Technical Sciences
Academy of Romania
www.jesi.astr.ro

Journal of Engineering Sciences and Innovation
Volume 7, Issue 1 / 2022, pp. 105-120
http://doi.org/10.56958/jesi.2022.7.1.105

H. Inter – and Transdisciplinarity in Science and Technology

Received ***30 September 2021*** *Accepted* ***7 March 2022***
Received in revised form ***14 December 2021***

Superposition and/or cumulation of loads in the case of linear behavior of matter, solved by a monodisciplinary or interdisciplinary approach

IONELA-MIHAELA ROŞU[1*] (MARIN), COSMIN JINESCU[2], ION DURBACĂ[2], IOLANDA CONSTANŢA PANAIT[2]

[1]National Environmental Protection Agency, Pollution Control and Permitting Directorate, Bucharest, Romania
[2]Politehnica University of Bucharest, Bucharest, Romania

Abstract. In the various chapters of science and especially in engineering sciences, there are many cases of superposition and/or cumulation of loads or their effects. The way one can solve such problems depends on the behavior of the material under load. The paper deals with the linear behavior of matter in which case the principle in physics, mechanics, mechanical engineering, electrical engineering, perfect gas statics, fluid flow etc. Case involving a monodisciplinary approach are separated from those involving an interdisciplinary approach to practical solutions. The interdisciplinary approach is also used to analyze the inverse problem where a certain load produces two or more effects, including the thermal one. One has found that, in the case of linear behavior, no synergistic effects can be detected. On that account, one can claim that, in the case of nonlinear behavior, solving the problems of superposition and/or cumulation of loads or their effects can now be done only by applying the principle of critical energy that allows – among other things – the introduction of the influence of matter damage, of the thermal effect, of the rate of load variation etc.

Key words: superposition/cumulation of the effects of loads, linear behavior, superposition principle, synergy, interdisciplinarity.

1. Introduction

In science, in general and especially in engineering sciences, there are numerous caspes of superposition and/or cumulation of loads. Finding a solution to these

[*] Correspondence address: marinmihaela2005@yahoo.com

loading cases asks for an interdisciplinary or transdisciplinary treatment. For this purpose, for example, one resorts to the principle of superposition.

The *principle of superposition in linear systems* was formulated by Daniel Bernoulli, in 1753: „*The general motion of a vibrating system is given by the superposition of its own vibrations*".

The principle was rejected first by Leonard Euler and then by Joseph Lagrange, being later accepted in the Joseph Fourier's work [1].

In general, these loads are different, they may be of a different nature or of the same nature but of different type. It is impossible to sum them up due to the different units of measurement.

Problems of superposition and/or cumulation of the loads applied to physical bodies are encountered in both living matter and inert matter, and in solving these problems one has to take into account the effects they can cause over time.

The physical body may be subjected to loads:

- by superposition the Y_i loads, which means the *simultaneous loading* with these loads;
- by *cumulating* the actions of the Y_i loads, which corresponds to the succesive loading with these loads;
- by *superimposing* the action of the Y_i loads and by *cumulating* the action of the loads, written Z_j.

In most chapters of science, the behavior of matter is considered linear and rendered by the general law,

$$Y = A \cdot X \tag{1}$$

wherein Y is the external load; X – the effect of the load; A – constant of the material.

The first law of linear behavior was enunciated by the English physicist Robert Hooke, in connection with the tensile stress of a body. The mathematical transposition of this statement is as follows:

$$\sigma = E \cdot \sigma \tag{2}$$

where σ is the applied mechanical stress; ε – the strain (effect); E – the modulus of elasticity of the physical body under load. It is used in *Deformable Solid Mechanics*.

Subsequently, general relation (1) were used in most chapters of science. For example, Ohm's law is used in *Electrical engineering*,

$$U = R \cdot I \tag{3}$$

where U is the voltage; R – electrical resistance; I – the intensity of the electric current.

In Fluid Mechanics one resorts to Newton's law,

$$\tau = \mu \cdot \dot{\gamma} \tag{4}$$

where τ is the shear stress; μ – viscosity of fluid; $\dot{\gamma}$ - shear strain.

The superposition and/or cumulation of the effects of several loads, in the case of linear behavior, is possible if the effects are of the same nature and are measured with the same unit of measurement.

In such cases one may resorts to the *principle of superposition*.

2. The principle of superposition in structures with linear behavior

The principle of superposition, also known as superposition property, states that, "*for a linear system the net response caused by two or more loads is equal to the sum of the responses caused by each load individually*."

In Figure 1, the Y_i loads applied to a physical body C produce their own effect (response) X_i, independently and without being affected by the other stresses.

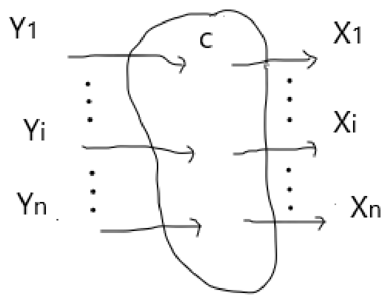

Fig. 1. Effects (X_1, ... , X_i, ... X_n) of applying loads (Y_1,..Y_i,...Y_n).

