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Best operation strategies for piezoelectric vibration energy harvesters. I. Theory

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Abstract. This paper is the first part of a three-part series. A general scheme of controlling the operation of piezoelectric vibration energy harvesters is proposed. It covers schemes proposed in literature as particular cases. The focus is on base-excited vibration energy harvesters.

Keywords: piezoelectric energy harvester; direct optimal control; general operation scheme

1. Introduction

Recent developments in emerging technologies require small-power sensors and actuators of long term energy-autonomy. They include biological implants [1], building climate control, structural health monitoring (automotive [2] and aerospace [3], wireless devices, vibration control systems [4], agricultural automation homeland security applications [5] and data transmitters. When these devices are of microscopic scale they are shortly called Microelectromechanical systems (MEMS). The needed power consumption for milli-scale commercial sensors is of the order of several to hundreds of μW [5]. When wireless sensors are considered, the power consumption depends on their subsystems. For example, in case of the computing unit it is of the order of 16 to 400 mW for the active mode, sleep mode: 160 μW for the sleep mode and 50 mW in idle mode and a range between 10 - 100 μW is expected [5]. The power consumption of the communication system depends on modulation type, operation mode and transmission rate. The sensing subsystems are numerous and their power consumption differs greatly. However, the power consumption is negligible in case

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of passive sensors (such as pressure and strain sensors, thermometers and accelerometers) [5].

Energy sources for sensors and actuators may be divided in traditional sources with fixed density of energy and sources with fixed density of power. The first category includes the electrical energy stored in batteries and the chemical energy in fuel cells and have limited lifetime before batteries are replaced or recharged and fuel is replenished. The second category ensures self-powering of the devices and includes renewable energies such as solar, thermal gradients, acoustic energy, kinetic fluid energy [6] and mechanical vibration energy [7] and are better suited for long-term implementation [5,8]. In some cases energy harvesting from such sources may be an alternative for traditional solutions taking into account the cost, environmental and health-related problems due to recharging or replacing and disposing of batteries [9].

The advantage of traditional sensors with external power supply is that they are very accurate. However, they are expensive and, as a consequence, just a single device is normally implemented. Therefore, a device failure has great influence on system performance. The self-powered sensors are less accurate but relatively low cost. This allows implementation of a large number of similar devices and this redundancy makes the system fault tolerant since failure of one device has not significant influence on the overall performance. Also, self-powered sensors can be used in areas far away from the electric grid or where batteries are difficult to be changed. Self-powered devices should obey some constraints dictated by application. The lifetime of a self-powered device should be ideally as long as the lifetime of the application. For instance, this means up to 30 years in case of building structural monitoring [5]. The devices size should be relatively small, allowing a large number of devices to be implemented. For most applications the size is in the *µm* - *mm* scale [5]. The cost of the device should be small enough to allow economical implementation of a network of devices. Notice that the cost of a battery is about 12-20% of an externally-powered system price [5]. Using a self-powered system will decrease the cost since there is no need of a battery and the associated interface. The self-powered devices are particularly useful in building environment control where the wiring cost is up to 90% of the total sensor cost [5].

Because many sources of mechanical vibrations are available in the environment, the conversion of their energy into electrical energy is a topic of current intense research [4]. Low-level vibrations are commonly found in the natural environment while high-level vibration sources are usually associated with human-made systems such as machineries and vehicles. Systems designed to convert the vibration energy into useful work are divided into resonant and non-resonant energy harvesters, respectively. In the first category, there is a matching between the device resonance frequency and the input vibration frequency, which is high (usually > 100 Hz), regular and has a small amplitude. The second category of energy harvesters applies to very low frequency vibrations (usually < 10 Hz) and irregular vibrations with very large amplitudes. Multi-band piezoelectric vibration energy harvesters for low-frequency applications have been also considered [10]. A low-cost, bi-stable piezoelectric energy harvester has been proposed, analyzed, and experimentally tested for the purpose of broadband energy harvesting over the frequency range of 9.0–14.0 Hz [11]. The two categories are not competing since they are most effective at different frequencies [5].

