted and the discs, a parabolic distribution (Fig. 6) equivalent to the total force of 1000 N was obtained. The value of the maximum contact pressure is not essentially influenced by the existence of the holes, the value of the maximum pressure being approximately constant, i.e. around of 18.27 MPa for the disc perforated through two holes and for the disc perforated through four holes compared to the same value of the maximum pressure of contact with the non-perforated disc. At the disc with 96 holes, the maximum contact pressure was p_{max} = 18.55 MPa. A slight tendency to decrease the value of the maximum contact pressure with the increase of the radius of the holes is also observed. For example, in the plate with 96 holes, the value of the maximum contact pressure drops from 18.55 MPa at r = 2 mm, to 18.48 MPa for r = 6 mm and to 18.07 MPa for r = 10 mm.

The stresses in the discs in the contact area are approximately constant at the value of 18.28 MPa in the case of the disc with two holes, regardless of the value of the hole radius, they vary between 18.28 MPa at r = 1 mm and 31.07 MPa at r = 50 mm for the disc with four holes and between 18.23 MPa at r =1 mm and 25.73 MPa at r = 10 mm for the 96-hole disc. Fig. 7 shows the variations of the maximum stress in the contact area of the disc with four holes and r = 1 mm and r = 50 mm.

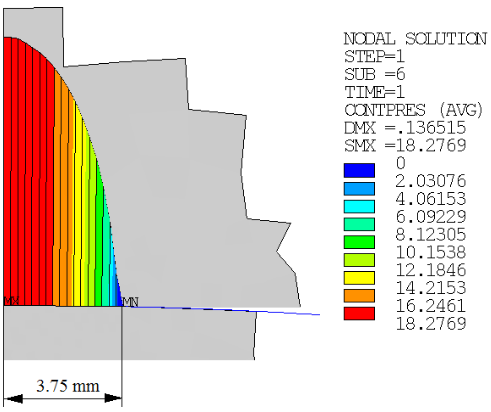
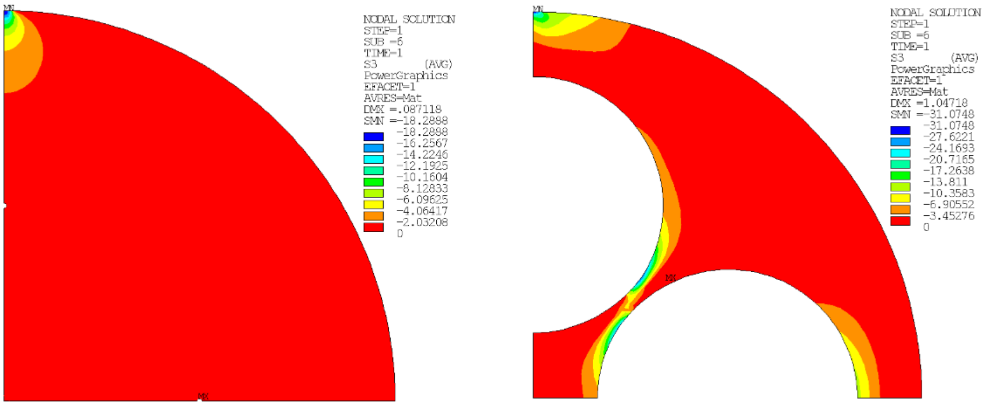


Fig. 6. Contact pressure distribution in the case of disc without holes.



(a) (b)

Fig. 7. Minimum principal stress distribution in the four-hole disc. (a) r = 1 mm; (b) r = 50 mm.

The stress state around the holes and respectively the values of the stress concentration factor are influenced by the proximity or distance from the load application area on the disc. This influence is all the greater the closer the holes are to this area. Thus, for the perforated disc by two holes located on the horizontal diameter of the disc, this influence is zero. For the disc with four holes located two each on both the horizontal and the vertical diameter, the stress distribution is noticeable only in the holes located on the vertical diameter. In the disc with 96 holes, the area comprising the first four rows of holes works as a column stressed in compression and the stresses on the contour of the holes closest to the place of application of the loads have important values. In Fig. 8 the stress distribution around the holes with radius r = 10 mm from the disc with 96 holes is given. On the contour of the hole closest to the place of application of the load, values of the maximum absolute principal stress is σ_3 = -11.44 MPa and a value of the stress concentration factor k_σ = 1.8 are found. On the contour of the hole furthest from the place of application load the principal stress values is around σ_3 = -2.85 MPa.