One may write that the X_i effect depends only on the Y_i load and that, in general, $X_i = f(Y_i)$.

The total answer, X_t, is obtained by algebraic summation *only if* all the X_i effects are measured with the same unit of measure, in which case,

$$X_t = \sum_{i=1}^{n} X_i \, . \tag{5}$$

In physics, the principle of "superposition" state that,

"*when two or more waves of the same type pass through the same point, the resultant of the displacement of that point is equal to the sum of the displacements produced, individually, by each wave".*

Relation (5) of the superposition principle shows that: "*the total effect is equal to the algebraic sum of the individual effects*".

This refers to effects of the same kind; the thermal effects accompanying any kind of stress are not considered.

A function F(x) that satisfies this principle is called a *linear function*. In linear systems, superposition is defined by two simple properties, *additivity* and *homogeneity*:

− additivity: $f(X_1 + X_2) = f(X_1) + f(X_2)$;

− homogeneity: $f(c \cdot X) = c \cdot f(X)$,

were *a* is a constant.

Because many physical systems can be modelled as linear systems, this principle has applications in physics and engineering. Linear systems can be easily analyzed mathematically by using, for example, the theory of linear operators, of the Fourier and Laplace transforms etc..

This superposition principle also applies to linear systems in mathematics, such as algebraic equation and systems of algebraic equation or differential equations, but also in the case of vectors, in which case superposition means the sum of vectors.

For example, in a Fourier analysis [3], the loading is considered to be a superposition of a high, infinite number of sinusoids. Each sinusoid is analyzed separately, by virtue of the superposition principle, and its particular effect can be calculated; it is also sinusoidal with the same frequency as the load but- in general – with different amplitude and a different phase. According to the superposition principle, the effect – or response – to the loading is the sum, or integral, of all sinusoids of individual effects. It is applied, for example, in the electromagnetic theory of light.

One faces the same problem when analyzing loading with an infinity of impulse functions by using the Green functions. The answer, or effect, is obtained by summing the individual answers corresponding to each load.

3. Some applications of the superposition principle

3.1. Solutions based on monodisciplinary approaches

- In *classical mechanics*, according to the superposition principle, it is stated that [4]:

”*If several forces act simultaneously on a body, each force produces its own acceleration independently of the other forces, the resulting acceleration being the vector sum of the individual accelerations*”,

$$\bar{a} = \sum_i \bar{a}_i \,, \tag{6}$$

where $\bar{a}_i$ is the vector of the acceleration produced by force $\bar{F}_i$, and the resulting acceleration $\bar{a}$ is the result of the total force $\bar{F}$. One can write that [4],

$$\left.\begin{aligned} \bar{F}_i &= m \cdot \bar{a}_i \\ \bar{F} &= \sum_i F_i = m \cdot \sum_i a_i \end{aligned}\right\} \tag{7}$$

where *m* is the body mass.

- *In mechanical engineering,* the superposition principle is used, for example, to solve problems such as bending bars subjected to several loads, if the response (effect) is linear. The principle applies provided that each load does not affect the result of loading by another load and the effect of each load does not significantly alter the geometry of the structure under load [4].

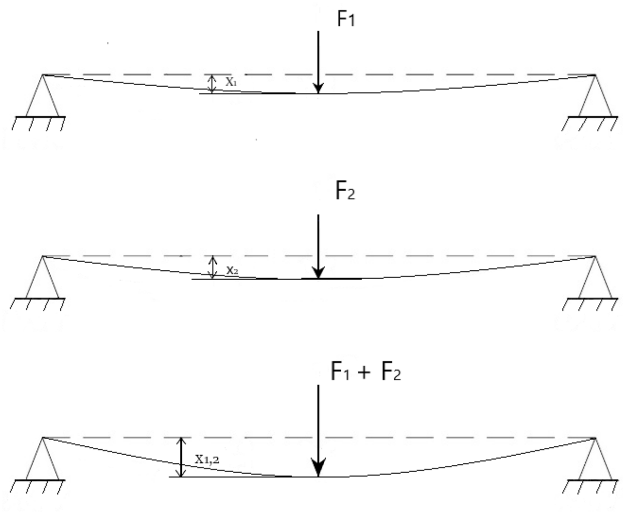

Fig. 2. Bending load on a bar.

In the case of a bending load of a bar placed on two supports (Fig. 2), we obtain: deflection x_1 when under force load F_1, deflection x_2 when under force F_2, deflection $x_{1,2}$ when under force load (F_1+F_2).

