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On unstable modes in graph-type circuits

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Abstract. The aim of this communication is to discuss the dynamics of a couple of graph-type circuits that exhibit one or more unstable modes. Examples of finding the sign of un unstable mode as well as mode competition are presented.

Keywords: graph-type circuits, unstable modes, mode competition.

1. Introduction

The concept of pattern is used in many domains with various significances. In this paper it is associated to architectures in which, for linear behavior, unstable modes exist and their development lead to a stable equilibrium point if the signals increase is limited by nonlinearities. Alternatively, the transient can be frozen by appropriate switches, the pattern being the final state of the network. Examples of pattern are the so-called Turing patterns [1] which have been initially introduced in order to model the spots on plants and animals. Analysis and circuit implementations have been reported in many papers among which we cite here only [2,3]. The main idea of the architectures able to exhibit Turing patters was the use of identical second order two-port cells sandwiched between two homogeneous resistive grids responsible for the reaction-diffusion interaction between cells. In the linear part of the cell resistive characteristic the equations can be linearized and, due to the homogeneities both of the cells and grid, using a change of variable mode decoupling can be obtained and analytical solutions are possible. The fundamental fact that allows mode decoupling is the existence of orthogonal spatial modes which are dependent on the boundary conditions [4]. Let us observe that the essence of the architecture able to produce Turing patterns is on the one hand that

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the cells, identical, are second order and the neighborhood generated by the homogeneous grid are first order, i.e., each cell is connected only with its nearest neighbors. In this way, it is possible to generate a band-pass region in the dispersion curve for the (harmonic) spatial modes.

Another type of architecture is the so-called homogeneous Y(s) configuration which consists of identical admittances grounded on one end and connected through controlled sources in a neighborhood at the other end. This time, the cells can be of any order including order one (a capacitor) and the neighborhood can be of any dimension, for capacitors and second order neighborhood, band-pass behavior for the harmonic spatial modes being possible. Moreover, in this case, any type of spatial frequency behaviors can be obtained, contrary to the case of Turing patterns. Results concerning applications of this behavior have been reported, among others, in [5-7]. The above architecture could also be analytically investigated using mode decoupling due to the symmetric templates used for interconnections, which implied the existence of an orthogonal set of eigenvectors associated with real eigenvalues.

A further generalization of the network discussed above has been proposed in [8] in relation to the concept of graph-type network. In the following we will develop the subject both with theoretical considerations as well as simulation examples.

2. The Y-type architecture

This type of architecture consists of identical grounded admittances Y(s) connected with the “hot” end to the nodes of a resistive undirected graph. The advantage of working with an undirected graph is that the eigenvectors continue to be orthogonal and the eigenvalues are real [9,10]. Inspired by the work of [11,12] concerning negative weight branches that ensure consensus, we discuss the case when negative resistances are allowed and, moreover, they could be designed such that one or more spatial modes be unstable. Another aspect with potential applications is that of designing graph-type circuits with imposed unstable eigenvectors. In what follows we will briefly present the main formula deduced in [10] for a circuit with the topology described above and for which we allow resistances in parallel with the admittances. In this case, the nodal equations corresponding to the extended graph (symmetric) matrix

$$[G] = \begin{bmatrix} G_{11} & G_{12} & \dots G_{1M} \\ \vdots & \vdots & \vdots \\ G_{M1} & G_{M2} & \dots G_{MM} \end{bmatrix}$$

are

$$Y(s)[V(s)] + [G][V(s)] = [J(s)]$$

where [J(s)] and [V(s)] are column vectors that correspond to the equivalent nodal currents due to the initial condition and nodal voltages respectively (in the Laplace domain). With the notation $[J(s)] = [X][A(s)]$ where [X] is the MXN matrix of initial conditions, [A (s)] of order 1xM transforms the initial conditions into currents sources (M=number of nodes and N= degree of Y(s)) the equations become

$$Y(s)[V(s)]+[G][V(s)]=[X][A(s)].$$

With a change of variables based on the matrix [Φ] of the orthogonal matrix of the eigenvectors we replace in the above equations $[V]=[\Phi]\left[\hat{V}\right]$ and $[X]=[\Phi]\left[\hat{X}\right]$ and get the differential equation in the spectral variables

$$Y(s)[\Phi]\left[\hat{V}(s)\right]+[G][\Phi]\left[\hat{V}(s)\right]=[\Phi]\left[\hat{X}\right][A(s)]$$

which becomes

$$Y(s)\left[\hat{V}(s)\right]+[\lambda]\left[\hat{V}(s)\right]=\left[\hat{X}\right][A(s)]$$

where the diagonal matrix $[\lambda]$ contains the eigenvalues. The spectrum component of each spatial mode m satisfies the differential equation

