Using CoMIFs in structural modal testing

MIRCEA RADEŞ*

Politehnica University of Bucharest, Splaiul Independenţei 313, Bucharest, Romania

Abstract: This paper reports experience in using two Componentwise Mode Indicator Functions to locate the modes of vibration of a real structure with sufficiently separated frequencies. The advantages of using selected sets of measurement points and different limited frequency bands are emphasized.

Keywords: Mode Indicator Functions, CoMIF, QCoMIF.

1. Introduction

In almost all structural testing procedures a decision has to be made as the number of modes that should be included in the analysis. Mode Indicator Functions (MIFs) are used to estimate the number of resonant modes active in given frequency band. MIFs are real-valued frequency-dependent scalars that exhibit peaks or dips at the natural frequencies of a structure.

Componentwise MIFs work on compound matrices encompassing all available complex Frequency Response Functions (FRFs), whose columns contain the values for each response/reference combination at all frequencies.

When the Compound FRF (CFRF) matrix is subjected to a Singular Value Decomposition (SVD), it is expressed as a sum of rank-one matrices, referred to as *principal components*. Each principal component matrix can be expressed as the product of a singular value, times a left singular vector (LSV) and the conjugate transpose of the corresponding right singular vector. The left singular vectors are linear combinations of the measured FRFs and have peaks at the natural frequencies.

The Componentwise Mode Indicator Function (CoMIF) is a plot of the diagonal elements of the projector onto the orthogonal complement of the subspace of the corresponding LSVs, i.e. on the plane perpendicular to the respective LSVs,

---

*Correspondence address: mircearades@gmail.com

against frequency. The component curves have one or more local minima at a natural damped frequency.

mircearades@gmail.com

Each CoMIF curve can be computed as the difference between a column vector of ones and the Hadamard product of the LSVs [1]

$$CoMIF_i = [1] - u_i \otimes u_i^*, \qquad (1)$$

where the star superscript denotes the complex conjugate and $\otimes$ denotes element-by-element vector product.

Alternatively, the CFRF matrix can be subjected to a QR or QLP decomposition. The Q-vectors, called Q-Response Functions, are linear combinations of the measured FRFs. They are orthogonal, hence independent, so that the minimization of the norm of the projection onto the complementary orthogonal subspace can be used as a criterion to locate the contributing modes.

The Q-Vector Componentwise Mode Indicator Function (QCoMIF) is based on the pivoted QLP decomposition of the CFRF matrix.

The QCoMIF is defined [2] by vectors of the form

$$QCoMIF_i = [1] - q_i \otimes q_i^*, \qquad (2)$$

where q_i are frequency dependent Q-vectors. Its plots versus frequency have dips at the damped natural frequencies.

The QCoMIF resembles the CoMIF. The QR decomposition is an orthogonal triangularization and is cheaper than the SVD which is an orthogonal diagonalization.

This paper compares the performance of these componentwise MIFs in the case of measurements made on a real structure used as a case study in the MODENT Suite of software for modal analysis developed at Imperial College London by ICATS [3].

2. Tested structure

The measurement data used in this paper were supplied on a diskette by the Centre of Vibration Engineering at Imperial College, London, from modal tests on the so called "1203 structure" [4]. The U-shaped structure is made of two parallel aluminum side plates connected by an end side plate and strengthened using two bars, as shown in Fig.1.

3. FRF measurements

The experimental data-base consists of 71 complex-valued FRFs measured in the range 0 to 800 Hz with a resolution of 1 Hz, using roving impulse excitation. An accelerometer was placed at node 24 while each of the 71 points was subjected to a hammer impact in turn.

Selected receptance frequency response functions $FRFjk$, for displacement at coordinate j produced by excitation at coordinate k are represented as magnitude (log scale) versus frequency (linear scale) plots in Fig.2. Note that most coordinates