According to the superposition principle for structures with linear behavior, the total deflection under a sum force (F_1+F_2) is equal to the sum of the individual deflections:

$$x_{1,2} = x_1 + x_2 \tag{8}$$

- Any *particle charged with an electric charge* creates an electric field in the environment around it. This electric field can be calculated with Coulomb's law [5].

The superposition principle applied to *electric fields* states that any load in space creates an electric field at a point, independent of other loads in that space. The resulting electric field is the vector sum of the electric fields produced by the individual loads.

- In the case of an *electrical circuit* Kirchhoff's laws are used, which represent applications of the superposition principle.

According to *first Kirchhoff's law*, the sum of the currents entering a node of an electrical network is equal to the sum of the currents leaving that node. Or, if you assign signs to the electrical currents (those that enter with a plus sign and those that leave the node with minus) one can write that the sum of all electric current in the network node is equal to zero,

$$\sum_{i=`1}^{0} I_i = 0 \tag{9}$$

Kirchhoff's second law states that into a closed loop the algebraic sum of the electromotive voltages of sources (E_i) is equal to the algebraic sum of the products of the current intensity (I_i) and the electrical resistance on each side (R_i),

$$\sum_i E_i = \sum_i R_i \cdot E_i \,. \tag{10}$$

- In the case of *perfect gas mixtures*, pressure p is the result of the superposition of the *partial pressures* p_i of its components.

Let volume V be occupied by a perfect gas mixture whose total pressure is p. The partial pressure p_i of the component i of the gas mixture represents the pressure it would have if it alone occupied the entire volume V.
Dalton law's states that [6],
" *the total pressure p of the perfect gas mixture is equal to the sum of the partial pressure p_i of its components*",

$$p = \sum_i p_i \,. \tag{11}$$

- *Amagat's law* refers to the correlation between the volume V occupied by a mixture of perfect gases and the volumes occupied by the gases that compose it. The gas mixture under pressure p occupies volume V. The partial volume V_j is the volume that the j gas species would occupy if it alone were under pressure p.

According to Amagat's law [6],
"The total volume V of a mixture of perfect gases is equal to the sum of the partial volumes of its components",

$$V = \sum_j V_j \tag{12}$$

Dalton's law and Amagat's law refer to perfect gases made up of "rigid" molecules, which do not deform under the action of pressure. These laws may be written in the form,

$$\sum_i \frac{p_i}{p} = 1 \quad \text{and} \quad \sum_j \frac{V_j}{V} = 1. \tag{13}$$

- The consideration of real gases composed of *deformable molecules*, using the *principle of critical energy*, led to relations [2]:

$$\sum_i \frac{p_{i,r}}{p} = P_{cr} \quad \text{and} \quad \sum_j \frac{V_{j,r}}{V} = P_{cr} \,. \tag{14}$$

$P_{cr} < 1$ is the critical participation,

$$P_{cr} = 1 - \frac{E_d}{E_p} \,, \tag{15}$$

E_d is the specific deformation energy of all molecules of the gas mixture (J/m^3), and E_p is the specific pressure energy (J/m^3).
By dividing, member by member, the first relations (13) and (14), one obtains,

$$\sum_i p_{i,r} = P_{cr} \cdot \sum_i p_i \tag{16}$$

from which it follows that $p_{i,r} < p_i$.
By dividing, member by member, the second relations (13) and (14), one obtains,

$$\sum_j V_{j,r} = P_{cr} \cdot \sum_j V_j \tag{17}$$

from which it follows that $V_{j,r} < V_j$.
Consequently, in real gases, due to the deformation of molecules:

- the actual partial pressure, $p_{i,r}$, is lower than for perfect gases, p_i;
- the partial volume of the actual gas, $V_{j,r}$, is smaller than that of the perfect gas, V_j.

- Ludwig Boltzmann's *principle superposition* (1878) extends the relation of stress or deformation additivity from linear-elastic bodies to bodies with linear viscoelastic behavior. The validity of this principle is limited to small deformations [7].

If the stress featuring tension σ_i produces the strain ε_i, and the sum of the specific deformations $\sum_i \varepsilon_i$, does not exceed the sum of the small deformations, then the principle of stress and strain additivity, respectively, is applicable to the viscoelastic body. Such a body – it is said – possesses linear viscoelasticity; this is a generalization of the linear elastic behavior.
Boltzmann's superposition principle states that [8]:
"The value of a characteristic function of a system is equal to the sum of all changes induced in the system by the driving functions which have been applied to it throughout its history."
Based on this principle, Boltzmann represented stresses and strains corresponding to the linear viscoelastic bodies by integrals in the 3-D space.

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The solutions in the cases presented so far are provided by concepts, theories and principles from a single discipline, or from a single chapter of science. For this reason they are assigned to "*monodisciplinary solutions*".

3.2. Solutions based on interdisciplinary approaches

- *The flow of a fluid* along a channel formed between two parallel plates can be achieved by [9]: - pressure action p (Fig. 3, a); - fluid entrainment, by moving one of the plates with speed v (Fig. 3, b).