$$Y(s)\hat{v}_m(s)+\lambda_m\hat{v}_m(s)=[\hat{x}_{m,1},\ldots,\hat{x}_{m,N}][A(s)]$$

or, for $Y(s)=\frac{Q(s)}{P(s)}$,

$$Q(s)\hat{v}_m(s)+\lambda_m P(s)\hat{v}_m(s)=P(s)[\hat{x}_{m,1},\ldots,\hat{x}_{m,N}][A(s)]$$

which shows that the characteristic equation for each spatial mode is $Q(s)+\lambda_m P(s)=0$, the stability of each mode being determined by the roots of this equation as shown in [10]. Let us observe that for the particular cases of Y(s)=s (Q(s)=s, P(s)=1), $\lambda_0=0$ and $\lambda_m<0$ for m=1,…,M-1 corresponds to consensus while $Y(s)=\frac{s^2+1}{s}\,(Q(s)=s^2+1,\quad P(s)=s)$ with the same constraints for the eigenvalues lead to synchronization. In both cases Y(s) are reactance functions with singularities on the imaginary axes so that a steady state solution, constant or oscillating exists as shown in [13].

3. Y-type architecture with imposed eigenvectors and eigenvalues

In what follows we discuss the possibility of designing Y-type architectures with imposed orthogonal eigenvectors and eigenvalues. A tool to generate such matrices can be found in [14]. Once the matrix is generated it is simple to associate to that matrix an Y-type circuit with resistances/conductances so chosen that the nodal equations correspond to the imposed resistive matrix. Contrary to previous papers on this subject, in the following we will consider a small Y-type architecture exhibiting two unstable modes with the same eigenvalue so that mode competition will be possible. The simulations have been done with capacitors and ideal positive and negative resistances. Such resistances can be sufficiently accurately actively simulated using CMOS operational transconductances; the results being sufficiently robust if we invoke Ostrowski's theorem that ensure continuity of the roots of a (characteristic) polynomial with respect to its coefficients. We decided to design an Y-type circuit based on capacitors (Y(s)=Cs) exhibiting two modes equal to 2 and two other modes equal to -2. To do this we used [14] to get a matrix whose (orthogonal) eigenvectors are on the lines of the array below each of then having on the left the corresponding eigenvalues.

-2	1	-1	1	-1
2	1	1	1	1
2	-1	-1	1	1
-2	1	-1	-1	1

The G matrix with the above eigenvectors is

$$[G] = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

With this matrix, the circuit described by the equations $s[V(s)] + [G][V(s)] = [V(0)]$ with C=10pF and conductanced coresponding to resistances of ±50KΩ has the eigenvalues and eigenvectors below

Table 1

Mode	Circuit eigenvalue	Eigenvector
1	2	1 -1 1 -1
2	-2	1 1 1 1
3	-2	-1 -1 1 1
4	2	1 -1 -1 1

is presented in Fig.1. Antiparallel diodes have been added in order to limit voltages on nodes.

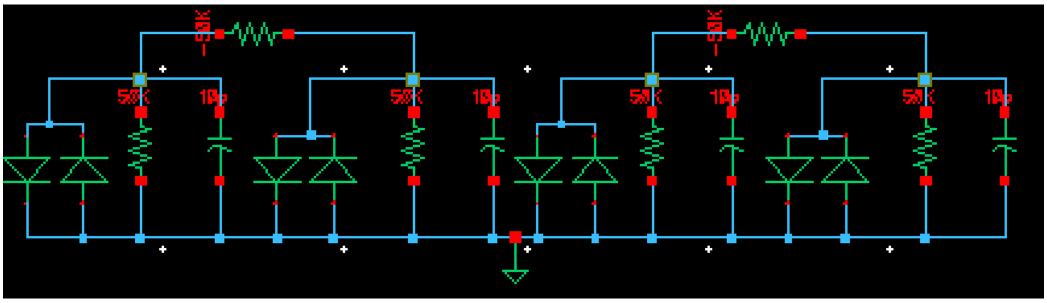

Fig. 1 Circuit with eigenvectors and eigenvalues listed in Table 1.

In what follows we will show several transient responses simulated with Cadence. The comments are in the figure captions.

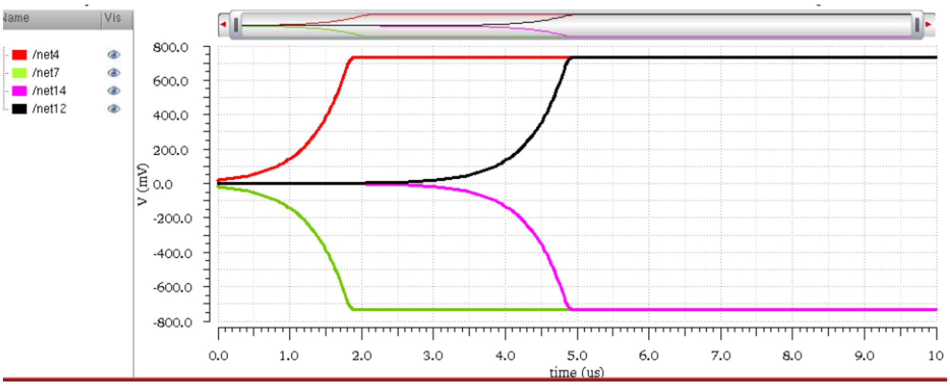

Fig. 2. Initial conditions: (20mV, -20mV, -100uV, 0mV) i.e., Sum of unstable Modes 1 and 4 with a perturbation of -100uV in favor of mode 4 who wins.