are along the horizontal X axis. As a result, the modal information is incomplete, modes having a relatively small contribution to the total response on this direction are less observable. Figure 3 presents plots of the real and imaginary parts and the Nyquist plot of a typical FRF.
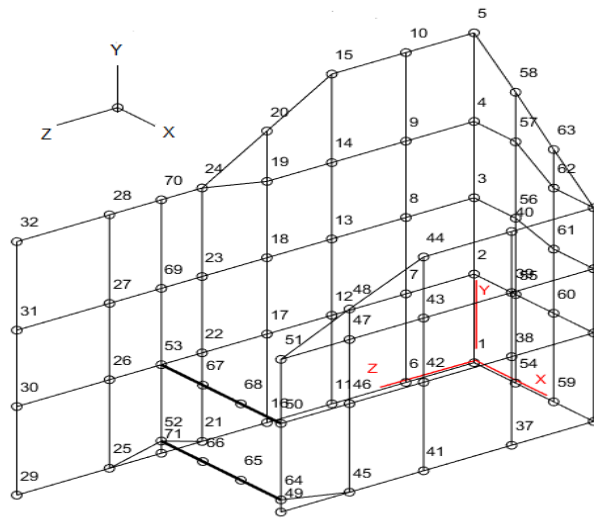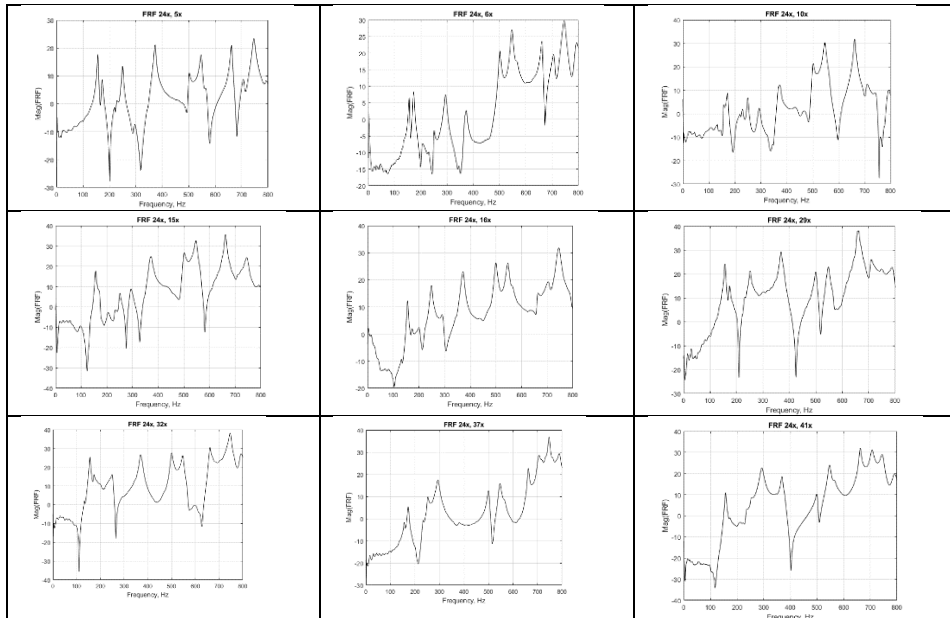

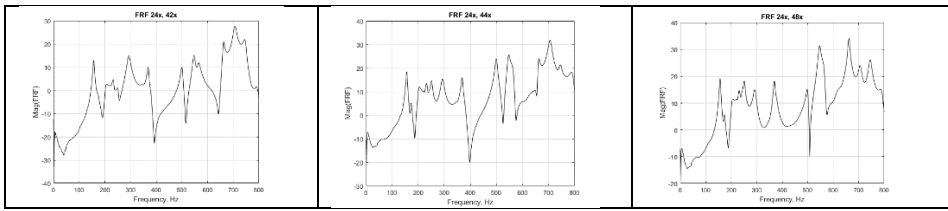Fig.1. Tested structure and measurement locations.
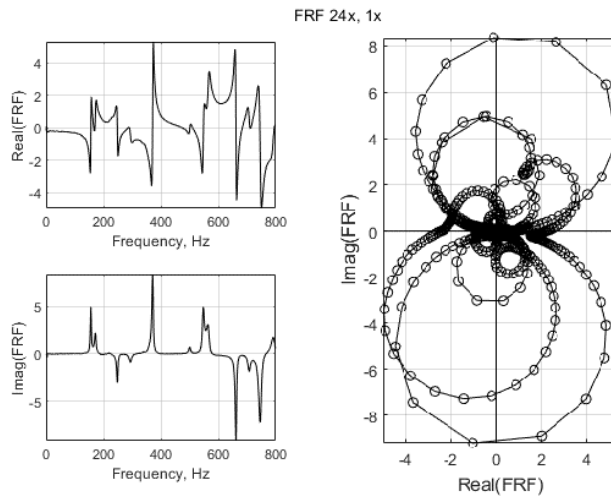
Fig. 2. FRF curves.