The pressure flow (Fig. 3, a) is called *Poiseuille flow*, and the drive flow (Fig. 3, b) is known as the *Couette flow*.

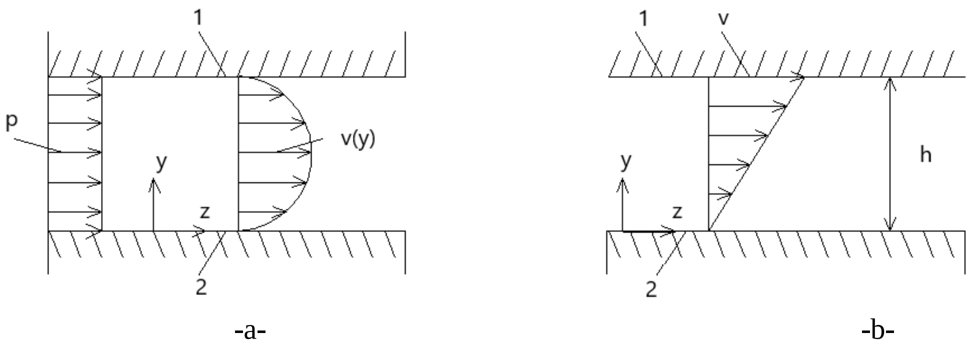


Fig. 3. The flow in direction z of a fluid between two parallel plates (1;2) produced by: a - pressure p; b – movement with speed v of the upper plate 1, parallel to the lower stationary plate 2.

In some practical cases the two types of flow superpose such as *in the screw channel of a polymer extrusion machine* (Fig. 4), where the pressure increases (dp/dz > 0), from p_0 to the maximum pressure, p_M, at distance L_{p_M}, after which it decreases (dp/dz < 0) on the distance $(L_s - L_{p_M})$ down to the pressure p_e (Fig.5).

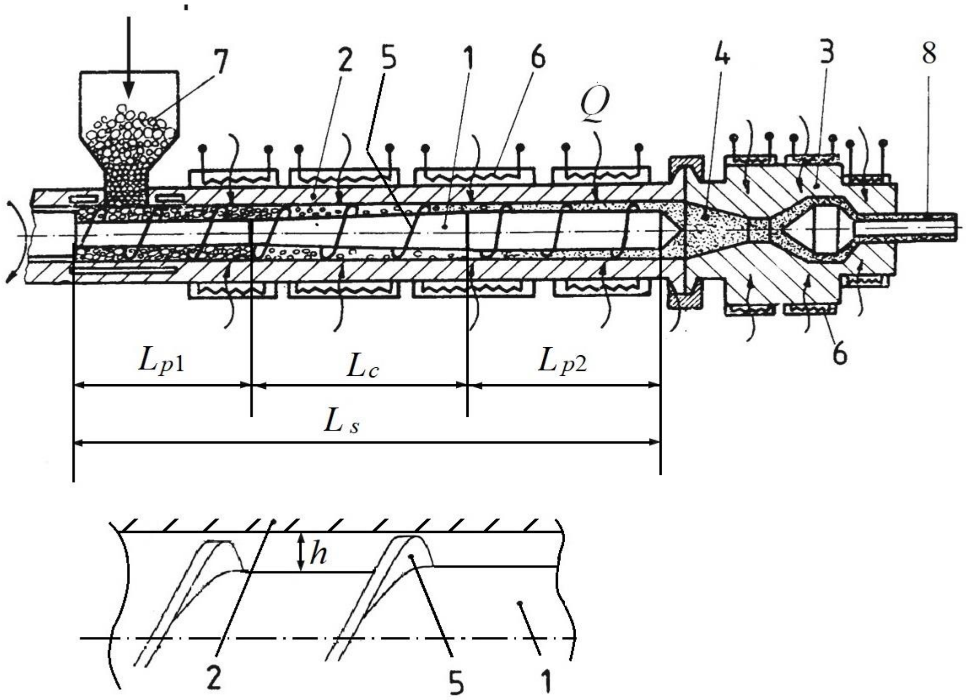


Fig. 4. The main part of a polymer extrusion machine [10;11]:
1 - screw; 2 - barrel; 3 - extrusion die; 4 - polymer melt; 5 - screw flight; 6 - heating electrical resistors; 7 - feed hopper with polymeric granules; 8 - extrudate.