A typical vibration energy harvester consists of a vibrating host structure (the energy source), a transducer and an electric load [9]. The transducer is based on some materials which can generate an electric potential in response to external mechanical stresses. These materials transform the energy of mechanical deformation into useful electrical power which can directly run low-power devices [12]. Three transduction mechanisms are usually considered: electrostatic, electromagnetic and piezoelectric. Different theories have been proposed for each mechanism. An unified approach for electromagnetic and piezoelectric transducers has been presented in [13]. A new vibration energy harvesting scheme based on the charging phenomenon occurring naturally between two bodies with different work functions has been proposed in [14]. A good review on harvesting of vibration energy has been published [15].

Most promising results are provided by vibration energy converters based on the piezoelectric effect [4] since they are feasible of micro-scale fabrication, provide large power densities and may be easy implemented [16]. Notice that the piezoelectric elements were used in the past in systems for passive or active vibration damping where the energy is dissipated through resistive heaters or other ways. Later on, the idea of not dissipating that energy but using it to power sensors has been suggested [5]. The disadvantage of the piezoelectric mechanism is that integration the transducer into micro-systems is rather complex since the material should be poled in a strong electric field (see details in [5]). However, there are many advantages, including its high energy conversion efficiency and the fact there is no need for a separate voltage source. Good reviews on piezoelectric energy harvesters and their applications have been published [17,18].

The voltage across the piezoelectric transducer is usually rectified by a bridge rectifier which is connected to the load, usually consisting of a resistor, a capacitor or a rechargeable battery [1]. Sometimes the diode bridge rectifier is cascaded by a DC/DC converter [19]. The maximum output power is provided when the input vibration frequency is matched by the resonant frequency of the transducer. In practice, the magnitude and frequency of the vibration depend on time and the power output decreases if the resonant frequency of the transducer is kept constant. Therefore, there is a need to dynamically adjust the transducer characteristics and load operation in order to obtain the best performance [4]. Many previous studies focused on the dependence of the output power on the transducer characteristics such as dielectric constant, polarization and geometry [1]. A geometric configuration studied extensively is the cantilever beam [20,21]. It allows low resonance frequencies and this is an important advantage. Both <3-1> – mode bimorph and <3-3> mode unimorph configurations have been treated [3,22]. The development, manufacturing, and testing of an advanced system based on three bimorphs has been reported in [23]. It has been shown that the performance depends on the scale of energy device [8]. Other geometric configurations are membrane structures used to harvest energy from walking or pulsing pressure sources [5]. It has been shown that L-shaped structure constitutes a promising harvester that supports multi-directional and multi-mode energy harvesting [24]. The effects of different geometrical design parameters of auxetic boosters on the performance of the energy harvesting system are investigated in [25].

The theory of vibration energy conversion is well established for a sinusoidal external force either for an exciting mass or an exciting base (see e.g. [26]). It has been shown that the maximum output power, derived from the energy harvested per cycle, depends on the natural frequency of the spring-mass system and the damping ratio, among other factors. Transition from open-circuit to short circuit conditions is associated with a shift in the natural frequency of the device, which may be neglected for macro-scale devices but become significant at micro-scale [8]. A general energy conversion theory covering piezoelectric, magnetostrictive, and electrostatic transducer technologies has been developed in [27] where an “effectiveness” for the vibration-based generator has been proposed. The theory of resonant energy harvesting has limitations since in practice the vibrations are distributed over a broadband of frequencies. Such sort of vibrations is sometimes modeled as Gaussian white noise and research concerning harvesting of its energy by using Duffing unimodal and bimodal oscillators has been performed (e.g. [28]). The main result is that non-linear oscillators are not significantly better than the linear oscillators but they may allow reducing the size of the harvesting device. In order to circumvent instabilities delayed self-excited harvester systems have been also studied [29]. More involved theories are using colored white noise instead of sinusoidal or Gaussian white noise [28]. Random ambient forces as well as white (band limited) Gaussian distributed noises have been considered in [30]. Real-world vibrations measured in trains cars and microwave ovens have been analyzed. Different strategies have been proposed to decrease the frequency bandwidth of the harvesting device [31].

Piezoelectric energy harvesters have been optimized from many points of view. The most usual optimization approach assume resonance operation and, as a consequence, relationships involved do not depend on time and the output power is maximized in respect to a parameter which in many cases is the electric load resistance (see [5],[30],[32] for instance). The optimal conditions are obtained in [13] by ensuring that the mechanical and electrical parts of the harvesting device operate in resonance and a matched electrical load is used for this purpose. The operation of a piezoelectric energy harvester was optimized in [8] for maximum output power for both the resonance and anti-resonance frequencies of the device and it has been shown the same maximum power is obtained for the two optima but the voltage and the current have significantly different values in the two cases.