Fig. 3. Plots of a complex FRF.

The imaginary part of the FRF appears to be a good substitute for the complex value to pinpoint the resonance frequencies.

4. Natural frequencies and mode shapes

Two modal shapes of the tested structure are shown in Fig.4 [3].
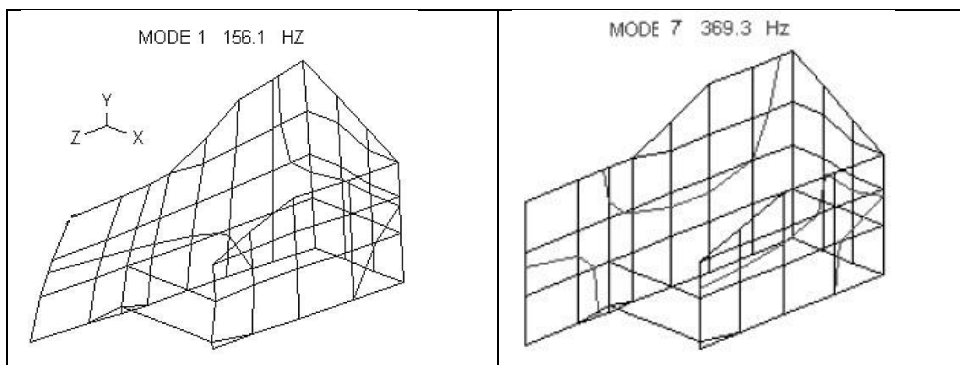

Fig. 4. Mode shapes.

The damped natural frequencies and modal damping ratios, calculated using the Global-M parameter extraction method provided in the MODENT software are given in Table 1 [3].

Table 1. Natural frequencies and damping ratios.

Mode	Natural frequency, Hz	Damping ratio, %	Mode	Natural frequency, Hz	Damping ratio, %
1	156.0	1.54	8	500.0	0.81
2	172.1	1.79	9	546.6	0.985
3	203.8	3.30	10	565.1	1.08
4	231.5	1.71	11	661.8	0.52
5	250.5	1.90	12	706.3	0.91
6	292.9	2.09	13	747.1	0.4
7	369.3	1.10	14	790.4	1.025

The original Global Method was developed by R. Fillod et al., at the University of Franche-Comté, Besançon [5, 6]. It is "based on matrices constructed as differences of receptances and mobilities, in order to eliminate the effect of the out-of-range modes assumed constant over the selected frequency range. In these matrices, the number of FRFs is overspecified with respect to the expected number of modes active in that range. Application of the SVD technique enables the determination of the genuine number of existing modes". The method is described in detail in the Ph.D. Thesis by N. Maia [7]. The Global-M version of the algorithm was developed by M. Imregun in the MODENT software released by ICATS [3]. A short presentation is given in the book by D. Ewins [8].

The aim of this paper is to document the ability of two componentwise MIFs to locate the modes of a real structure with sufficiently separated natural frequencies. Their performance depends on the right selection of the number of components.

5. The CoMIF

The *Componentwise Mode Indicator Function* (CoMIF) can be presented either in a format with subplots for individual components or with overlaid curves.

In the basic formulation [1] the number of curves in the CoMIF plot was taken equal to the effective rank of the CFRF matrix, i.e. to the truncated number of its principal components. Usually, each curve has a local minimum at the natural frequency of the corresponding dominant mode.

Experience has shown that this way the CoMIF pinpoints only the modes with dominant contribution to the system dynamics in the frequency range of measurement data. Modes with less contribution might be omitted.