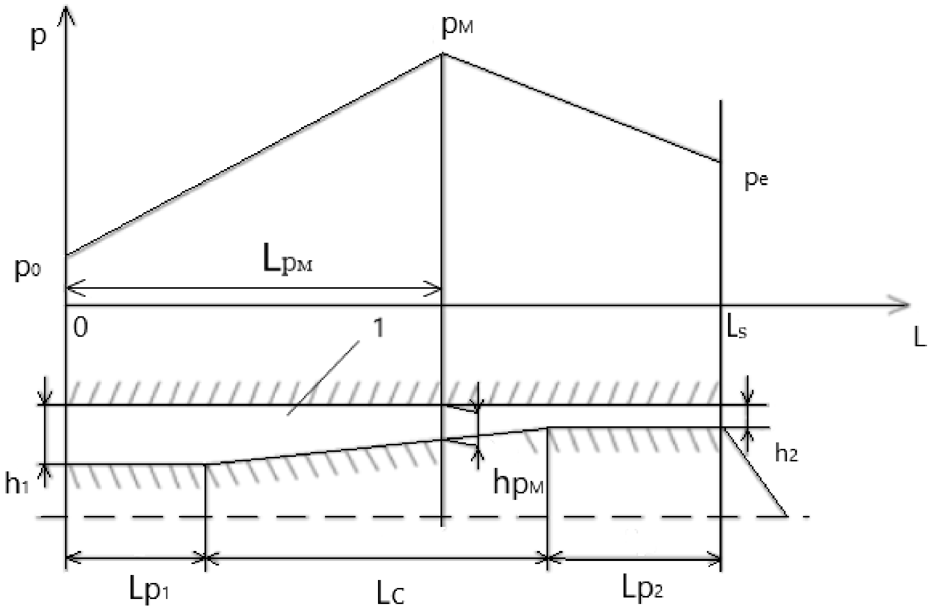


Fig. 5. Pressure evolution along the screw channel, 1, provided with a compression zone, L_c, located between the flat zones, L_{p_1} and L_{p_2}, with depths h_1 and h_2.

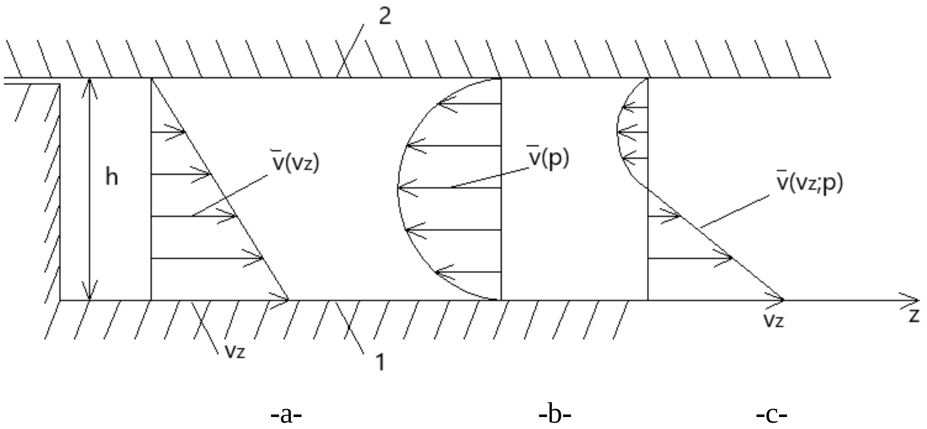


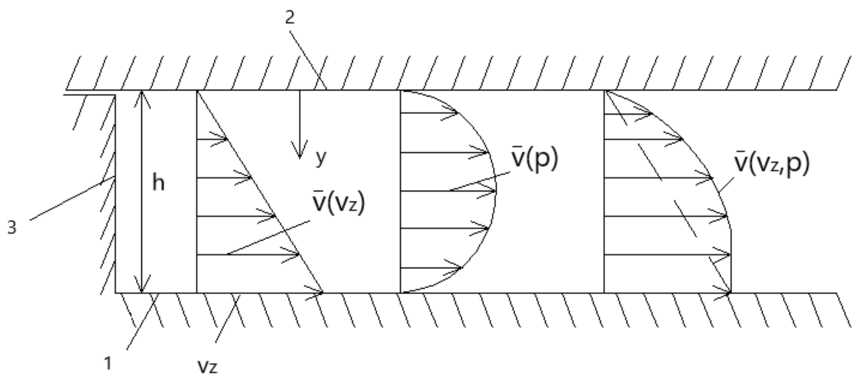
Fig. 6. Velocity profile: a- produced by the melt entrainment caused by the screw surface (1), with velocity v_z; b – produced by the pressure p increasing along the screw channel ($dp/dz > 0$) and written v(p); c – result of two velocity profiles (a and b) superposition.

Before the maximum pressure section, p_M, the Poiseuille flow ("reverse" flow) opposes the flow generated by the Couette-type flow (Fig. 6).

The velocity profiles corresponding to the two types of flow superpose and the final profile is obtained (Fig. 6, c), by vector summation,

$$\bar{v}(v_z; p) = \bar{v}(v_z) + \bar{v}(p). \tag{18}$$

After the maximum pressure section, along the length L_{p_M} the pressure decreases continuously down to p_e and acts in the direction of increasing the velocity. The two velocity profiles are added together and one obtains the final profile in Figure 7, c.