Some authors maximized the output power by controlling the input resistance of the harvesting device [32] or adjusting the piezoelectric energy harvester [4],[33].

Optimization may focus on the electronic circuits. For instance, the output power has been maximized by using different approaches such as adjusting the natural frequency of the energy harvester, improving the extracted power by using adaptive circuits, minimizing power losses in the rectifying diodes and controlling the rectified DC voltage [4]. Also, the maximization of the energy transfer from the transducer to the storage device may be obtained by the optimization of power electronics operation [5]. The design of a back-controller and the parameters of the electronics (bus voltage and switching frequency) are optimized for piezoelectric energy harvesting [34].

Other studies are more specific in techniques and methodologies. Power maximization has been treated in literature for both unconstrained and constrained proof mass displacement and for harmonic, double sinusoid input and frequency-swept sinusoid input. Also, distributed-parameter optimizations and two-stage procedure allowing optimization of geometry, resonance frequency and electrical load for linear electromagnetic micro-power generator have been proposed (for a short review, see [32]). Power Extracted From Piezoelectric Harvesters Driven by Non-Sinusoidal Vibrations has been treated in [35]. Tuning both the electrical and mechanical impedances is simple and very efficient while using a variable capacitive load can be used to match the frequency of the external vibration in real time (see [12] for discussions). The variance of the output power has been minimized in some papers while the average power across the transducer has been maximized in [36]. The optimal feedback gains has been found by solving two nonlinear coupled algebraic relationships similar to standard Lyapunov and Riccati equations. A generalized form of the method of Lagrange multipliers has been used in [12] to maximize the output power.

Previous results showed that the change of the input frequency decreases significantly the output power and a tuning mechanism should be incorporated. Another motivation for a tuning device is the variability in the manufacturing process of the transducer, which may results in up to ten percent size variation. It is suggested that the tuning may be made with a variable capacitor but this research line is not followed in [5].

The time-dependent optimization of piezoelectric energy harvesters operation has been also treated in previous studies. Optimal control methods have been already used in this context. Some authors are using rigorous indirect optimal control procedures. For instance, the energy harvesting of random vibrations was treated by indirect optimal control methods in [1,34]. The electrical time constant and the stiffness value have been found in [32] by using the Pontryagin's maximum principle. The control functions are the open-circuit stiffness and the load electrical resistance [32]. A semi-active control approach of the damping electromagnetic coefficient is proposed in [31]. Usage of indirect optimal control methods (the Pontryagin Maximum Principle) outperforms the classical harvester device with constant parameters. Other optimal control approaches have been also used. Some authors treated the case of linear quadratic Gaussian (LQG) optimal control. It is known that the LQG controller is a simple combination of a Kalman filter (a linear–quadratic state estimator) together with a linear–quadratic regulator. Further details are found in [9] where the Pontryagin Maximum Principle has not been used. An original model has been proposed in [32] where a lumped-element circuit replaces the differential equations when the energy harvester is described. The optimal control of a time-invariant linear system perturbed by Gaussian white noise is treated in [9] where the maximization of a quadratic cost function without control penalty and passive controls constraints have been considered. Previous results obtained by using optimal feedback control theory pointed that for linear harvesters with monochromatic excitation input there is an optimal energy harvesting circuit which consists of a discontinuously-conducting buck-boost converter driven at constant duty cycle, in combination with a passive linear reactance [34]. The harvester oscillator is described in [9] as a system whose dynamics is driven by the external vibration source and the electromechanical coupling force. In addition, a control force is added. Also, a control voltage is added to the voltage created by the transducer. The optimal operation of these two controls is treated in a generic way. Optimal control with unconstrained and constrained proof mass displacement has been treated in [32].

Now, we present the objectives of this study. One of the ways of improving the performance of piezoelectric energy harvester is by controlling their operation and this has been treated by several authors. As seen, a diversity of particular cases has been studied. A large amount of details may be found in literature but our knowledge is not yet consolidated. For this, we would need to extract what is essential from the accumulated information. Therefore, there is a need for a more systematic description of the control problem and, when the results are presented, for abstracting some rules of rather general validity. This study has two main goals. First, to describe the problem of the energy harvesting systems control in a general way, covering most of previously treated particular cases. This goal is covered in the present part 1 of the paper. Second, to extract some common patters from the multitude of control strategies used when treating several particular cases. This goal is covered in the next parts 2 and 3 of the paper.