Indeed, figure 5 illustrates a CoMIF plot with 14 subplots computed based on all available 71 FRFs. The effective rank of the CFRF matrix, estimated from the decrease of its singular values, is $N_r = 11$ (Fig. 6).
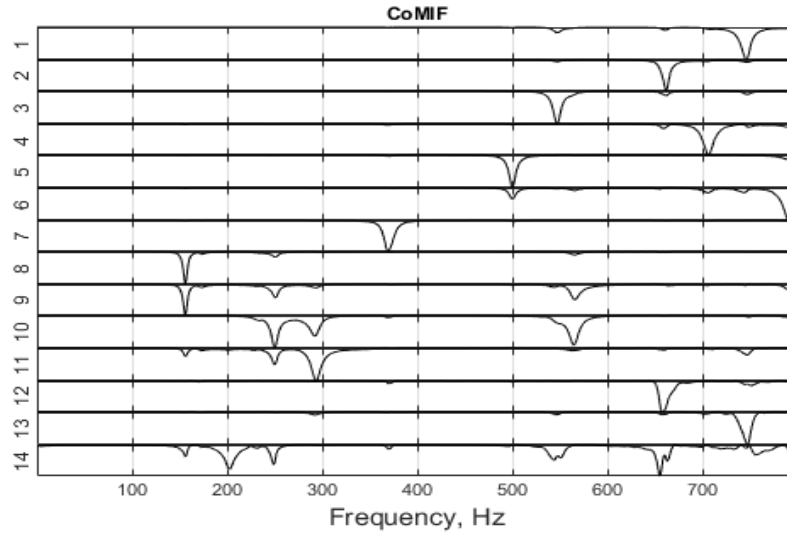

Fig. 5. CoMIF plot for the first 14 principal components of the CFRF matrix.

In the CoMIF plot (Fig.5), the first 11 subplots locate a natural frequency at the deepest minimum. Modes 2 and 4 are not revealed. So it is good practice to calculate more component curves then the effective rank of the CFRF matrix.
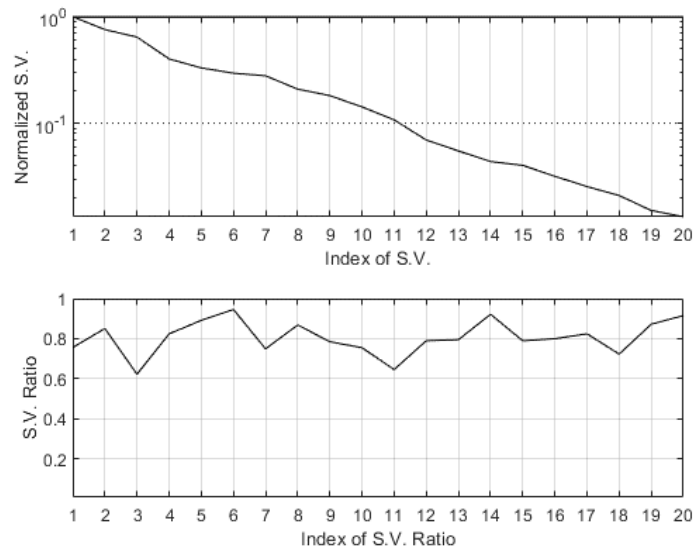

Fig. 6. Singular Values Ratio plot.

The CoMIF plot with overlaid curves is shown in Fig.7. Mode 4 is still missing. It can be used as an indicator of the natural frequencies, but can be misleading having several curves exhibiting a minimum at the same frequency even for this structure which has no repeated values. A better substitute is the Aggregate MIF [9].
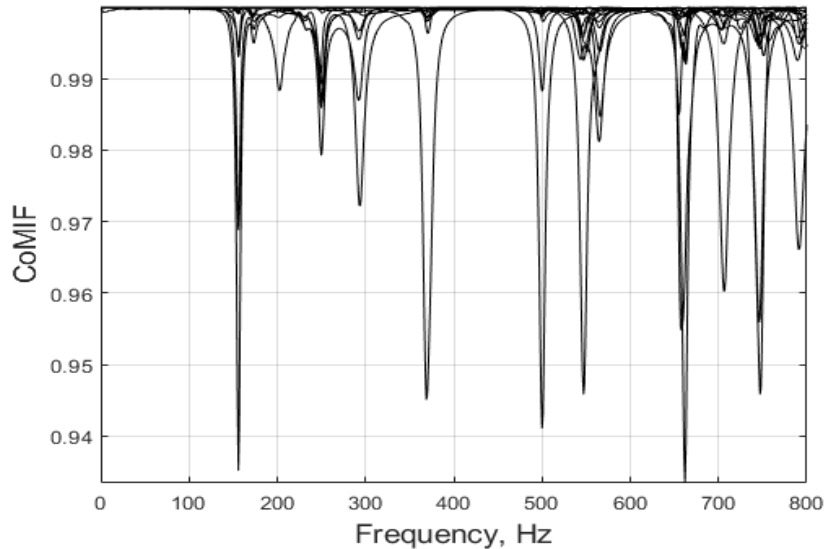

Fig. 7. CoMIF overlay based on 71 FRFs.