-a- -b- -c-

Fig. 7. Velocity profiles: a – produced by the melt entrainment caused by the screw 1 with velocity v_z; b – produced by the pressure decreasing along the screw (dp/dz < 0) and denoted v(p); c – the result of summing the two velocity profiles (a+b); 2 – the extruder barrel; 3 – helical spiral of the screw.

- When calculating the temperature variation on the screw channel depth (h), the heating from the outside and the heating by internal friction superpose.

One has found that the temperature in the melt layer is the *result of the superposition of the temperature increase* produced by the external heating, ΔT_e, due to external heating flux Q, and the temperature increase caused by the internal friction of the polymer melt, ΔT_f . In a certain section of the melt layer in the screw channel, the temperature is calculated with the relation [9],

$$T(\zeta) = T_{m,s} + \Delta T_e(\zeta) + \Delta T_f(\zeta), \tag{19}$$

wherein $T_{m,s}$ is the melt temperature at the edge of the boundary layer in the screw, and $\zeta = y/h$, is the relative depth of the point at which the temperature is calculated; y is measured from the surface of the barrel to the surface of the screw channel (Fig. 7); h – the screw channel depth near the point where the melt temperature $T(\zeta)$ is calculated.

The increase in temperature by internal friction $\Delta T_f(\zeta)$ depends on the rheological behavior of the melt. The increase in temperature by heating from the outside (usually from the barrel, depends on the temperature at the edge of the boundary layer next to the barrel $T_{b,s}$ and next to the screw, $T_{m,s}$, according to the relation,

$$\Delta T_e(\zeta) = (T_{b,s} - T_{m,s})(1 - \zeta), \tag{20}$$

The evaluation of the polymer melt flow through the screw channel (the active organ of the extruder) is *a case of interdisciplinary solution* because it brings together concepts and knowledge belonging to rheology, fluid mechanics and heat transfer.

- *Superposition vibration* when polymer melts flow under pressure [12].

When a thermoplastic polymer melt flows, the superposition of vibrations over the stress state has the effect of reducing its viscosity. Consequently, in the case of injection molding machines, the pressure required to move the melt through the channels of the injection mold is reduced. At the same time, the critical velocity corresponding to the unstable flow of the melt is increased, which can have the effect of improving the quality of the injected product, as well as of the extruded product.

This is another case of *interdisciplinary solution* based on the transfer of concepts between fluid mechanics, vibration mechanics and rheology.

4. The inverse problem, the separation of the effects produced by a certain load, treated interdisciplinarily

When cumulating the effects, one should take into account the mode of load application, whether it is *simultaneous or successive*, because the total effect is influenced by it [2;13]. For example, if Y_1 is applied by shock and at the same time Y_2 is applied statically, the total effect is different if these loads are applied successively. After the shock application of Y_1 its action becomes static, and the effect is that of a static stress, in this case – in the end – two static effects are combined.

Unlike the previous cases when one analyzed the superposition of loads (Y_1;Y_2...Y_n) that determines the same kind of effect, X (e.g. strain, or acceleration etc.), (Fig. 8,a), one encounters situations where a certain stress Y produces two or more different effects. For example, when a viscoelastic fluid is acted upon with pressure p, two different effects result: the shear stress τ, corresponding to the viscous behavior and the normal stress σ, corresponding to the elastic behavior of the melt (Fig. 8, b).

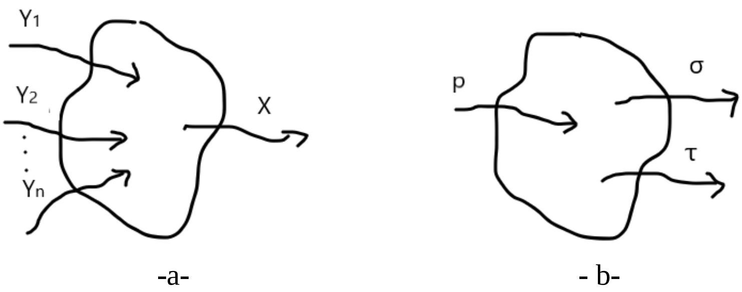


Fig. 8. a – Several external loads (Y_1, Y_2...Y_n) determine a single type of effect, X;
b – a single load (pressure p) determines two effects: the tangential stress τ and the normal stress σ, of an elastic nature.

In other words, a Y load can produce two or more different effects (X_1, X_2...), which being of a different nature, cannot be added algebraically even if the behavior of the mass is linear.
The solution of the problem of superpositing effects in such cases is done by separating the effects generated by the same load.
In general, when using relationship (1) for the behavior of matter, only one effect is taken into account, the main effect X_i. But, in reality, in each case one should add the thermal effect, X_{th}, which is sometimes very important, even prevailing.
If one introduces the individual thermal effect, corresponding to each load, Figure 9 is obtained.