The influence of the holes on the stress state in the center of the discs perforated by 2 or 4 holes with a radius of less than 5 mm can be appreciated to be very small compared to the stresses in the non-perforated disc. The state of stress changes as the hole radius r or ratio r/R increase. Thus, if for the radius r =1 mm the principal stresses in the center of the disc perforated by two or four holes are σ_1 =2.1 MPa and σ_3 = -6.37 MPa; if the radius of the holes is r = 50 mm the principal stresses are σ_1 = 3.9 MPa and σ_3 = -14.9 MPa at the disc perforated through 2 holes and σ_1 =2,379 MPa and σ_3 = -2,3MPa at the disc perforated through 4 holes with r = 50 mm.

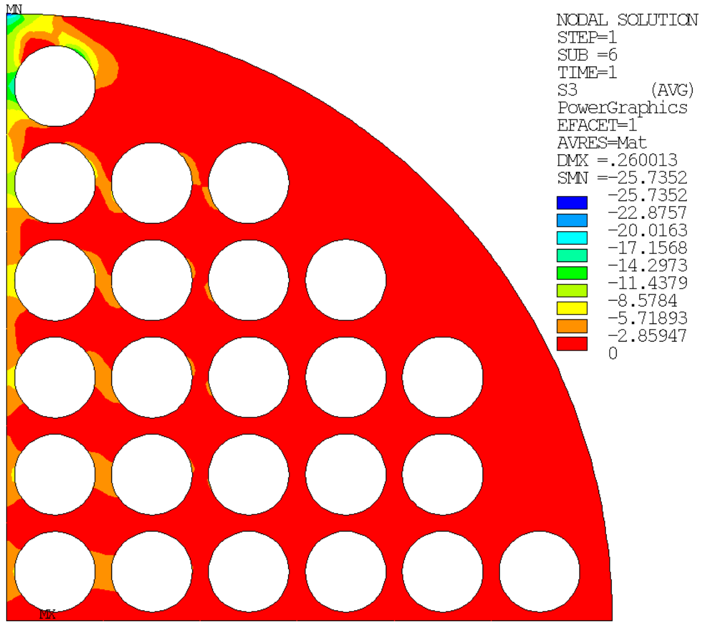


Fig. 8. Minimum principal (max. absolute) stress distribution around holes.

For the perforated disc by 96 holes, it is found that the principal stresses in the center of the disc increase with the increase of the radius of the holes and the reduction of the distance between the holes. Thus, for the case where the radius of the holes has a small value in relation to the radius of the disc r = 2 mm and r/R = 0.013 the principal stresses have the values σ_1 = 2.59 MPa and σ_3 = -6.98MPa and thus k_σ =1.37. At the value r = 6 mm and r/R = 0.04 the principal stresses increase to values σ_1 = 5.2 MPa and σ_3 =-12.03 MPa and thus k_σ = 2.11 and at r = 10 mm and r/R = 0.06 to σ_1 = 9.44 MPa and σ_3 = -29.5 MPa and k_σ = 4.63.

4. Stresses in discs loaded with uniformly distributed forces on the contour

The analysis of the stresses and displacement state in the discs was also carried out for the case where the concentrated radial force F = 1000 N is distributed on the circumference of the discs of radius R = 150 mm, as a radial force uniformly distributed on the contour, of value

$$p_\ell = \frac{F}{2\pi R} = 1.061 \text{ N/mm.}$$

This load is distributed over the entire thickness of the disc t=10 mm, resulting p=0,1061MPa.