The only main drawback of CoMIF appears to be the arbitrary selection of the number of subplots or component curves.

To test the robustness of CoMIF, it was decided to use a smaller set of FRFs. The selection of a suboptimal reduced set of columns in the CFRF matrix was carried out using the Effective Independence (EfI) concept developed by Kammer [10]. Let

$$[P_C] = [C][C]^+ \tag{3}$$

be the orthogonal projector onto the column space of the CFRF matrix $[C]$ where + denotes the pseudoinverse. Each diagonal element P_{jj} represents the fractional contribution of the j-th FRF to the rank of $[P_C]$. Elements P_{jj} are sorted based on magnitude. The excitation location with minimum contribution to the rank of $[P_C]$, indicated by the smallest element, is removed. The matrix $[P_C]$ is then recomputed, and the process is repeated, FRFs being deleted one at a time until the prescribed number is attained. If several functions have the same P_{jj} value, then they are deleted simultaneously.

Figure 8 shows the CoMIF based on only 14 FRFs selected by EfI, ranked as

$$P_k = (70\ 41\ 48\ 6\ 37\ 44\ 15\ 42\ 5\ 32\ 29\ 71\ 10\ 16). \tag{4}$$

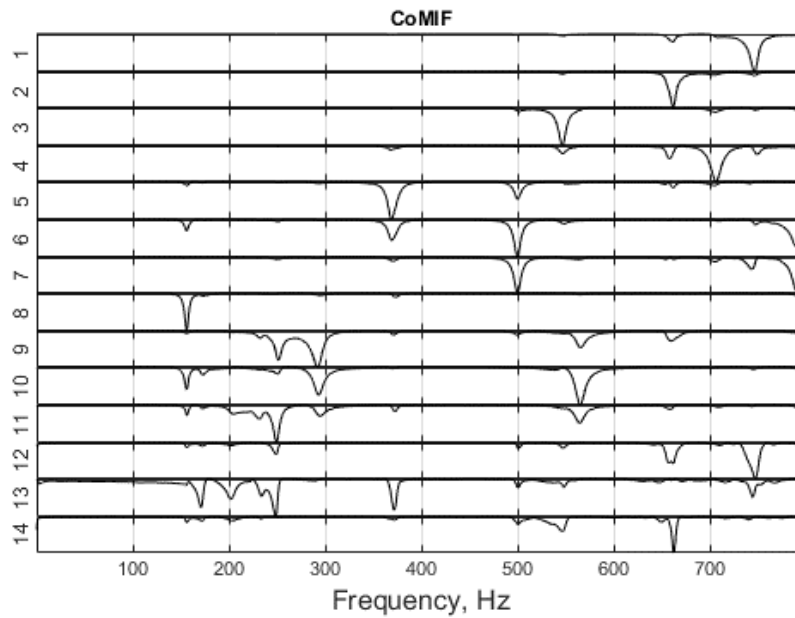As expected, the selected points are located on the upper and lower edges of the side plates.

Fig. 8. CoMIF plot based on 14 FRFs selected by EfI.

The corresponding CoMIF with overlaid curves is given in Fig.9.
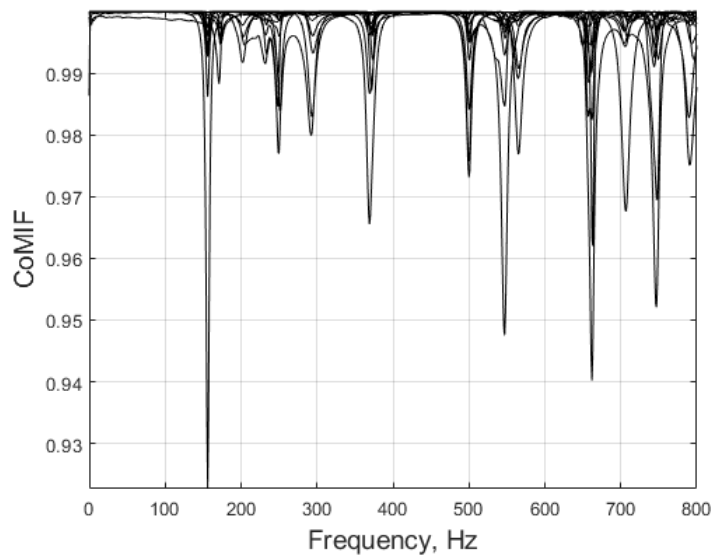Mode 2 at 172.1 and mode 4 at 231.5 are better pinpointed.