Here we focus on piezoelectric energy harvesters whose electrical load consists of a variable resistor or a capacitor whose operation is optimally controlled. The objective is to maximize the energy extracted. We are using an original approach based on powerful direct optimal control techniques. This approach is more versatile than the indirect approach.

2. Controlling energy harvester operation

A generic architecture of energy harvesters is described in [9] where the key challenges are discussed. It consists of a mechanical domain an electrical domain. A controller drives mechanical and electrical actuators to increase the energy transfer in the harvester. Wireless microsensor node networks became a field of intense research in the last decades. A node usually consists of a sensor, a transceiver and associated electronics [8]. Electric interface circuits are necessary since the voltage at transducer output should be made compatible with the electric load or energy storage element in order to allow energy transfer [4].

A large diversity of particular configurations of vibration energy harvesters has been proposed and analyzed. An oscillator coupled with an electromagnetic transducer which is connected to electronic circuits and energy storage is described and analyzed in [36]. The electromagnetic transducer is based on linear-to-rotation motion conversion. Since the electronics must inject as well as extract power an H-bridge is used. The tracking of the current command signal is accomplished through high-frequency pulsewidth modulation switching control of four MOSFETs in connection with a proportional-integral controller (for details see [36]). The energy harvester considered in [1] consists of a flexible mechanical structure and embedded transducers. The input consists of a time-dependent acceleration while the output consists of currents and voltages at transducer terminals, which are received by a power-electronic network allowing power to be delivered to a power bus. An adaptive active piezoelectric harvester is proposed and analyzed in [4]. The harvester consists of a cantilever beam with an attached inertial mass. The voltage feedback to the piezoelectric element is defined as the control in [4]. A real-time resonant frequency tuning system was proposed in [37] and a microcontroller was implemented on a wireless sensor. The average harvesting power output increased up tot 30% under random frequency excitation. Mechanical tuning techniques have been reviewed, classified and compared [38]. Control by active power-electronics has been considered in [34].

Three type of energy harvesting electronic circuits are commonly in usage [4]: (i) the passive diode-rectifier circuit (which is the simplest but has lower efficiency), (ii) the semi-active circuit (where the output voltage may be processed to increase its magnitude and change its phase in order to maximize the output power) and (iii) the active circuit (where appropriate electrical boundary conditions are applied to the piezoelectric element to maximize the effectiveness of the harvester). Several particular topologies of using a rectifier may be considered. They include the classic interface circuit, the synchronous electric charge extraction and the synchronized switch harvesting on inductor [4].

Distributed-transducers configurations are expected to be more effective than single-transducer schemes since random vibrations may cause resonance in different modes and generate charge in the piezoelectric patches and appropriate power-electronic control may allow energy collection from these patches (for details see [1]). However, the control of several transducers implies proper control of each transducer and this elementary approach is considered here.

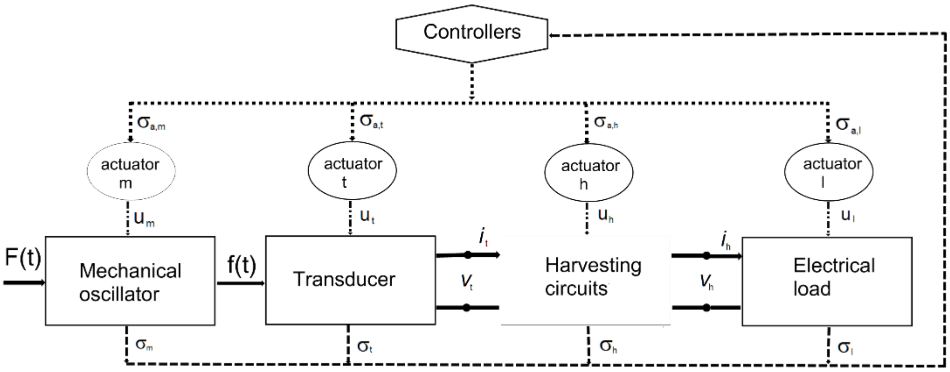

Fig. 1. General scheme of energy harvesting. F(t) and f(t) – vibration source force and state of oscillator ; v_t and i_t – voltage and current from transducer; v_h and i_h – voltage and current at harvesting circuits output. $\sigma_{a,i}$ (i=m,t,h,l) – control signals to various actuators; σ_j (j=m,t,h,l) – signals from system components to the controllers; u_k (k=m,t,h,l) – input from various actuators to system components. m,t,h,l denote mechanical oscillator, transducer, harvesting circuits and load, respectively.