To eliminate rigid body displacements, full model of discs were considered articulated and simply supported on two nodes on the vertical diameter. The problem is reflective symmetrical along two planes and only a quarter of the disc could be used in the analysis of the of stress state in the discs. After the discretization of the perforated discs with two, four and 96 holes with a radius between 1 and 50 mm and their loading with the uniform radial load *p* (Fig. 9), the stress state in the discs was analyzed.

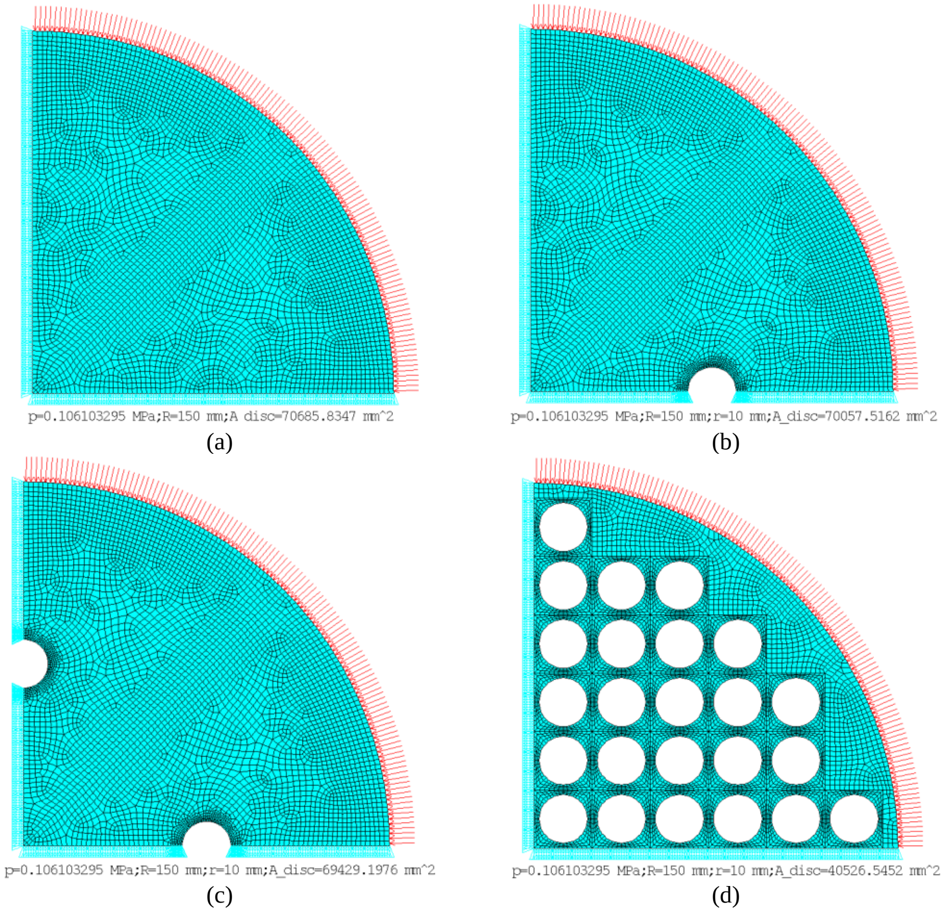
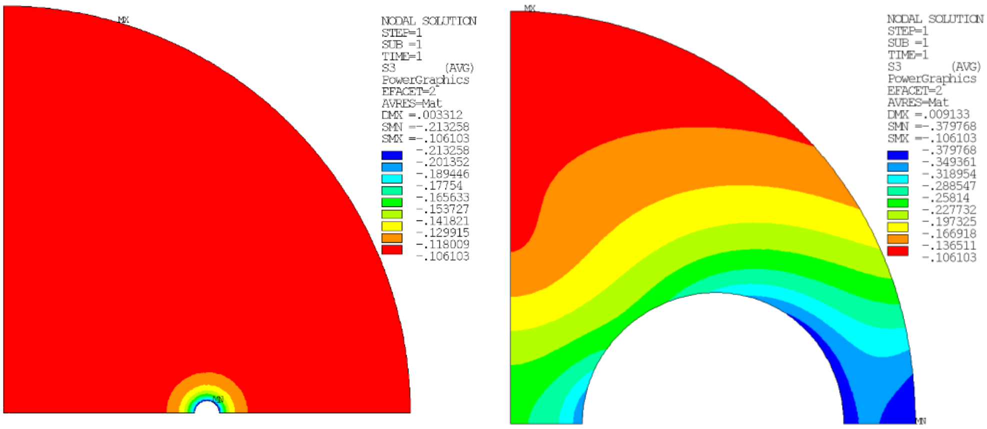