Fig. 9. CoMIF overlay based on 14 FRFs selected by EfI.

For the location of the 'weak' modes it is recommended to plot the CoMIF for restricted frequency bands. Figure 10 shows the CoMIF plot for the frequency band 100-300 Hz.
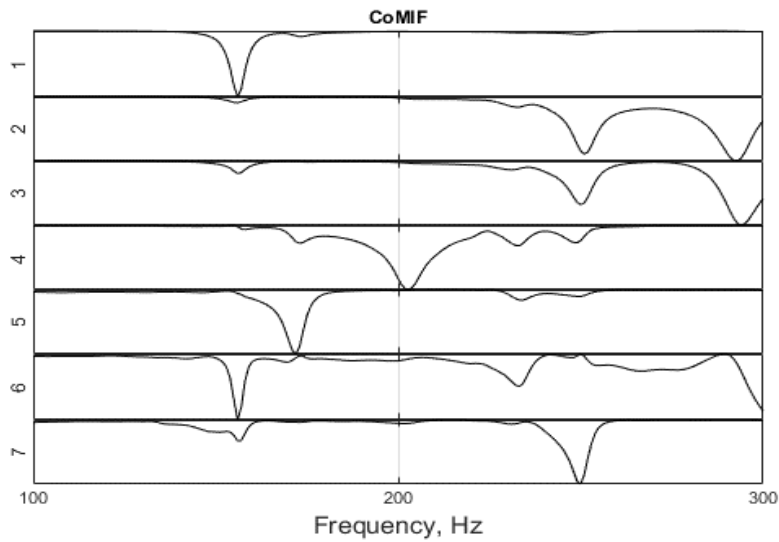

Fig. 10. CoMIF plot for 100 to 300 Hz.

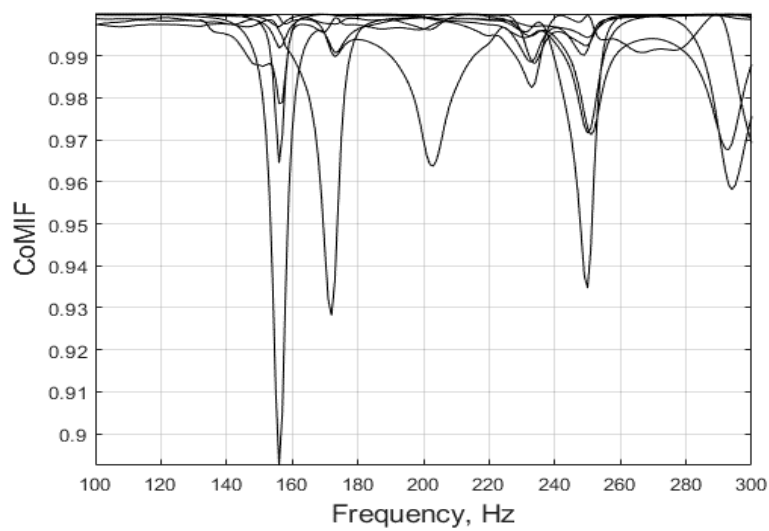The corresponding overlay plot is depicted in Fig.11.

Fig. 11. Overlay CoMIF plot for 100 to 300 Hz.

All six modes in the selected frequency interval are clearly located.

6. The QCoMIF

The *Q-Vectors Componentwise Mode Indicator Function* (QCoMIF) is constructed using *Q*-vectors obtained from the QLP decomposition with column pivoting of the measured complex CFRF matrix. They are linear combinations of the measured FRFs and orthogonal, hence independent.