A general scheme of energy harvesting system covering many of the existing schemes is proposed here (see Fig. 1). It consists of two parts: a supporting part for the energy flow and a control part. The first part consists of four components, namely a mechanical oscillator, a transducer, harvesting circuits and an electrical load. The second part consists of controllers which are not usually analyzed in the context of the energy harvesting system.

The mechanical oscillator is in contact with a source of vibration energy. The input force $F(t)$ from the energy source is acting on the mechanical oscillator and changes its state $f(t)$. Mechanical energy is transferred from the oscillator to the transducer where it is converted into electrical energy. The transducer output consists of the current i_t and the voltage v_t. This output is not always compatible with the electrical load characteristics. Therefore, the interface between the transducer and the load is ensured by harvesting circuits, whose output consists of the voltage v_h and the current i_h which is delivered to the load.

The controllers are generically able to drive all sections of the first part of the system. They receive the signals σ_j (j=m,t,h,l) from mechanical oscillator, transducer, harvesting circuits and load, respectively, and send signals $\sigma_{a,i}$ (i=m,t,h,l) to various actuators, which, in turn, drive the four components of the system in an active, passive or hybrid fashion. The controllers are designed to improve the effectiveness of harvester components. In practice, controllers are usually driving only one part of the four components, for instance the mechanical oscillator, the transducer, the harvesting circuits or the load. Here the objective is to maximize the harvested energy by the optimal control of the load.

3. Optimal control procedure

Several studies on vibration energy harvesting used indirect optimal control techniques which have the advantage that may be based on a powerful, rigorous, theoretical tool (the Pontryagin Maximum Principle). Despite being usually less precise as the indirect optimal control methods, the direct methods have, however, some advantages. For instance, they do not need defining the Hamiltonian and to derive the adjoint equations, which are necessary steps when indirect methods are used. Also, they are easy to use in case of complicated switching structures coming from constraints on controls and state variables and they are more robust during the initialization phase. This explains the wide use of direct optimal control methods in industrial applications. For instance, the optimal control of energy harvesting systems has been performed in [32] by using the open-source software tool CasADi with Python interface. The ODE system has been solved with CVODES from the SUNDIALS integrator suite while the obtained optimization problem has been solved with IPOPT.

The direct optimal control methods are based on ordinary differential equations (ODE) for the state variables. The ODE systems are solved by using appropriate boundary conditions while the state variables and controls are subjected to several constraints coming from the nature of the physical problem or from several space and time restrictions.

Here the optimal control problem (OCP) is solved in several steps, which are shortly explained in the following. First, the dynamics of the energy harvester is described in terms of ODEs for the state variables and controls. Next, the objective function is proposed. The objective function has to be extremized and the ODEs constitute constraints during the extremization procedure. At this stage the OCP is infinite dimensional since it involves functionals. Next, the state and control variables, as well as the dynamics equations, are discretized in the space of the independent variable. This way, the infinite dimensional OCP is transformed into a finite dimensional non-linear problem (NLP). This process is performed here by using the BOCOP package [39]. Further details on direct transcription methods and NLP optimization algorithms are given in [40,41].The IPOPT package performs the optimization outside BOCOP, which constitutes the interface for other packages written in different programming languages (MUMPS for linear algebra procedures, ADOL-C for automatic differentiation and COLPACK for Graph Coloring Algorithm Package).

A few practical aspects follow. We used three discretization procedures: Euler (implicit, first stage, order 1), Midpoint (implicit, first stage, order 1) and Gauss (implicit, second stage, order 4). Midpoint discretization usually provides more precise results. By default, the objective function is minimized by BOCOP. When maximization of a specific objective function is needed, a new objective function is defined, which is the negative of the default objective function. The convergence of the optimization algorithm is slower or faster, depending on the initial guess distributions of state variables and control. These distributions are usually found by trial procedures.