Fig. 9. Discs loaded with uniformly distributed forces on the contour.

The stress state in the non-perforated disc was also studied with the same loading and stress conditions as in Fig. 9, to be able to make a comparison with the stress state in the perforated disc and calculate the value of the stress concentration factor. The relations for stresses calculating in the non-perforated disk can be obtained by using the stress function or by customizing the relations from the thick-walled tubes required by external pressure [8]. Considering that the inner radius of the tube is zero, it results in the direction of the radius and the direction of the tangent: $\sigma_r = -p_l$, $\sigma_t = -p_l$. Applying these relations, the stresses at any point on the nonperforated disc are: σ_r = -0.1061 MPa and σ_t =-0.1061 MPa. Using the finite element method [6] the results were: σ_r = -0.106103 MPa and σ_t = -0.106103 MPa, values extremely close to those obtained analytically. In this case of stress in the disc, there is no more stress concentrator, the stress concentration factor k = 1 and the stress at any point is σ =0.106 MPa. In the first case of concentrated load, the maximum stress on the contour of the non-perforated disc in the contact area: σ = 18.28 MPa and for the stress concentration factor k = 2.87 were obtained.

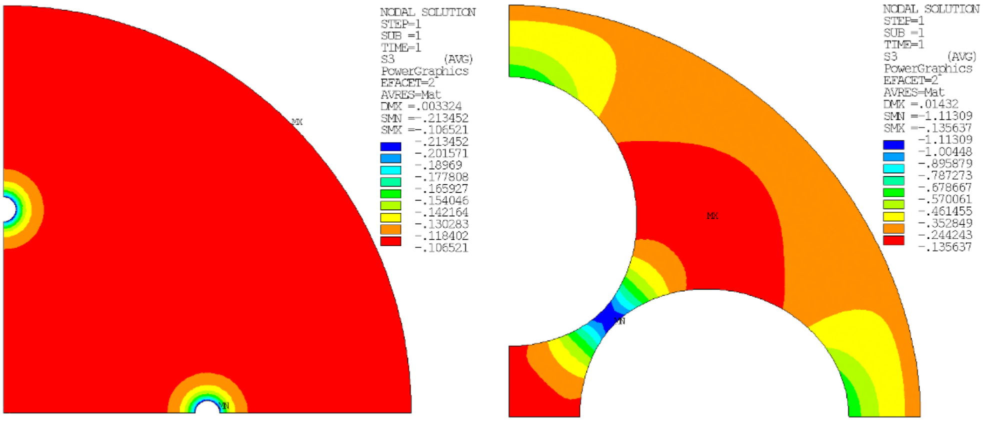
Software Ansys gives for each case studied the displacements and principal stresses distribution. The multitude of cases does not allow all the results to be reproduced in the work. Here are some results:

- for the disc with two holes with radius r =5 mm the stress on the contour of the holes σ = -0.2125 MPa resulting in k = 2 and the stress between the holes σ = -0.10627 MPa (Fig. 10,a);
- for the disc with two holes with radius r = 50 mm the stress on the contour of the holes σ = .-0.379691 MPa resulting in k = 3.578 and the stress between the hole in the center of the disc σ =.-0.227 MPa (Fig. 10,b);
- for the disc with four holes with r = 5 mm the stress on the contour of the holes σ = -0.213 MPa resulting in k = 2 and the stress between the hole in the center of the disc σ = -0,106 MPa (Fig. 11,a);
- for the disc with four holes with r = 50 mm the stress on the contour of the holes σ = -1.11 MPa resulting in k = 10.47 and the stress between the hole in the center of the disc σ = -0.135 MPa (Fig. 11,b);
- for the disc with 96 holes with r=6mm the stress on the contour of the holes σ = -3.05 MPa resulting in k = 2.877 and the stress between the holes, in the center of the disc σ = -0.122MPa (Fig. 12,a);
- for the disc with 96 holes with r=10mm the stress on the contour of the holes σ = -0.436MPa resulting in k = 4.11 and the stress between the holes, in the center of the disk σ = -0.148Pa (Fig. 12,b).