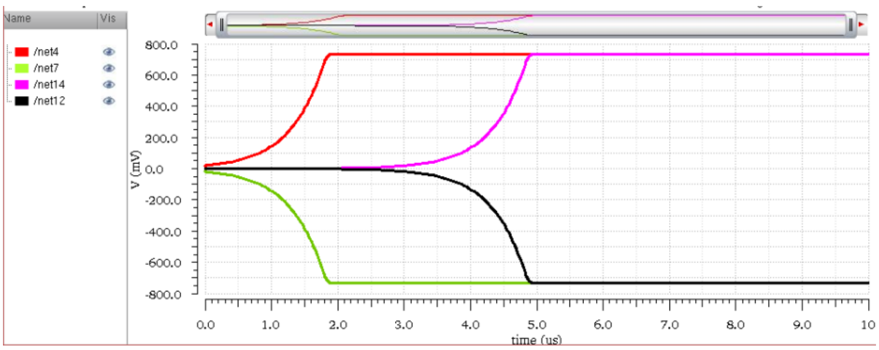

Fig. 3. Initial conditions: (20mV, -20mV, 100uV, 0mV) i.e., Sum of unstable Modes 1 and 4 with a perturbation of 100uV in favor of mode 1 who wins.

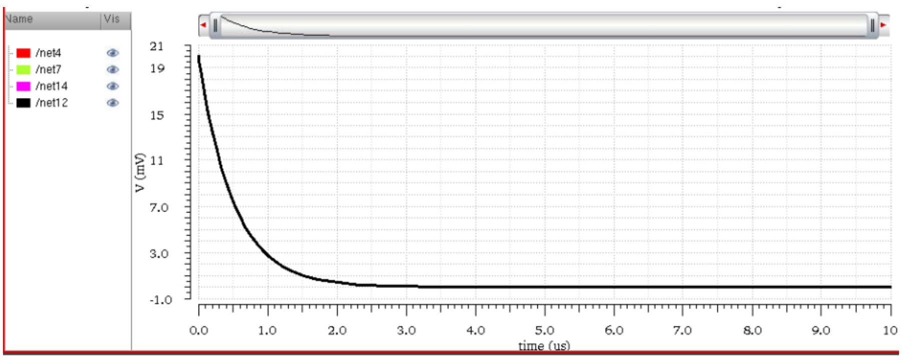

Fig. 4. Initial conditions: (20mV, 20mV, 20mV, 20mV) i.e., Mode 2 (stable).

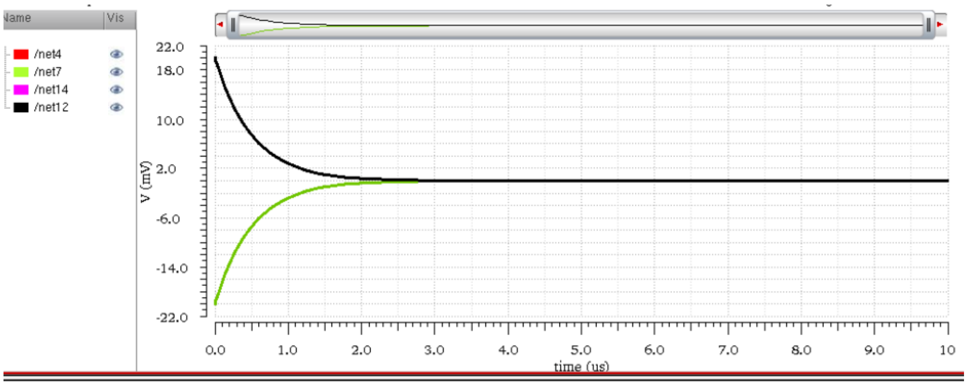

Fig. 5. Initial conditions: (-20mV, -20mV, 20mV, 20mV) i.e., Mode 3 (stable).

Below we seed the sum of two unstable modes with identical strength.