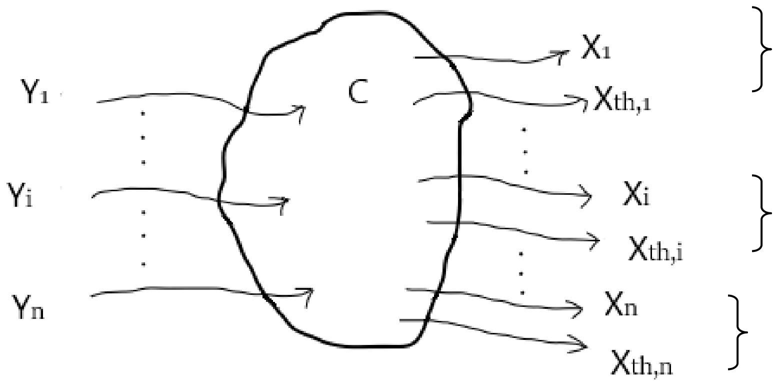


Fig. 9. The action of (Y_1,..Y_i,...Y_n) with effects (X_1,...X_i,...X_n) and the individual thermal effects corresponding to each load ($X_{th,1}$,...$X_{th,i}$,...$X_{th,n}$).

When writing relation (1), the thermal effect is neglected, no matter how great.
It is necessary to *separate the two effects, X_i and $X_{th,i}$*, respectively, and their correlation with Y_i. That can only be done by using the concept of energy. The amount of total heat produced by the load is calculated by the relation:

$$Q_t = Q_1\left(X_{th,1}\right) + ... + Q_i\left(X_{th,i}\right) + ... + Q_n\left(X_{th,n}\right). \quad (21)$$

For example, when a fluid flows through a pipe under the action of pressure, the pressure applied at the pipe inlet, p_1, is the stress ($Y_1 \equiv p_1$), which has the following effects:

- compresses and elastically deforms the fluid (X_1). In the case of viscoelastic fluids, the elastic energy relaxes at the pipe outlet and causes to swell of the fluid;
- moves the fluid along the pipe and generates the kinetic energy, corresponding to the average velocity per section (X_2);
- creates the velocity profile due to the propagation towards the pipe center of the influence of the pipe wall adhesion. As a result, internal friction occurs between adjacent fluid layers, which results in the dissipation of the corresponding energy in the form of heat ($X_3 = X_{th}$). In some cases the fluid is sliding in relation to the solid surface. The corresponding energy dissipates as heat.

The single loading $Y_1 \equiv p_1$ has three effects: X_1, X_2 and $X_3 = X_{th}$.

Basically, one faces the problem of calculating the thermal effect. By abstracting it from the energy corresponding to Y_1, one obtains the energy consumed for generating the effects X_1 and X_2.

With viscoelastic fluids, as well as with elastoviscos solids, one has to solve the problem of separating the viscous behavior from the purely elastic behavior. The two properties coexist and manifest themselves simultaneously.

Consider the linear behavior of:

- *the viscous component*, according to Newton's law (4);
- *the elastic component*, according to Trouton's law [14],

$$\sigma = \mu_e \cdot \dot{\varepsilon}\,, \qquad (22)$$

where σ is the normal, elastic stress; $\dot{\varepsilon} = d\varepsilon/dt$ – strain rate velocity (ε – strain; t – time); μ_e – extensional (constant) viscosity;

- *the friction shear stress*, τ_f, between the fluid and a solid surface,

$$\tau_f = f \cdot p\,, \qquad (23)$$

where *f* is the friction coefficient; p – the local pressure.

The separation of the effects can be done by using the concept of *specific energy* (energy density) or *specific power* (power per unit volume), applied to the viscous component and the elastic component, respectively. You get:

- the specific power of the viscous component,

$$P_{s,v} = \int_0^{\tau} \tau \cdot d\dot{\gamma} = \frac{\tau^2}{2\mu}; \qquad (24)$$

- the specific power of the elastic component,

$$P_{s,el} = \int_0^{\sigma} \sigma \cdot d\dot{\varepsilon} = \frac{\sigma^2}{2\mu_e}. \qquad (25)$$

Due to *external friction*, for example between a fluid which slips on a solid surface, the following specific power is used,

$$P_{s,fr} = \frac{F_{fr} \cdot v}{V} = \frac{\tau_f \cdot v}{\delta}\,, \qquad (26)$$

where $F_{fr} = \tau_f \cdot S$ is the frictional force, *v* the velocity of the fluid in relation to the surface S and $V = S \cdot \delta$ is the fluid volume of thickness δ involved in the frictional process.

The total specific power is obtained by summing the partial specific powers,

$$P_{s,t} = P_{s,v} + P_{s,el} + P_{s,fr}. \qquad (27)$$

• The problem is the same when has to deal with an *electromagnetic circuit* crossed by an electric current of intensity I. Around this circuit an electric field and a magnetic field are created, which coexist and superpose. Their effects cannot be summed algebraically, but only by using the concept of specific energy (energy density), E_s, which is expressed in units of energy per unit volume (J/m^3).