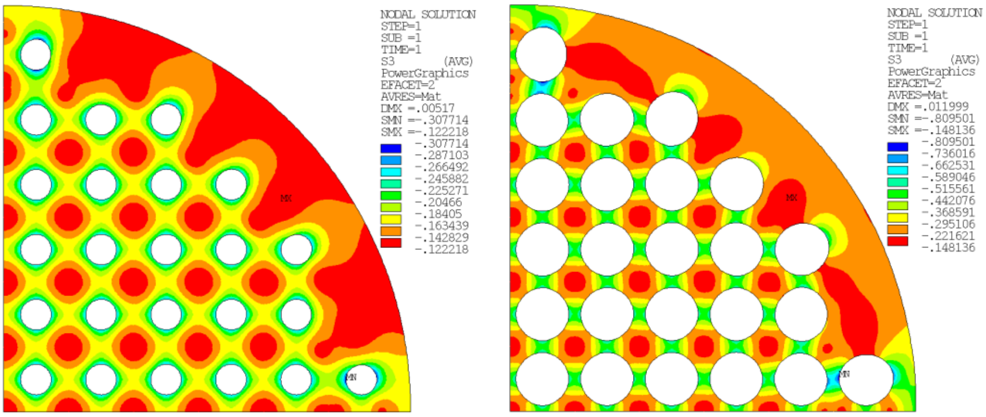
(a) (b)

Fig. 10. Stress distribution in the disc with two holes. (a) r = 5 mm; (b) r = 50 mm.



(a) (b)

Fig. 11. Stress distribution in the disc with four holes. (a) r = 5 mm; (b) r = 50 mm.



(a) (b)

Fig. 12. Stress distribution in the disc with 96 holes. (a) r = 6 mm; (b) r = 10 mm.

5. Conclusions

Following the research carried out on the two variants of loading the studied perforated discs, some conclusions can be drawn:

- the value of the stresses and the stress concentration factors increase with the decrease of the stiffness of the discs. The stiffness of the discs is reduced by increasing the number of holes and their radius;
- the stress distribution and values of the stresses in the discs are a function of the force application mode;
- the application of a uniformly distributed load on the outer contour of the discs, equivalent to the concentrated force or the force applied on a small area of the contour, produces a change in the stress state by reducing the stress values on the

contour of the discs and holes. With the non-perforated disc, the production of a homogeneous stress state is observed, a situation also confirmed by the analytical study.

References

[1] Flugge S., *Elasticity and Plasticity*, Springer-Verlag, 1958.
[2] Frocht M., *Photoelasticity*, vol. I, II, New York, John Wiley&Sons, 1957.
[3] D'Sylva R., *Bending of Perforated Plates*, Transactions of ASME, Serie E, December, 1992.
[4] Iliescu N., Atanasiu C., *Metode tensometrice în inginerie*, București, Editura AGIR, 2006.
[5] Zienkiewicz O. C., *La méthode des éléments finis. Appliquée a l art de l ingenieur*, Paris, New York, Groupe Mc Graw-Hill.
[6] Sorohan Șt., *Elemente finite în inginerie*. București, Editura Politehnica, 2015.
[7] Voinea R., Voiculescu D., Simion P., *Introducere în mecanica solidului cu aplicații în inginerie*, București, Editura Academiei, 1989.
[8] Atanasiu C., Jiga G., *Comportement mécanique des matériaux*, București, Editura AGIR 2017.