4. Vibration energy harvesting system

Several systems of vibration energy harvesting have been proposed and analyzed in literature. The case of an energy harvesting system with the harmonic excitation force applied directly to the central mass has been treated in [42]. However, the harmonic excitation is applied in most cases to the base (see e.g. [29]). Other cases of base-excited piezoelectric energy harvesting systems have been also considered (see e.g. [33],[31],[5],[8],[26]).

The base-excited energy harvesting system proposed in [5], which is simple and shows all characteristics of more complex systems, is considered here. It consists of a base and a proof mass M with a transducer placed between (Fig. 2). The transducer consists of a piezoelectric element and two electrodes. The thickness of the undistorted piezoelectric element is t_p and one electrode thickness is t_e. It has been shown that that neglecting the electrodes and the protective layers may affect the accuracy of the model [43]. Therefore, we include the thickness t_e in our model.

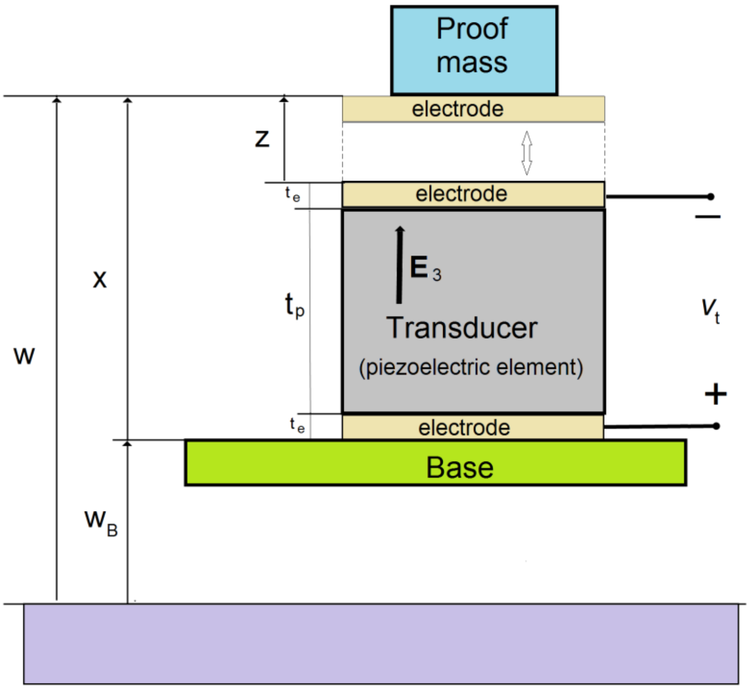

Fig. 2. Piezoelectric energy harvester with base excitation.

A basic 1-D accelerometer-type closed-form model is proposed in this paper. Its core has been presented in [5] and was used as a framework in other papers since it has all characteristics of more involved 2-D models [8]. The main assumption is the dynamics of the structure is not affected by the dynamics of the harvester. This makes sense since the harvester mass is much smaller than that of the structure. Moreover, most of the kinetic energy of the structure is carried by its first structural mode (the dominant mode). Therefore, the structure is a sort of low-pass filter between the excitation input and the harvester [9].

The displacement of the base with respect with a (fix) reference system is described by the variable $w_B(\tau)$ where τ denotes time. The displacement of the top of the transducer in respect with the base and the fix reference system is described the variables x and w, respectively. The variable z defines the displacement of the top of the transducer with respect to its un-deformed position. The variables z and x are related by:

$$z = x - t_p - 2t_e \tag{1}$$

The thickness of the piezoelectric element in un-deformed and deformed state is t_p and $x - 2t_e = \left(z + t_p\right)$, respectively.

The stress T_3 and the electric displacement D_3 for the piezoelectric element depend on the strain S_3 and the electric field E_3 as follow [5]:

$$T_3 = c_{33}^E S_3 - e_{33} E_3 \tag{2}$$

$$D_3 = e_{33} S_3 + \varepsilon_{33}^S E_3 \tag{3}$$

where c_{33}^E is the stiffness coefficient (in short circuit conditions), ε_{33}^S is the permittivity of the piezoelectric element and e_{33} is the piezoelectric constant. Equations (2) and (3) apply for the linear elastic theory (small deformations). Usage of Fig. 1 allows to write the relationship between the electric field E_3 and the voltage v_t across the transducer:

$$E_3 = -\frac{v_t}{x - 2t_e} = -\frac{v_t}{z + t_p} \tag{4a,b}$$

The piezoelectric element deformation is very small as compared with its un-deformed thickness ($z << t_p$). Therefore, Eq. (4b) is well approximated by:

$$E_3 \cong -\frac{v_t}{t_p} \tag{4c}$$

The strain S_3 is defined in respect with the un-deformed thickness t_p of the piezoelectric element:

$$S_3 = \frac{x - 2t_e - t_p}{t_p} = \frac{z}{t_p} \qquad \text{(5a,b)}$$

One denotes by A_p the cross-section surface area of the piezoelectric element and by q_t the electric charge separated on the extremities of the piezoelectric element. Therefore, the electric displacement is simply defined as the charge on the electrodes per unit area:

$$D_3 = \frac{q_t}{A_p} \qquad \text{(6)}$$

The base vibration is transmitted to the proof mass which vibrates under specific frequencies. The piezoelectric element, of mass M_p, is also vibrating. The usual approach is the treat only the vibration of the proof mass. However, an effective proof mass M_T is considered to take into account the influence of piezoelectric element vibration. When longitudinal vibrations are considered, lumping one third of the mass of the rod is commonly assumed for resonance frequency estimation (see Table 1)[5]:

$$M_T = M + \frac{1}{3} M_p \qquad \text{(7)}$$

The stress in the piezoelectric element is given by the ratio between the interaction force with the proof mass and the cross section surface area of the element:

$$T_3 = -\frac{M_T \ddot{w}}{A_p} = -\frac{M_T\left(\ddot{w}_B + \ddot{x}\right)}{A_p} = -\frac{M_T\left(\ddot{w}_B + \ddot{z}\right)}{A_p} \qquad \text{(8a,b,c)}$$

Equation (1) has been taken into account when Eq. (8c) has been written. In Eq. (8b) $\ddot{w}_B$ is base acceleration. Harmonic base displacement is assumed here, i.e:

$$w_B(\tau) = \overline{w}_B \sin(2\pi f \tau) \qquad \text{(9)}$$

where $\overline{w}_B$ and f denote the magnitude and frequency of the base excitation. Therefore, the time dependent base acceleration is given by:

$$\ddot{w}_B(\tau) = a \sin(2\pi f \tau) \qquad \text{(10)}$$

where the magnitude a of the base acceleration is given by:

$$a = (2\pi f)^2 \qquad \text{(11)}$$

Usage of Eqs. (2),(3), (6) and (8c) yield:

$$-\frac{M_T\left(\ddot{w}_B + \ddot{z}\right)}{A_p} = \frac{C_{33}^E}{t_p} z + \frac{e_{33}}{t_p} v_t \qquad \text{(12)}$$

$$\frac{q_t}{A_p} = \frac{e_{33}}{t_p} z - \frac{\varepsilon_{33}^S}{t_p} v_t \qquad \text{(13)}$$

The notation of Table 1 is adopted and the viscous damping force $2\zeta_m \omega_N \dot{z}$ is added in Eq. (12) , where ζ_m is the mechanical damping ratio.

Table 1. Notation adopted.

Quantity	Notation
Electromechanical coupling coefficient	$\theta \equiv -\frac{e_{33} A_p}{t_p}$
Capacitance of the piezoelectric element	$C_p \equiv \frac{\varepsilon_{33}^S A_p}{t_p}$
Effective stiffness	$K \equiv \frac{c_{33}^E A_p}{t_p}$
Proof mass resonance frequency	$\omega_N \equiv \sqrt{\frac{K}{M_T}}$

Then, Eq. (12) becomes:

$$\ddot{z} + 2\zeta_m \omega_N \dot{z} + \omega_N^2 z - \frac{\theta}{M_T} v_t = -\ddot{w}_B \tag{14}$$

Usage of Eq. (13) and notation of Table 1 yield:

$$q_t + \theta z + C_p v_t = 0 \tag{15}$$

Notice that the components of Fig. 2 are fully electromechanically coupled while some energy harvesters have part of their structure inactive.

5.Conclusions

Energy harvesting systems may be controlled in different ways. A general approach is shortly proposed in section 2 of this paper. Different types of energy harvesting systems have been treated. Here we focus on the base-excited system with a particular case very often studied in literature [5]. Applications of the present part 1 of the paper are presented in two following parts 2 [44] and 3 [45], where direct optimal control techniques are used in a systematic way.

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