The total specific energy of the two fields is the sum of the specific energies of the electric field, $E_{s,e}$, and the magnetic field, $E_{s,m}$,

$$E_s = E_{s,e} + E_{s,m}\,. \qquad (28)$$

In the case of the linear behavior:
- of the electric field, the general relation (1) becomes,

$$D = \varepsilon \cdot E \,, \tag{29}$$

where D is the electrical induction; E – electric field intensity; ε – the electrical permittivity of the environment where the electric field is created;
- of the magnetic field, the general relation (1) becomes,

$$B = \mu_m \cdot H \,, \tag{30}$$

where B is the magnetic induction; H – magnetic field intensity; μ_m – the magnetic permeability of the environment where the magnetic field is created.
The specific energies in relationship (28) have the expressions,

$$\left.\begin{aligned} E_{s,e} &= \int_0^E D \cdot dE = \frac{\varepsilon \cdot E^2}{2} = \frac{D^2}{2 \cdot \varepsilon}\,; \\ E_{s,m} &= \int_0^H B \cdot dH = \frac{\mu_m \cdot H^2}{2} = \frac{B^2}{2 \cdot \mu_m}\,. \end{aligned}\right\} \tag{31}$$

However, out of the total energy of the electric current that passes through the conductor that forms the electromagnetic circuit, a part is transformed into heat, corresponding to the overcomes its electrical resistance.
In the case of a conductor linear behavior, according to Ohm’s law (3), the power dissipated in the conductor of electrical resistance R, has the expression,

$$P_r = U \cdot I = \frac{U^2}{R}\,, \tag{32}$$

while the specific energy,

$$E_{s,R} = \frac{P_R \cdot t}{V} = \frac{U^2 \cdot t}{V \cdot R}\,, \tag{33}$$

wherein V is the volume of the conductor traversed by the electric current; t – the duration of the passage of the electric current through a circuit section.
Consequently, the total specific energy corresponding to the electrical circuit is,

$$E_{s,t} = E_{s,e} + E_{s,m} + E_{s,r}\,. \tag{34}$$

The problem considered has an *interdisciplinary solution* based on the transfer of concepts through the boundary between the fields of electricity and magnetism. This transfer engendered the distinct chapter of electromagnetism.
The following should be noted:
− in the case the of nonlinear behavior of matter, *relation (4) is not valid*, but relation,

$$X_t \neq \sum_i X_i \,, \tag{35}$$

is valid, as shown – for example – in works [2;13];
− synergistic effects are not obtained in the case of linear behavior, but only when the relation (35), corresponding to the nonlinear behavior, is valid, a fact that was demonstrated in the work [2;13;15;16];

– in the case of the linear behavior of matter, deterioration cannot be introduced in the assessment of its strength and, in general, the thermal effect is neglected.
The shortcomings reported above can be overcome by using *the principle of critical energy* a transdisciplinary principle [2;13;15].

5. Some comments on the superposition principle

The superposition principle is applicable only to systems with linear behavior rendered by relation of type (1).
As to superpose loads in the case of nonlinear behavior of matter or in order to separate the effects of a load one should use *the principle of critical energy* [2;13;15].
For example, paper [17] highlighted the separation of viscoelastic behavior from elastoviscos behavior, based on the principle of critical energy [2;18].
Many other problems of loads superposition, or superposing effects have been solved with the help of the principle of critical energy [2;13;15;16;19-21]. The use of this transdisciplinary principle has allowed the unitary treatment of mechanical structures and living organisms under several loads of the same or different nature [21;22].
In general, physical systems behave only approximately linearly, so the application of the superposition principle is approximate in the case of real systems. This accounts for the development of the chapters on nonlinear optics and nonlinear acoustics.The recourse to the principle of critical energy can solve the problems of real systems, generally nonlinear.

6. Conclusions

In science, in general and – especially - in engineering sciences, there are many cases of load superposition and/or cumulation of loads. The loads can be of the same nature but of different type, or of different nature (mechanical, thermal, electromagnetic, chemical etc). The effects of these loads may also be of the same or different types. On the other hand, the behavior of matter under load can be linear or nonlinear. One should add to this the cases where a particular load may produce two or more different effects.
The question is to find the procedure by which, in a given loading case, one might find the total effect. Given the many possible effects, problems of this kind are interdisciplinary, multidisciplinary or transdisciplinary.
The paper analyses and systematizes the simplest case of superposing loads, namely the one that refers to matter with linear behavior. To this end, the superposition principle is presented and its application is exemplified in several practical cases. One examines both the cases that can be solved by monodisciplinary evaluation and some cases that can be solved by interdisciplinary approach.

For the first time, there are clearly some cases where two or more different effects may correspond to a particular load.

One has also shown that finding a solution to such problems like the superposition and/or cumulation of loads/effect with matter featuring nonlinear behavior, can be done – at present – only by resorting to the principle of critical energy.

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