In the basic formulation [2] the QCoMIF plot has as many curves (subplots) as the rank of the CFRF matrix. The rank is estimated from the decrease of the diagonal elements of the *L* matrix, which is similar to the decrease of the singular values.
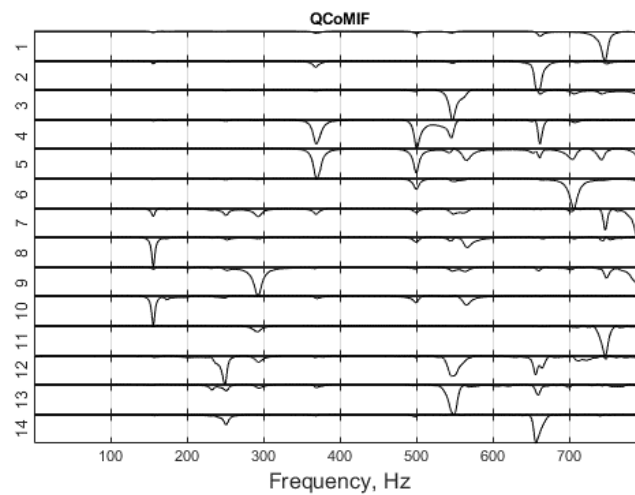

Fig. 12. QCoMIF plot based on 71 FRFs.

The QCoMIF plot from Fig.12 is based on the full CFRF matrix, with 71 columns. Again modes 2 and 4 are indiscernible.
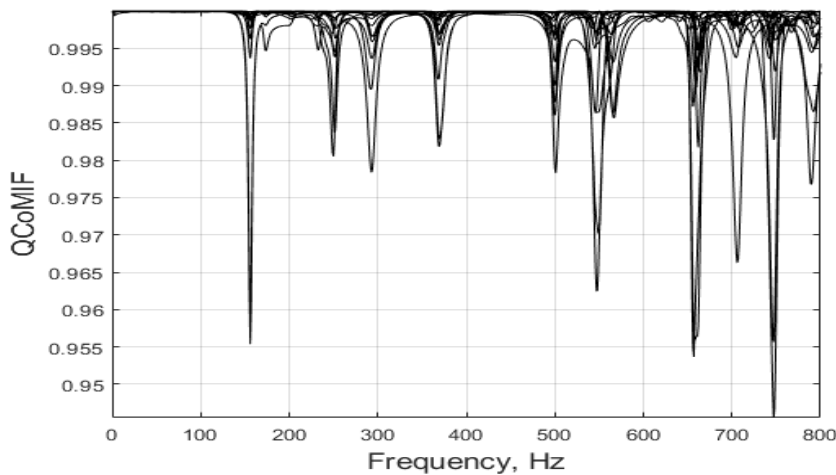

Fig. 13. Overlay QCoMIF plot based on 71 FRFs.

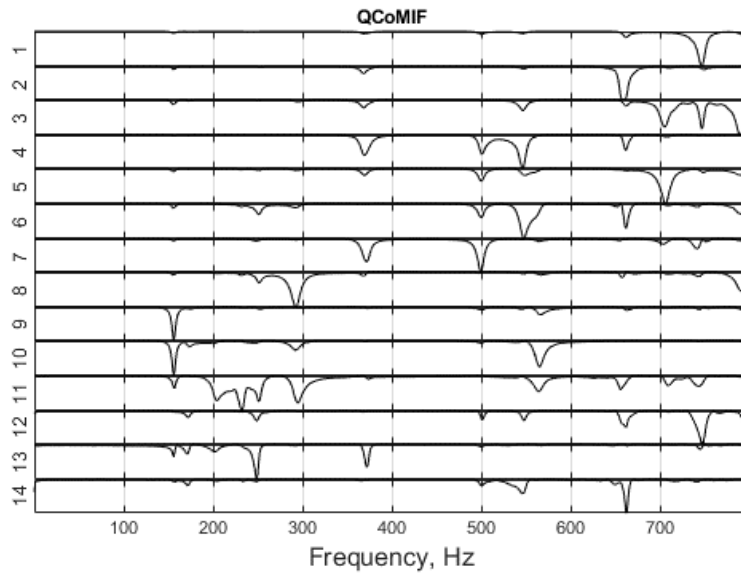The corresponding overlay plot is shown in Fig.13. It fails to locate mode 3.

Fig. 14. QCoMIF plot based on 14 FRFs selected by EfI.