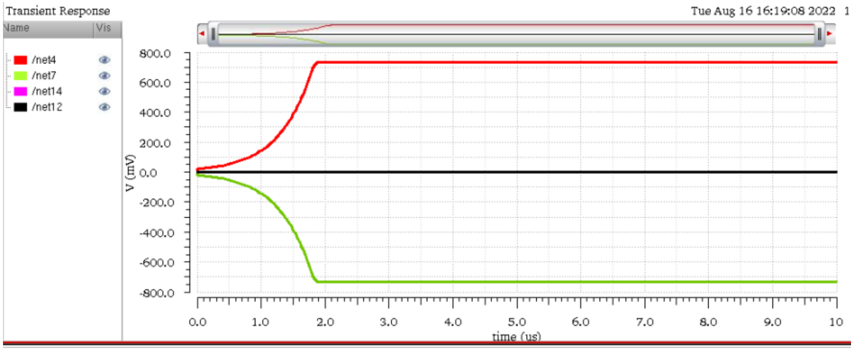

Fig. 6. Initial conditions: (20mV, -20mV, 0mV, 0mV), i.e., Mode 1 (10mV, -10mV, 10mV, -10mV, unstable) + Mode 4 (10mV, -10uV, -10mV, 10mV, unstable). The unstable modes are theoretically of equal strength so their sum wins.

However, if the strength of the unstable modes is not the same, the strongest wins as seen in Fig.7.

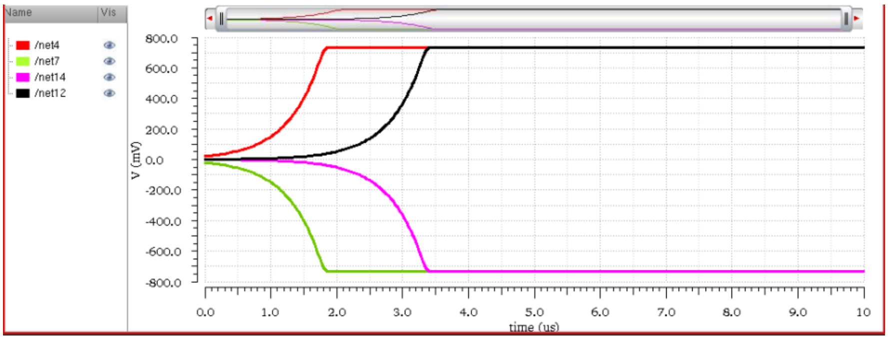

Fig. 7. Initial conditions: (21mV, -21mV, -1mV, 1mV), i.e., Mode 1 (10mV, -10mV, 10mV, -10mV, un stable) + Mode 4 (11mV, -11mV, -11mV, 11mV, unstable). The strongest unstable mode (Mode 4) wins with appropriate polarity (mode competition).

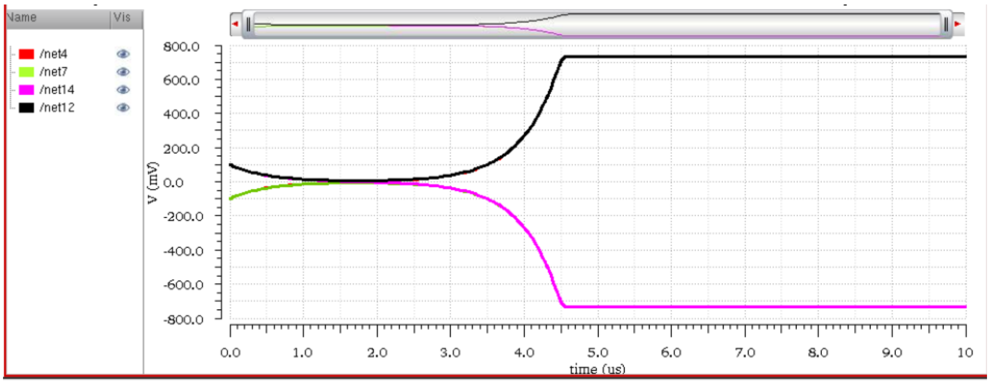

Fig. 8. Initial conditions: (-99.9mV, -100.1mV, 99.9mV, 100.1mV), i.e., Mode 3 (-100mV, -100mV, 100mV, 100mV, stable) + Mode 4 (100uV, -100uV, -100uV, 100uV, unstable). The unstable mode wins with appropriate polarity.

4. Concluding remarks

We have reviewed the mechanism of generating patterns in certain architectures i.e., the existence of unstable modes in the linearized model an array exhibiting certain homogeneities. Such an architecture is the so-called Y-type graph circuits with undirected branches and identical grounded admittances. The main mechanism to analyze the circuit behavior is mode decoupling based on the orthogonality of the eigenvectors of the network. The new aspect discussed in this communication is to suggest applications based on the possibility of designing circuits with imposed eigenvalues and orthogonal eigenvectors. It is possible to use such architectures in order to detect the sign of a “hidden” mode or to use mode competition to detect minor differences in the unstable modes found in the initial conditions.

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