The QCoMIF based on only 14 FRFs selected by EfI as in (4), is shown in Fig.14. It pinpoints all 14 modes.
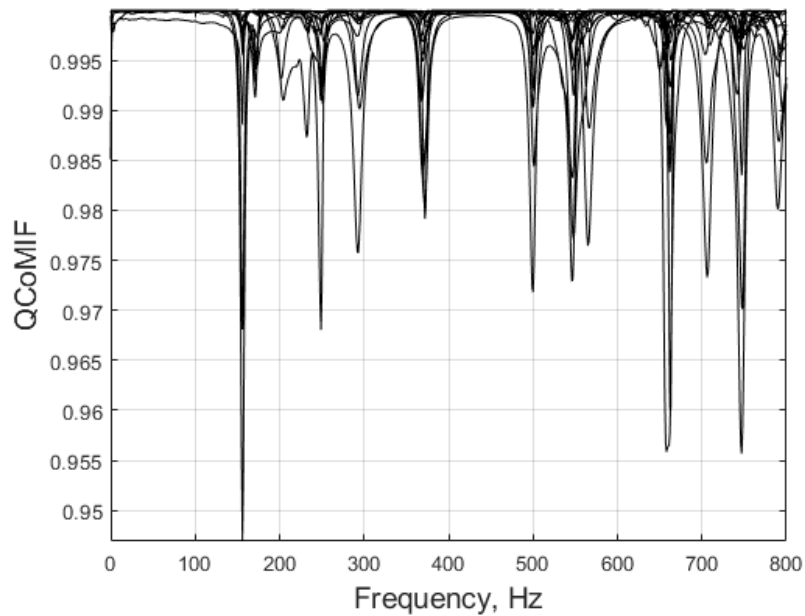

Fig. 15. Overlay QCoMIF plot based on 14 FRFs selected by EfI.

Figure 15 shows the corresponding overlay plot. All 14 modes are clearly indicated by troughs.

7. Concluding remarks

The aim of this paper was to assess the performance of two componentwise mode indicator functions in the analysis of single reference test data with roving excitation. A real structure with sufficiently separated frequencies has been selected, to avoid any misinterpretation due to existence of double modes. Measurements along a single direction produced 'weak' modes, with a relatively small contribution to the total response, hence less observable.

It turned out that, with adequate interpretation, CoMIFs are reliable tools for the determination of the vibration modes active in a given frequency band. The key point is the appropriate selection of the number of component curves (individual plots) which has to be larger or equal to the effective rank of the CFRF matrix.

Problems raised by the existence of weak modes can be overcome by an efficient selection of FRFs, hence of coordinates used in measurements. Best results have been obtained using either a reduced set of FRFs measured at points located at the upper and lower margins of the side plates of the tested structure or performing the analysis in limited frequency bands.

References

[1] Radeş M., Ewins D.J., *The Componentwise Mode Indicator Function*, Proc. 19th Int. Modal Analysis Conf., Kissimmee, Florida, Feb 2001, p. 903-908.

[2] Radeş M., Ewins D.J., *MIFs and MACs in Modal Analysis*, Proceedings of 20th IMAC Conference on Structural Dynamics, Los Angeles, California, Feb 2002, p. 771-778.

[3] A Guided Tour. The MODENT Suite of Integrated Software for Structural Dynamics, ICATS, 1988-99.

[4] MODENT Users Guide, Imperial College Analysis and Testing Software (ICATS), 1988-2008.

[5] Fillod R., Lallement G., Piranda J., Raynaud J.L., *Méthode Globale d'Identification Modale*, Mécanique, Matériaux, Électricité, Mar.-Apr. 1984.

[6] Fillod R., Lallement G., Piranda J., Raynaud J.L., *Global Method of Modal Identification*, Proc. 3rd Int. Modal Analysis Conf., Orlando, Florida, 1985, p. 1145-1151.

[7] Maia N.M.M., *Extraction of Valid Modal Properties from Measured Data in Structural Vibrations*, Ph.D. Thesis, Imperial College, London, Dec. 1988.

[8] Ewins D.J., *Modal Testing: Theory, Practice and Application*, 2nd ed., Research Studies Press Ltd., Baldock, Hertfordshire, England, 2000.

[9] Radeş M., Ewins D.J., *The Aggregate Mode Indicator Function*, Proc. 18th Int. Modal Analysis Conf., San Antonio, Texas, Feb 2000, p. 201-207.

[10] Kammer D.C., *Sensor Placement for On-Orbit Modal Identification and Correlation of Large Space Structures*, J. Guidance, Control and Dynamics, **14**, 1991, p.251-259.