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The magnetic flux-voltage characteristic computation for a magnetic circuit branch

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Abstract. In many technical problems, the rapid magnetic field problem computation can be performed by adopting a magnetic circuit model. Beside some very simple Kirchhoff equations, there are also necessary the constitutive relationships $\varphi - u_m$ corresponding to the magnetic circuit branches (MCB), where φ is the fascicular magnetic flux and u_m is the magnetic voltage. This paper presents a procedure to determine these relations, by solving the magnetic field problem under certain boundary conditions for MCB. The nonlinearity of the **B-H** characteristic is treated using the polarization method and the iterations are performed in terms of magnetic field strength **H** correction.

Keywords: nonlinear magnetic circuit branches, polarization method, scalar magnetic potential.

1. Introduction

The magnetic field problems computation takes place, in general, by using differential, integral or hybrid numerical procedures. For 3D structures the computation effort can be great, especially if some modifications in field sources or problem geometry are taken into account. The nonlinearity is addressed by using the Newton-Raphson method (NRM) or by the polarization fixed point method (PM) [1]. For NMR, if this method is convergent, then the convergence speed is bigger than that corresponding to PM. More, for PM the convergence is always

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provided and the convergence speed can be greatly accelerated by using a dynamic over-relaxation procedure [1, 2].
In some structures, the magnetic field computation can be simplified if we adopt a magnetic circuit model. In some cases, the $\varphi - u_m$ characteristic of the MCB is determined, by solving the magnetic field problem only in the domain corresponding to each MCB. For this reason, sometimes, it is sufficient to adopt a 2D or 1D model.
The advantage of using the MCB is even more important if the same MCB is used in many structures, in which the sources or other branches can be different. For example, in [3–5], a new equipment is proposed. This equipment allows the efficient determination of the static **B-H** characteristic.
The procedure for B-H characteristic determination implies the computation of a magnetic field inverse problem. The use of a numerical procedure for stationary magnetic field problems necessitates a huge effort and, sometimes instabilities can occur.
The use of a magnetic circuit model, for the case of measurement on samples, reduces a lot the computation effort [6]. The equipment analyzed in [6] admits MCB whose $\varphi - u_m$ relations can be obtained using 3D models. To increase the equipment's performances, it is useful to use field concentrators, which cannot admit 2D models.
In this paper, we present the procedure for determining $\varphi - u_m$ for MCB with 3D structures.

2. The magnetic circuit branch

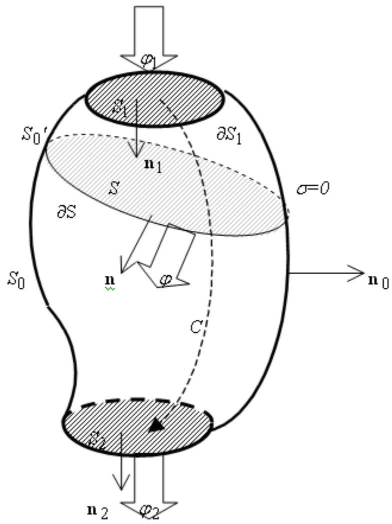

Fig. 1. The magnetic circuit branch.

Let us consider the domain Ω, without electric current (**J**=0), with boundary $\partial\Omega$ (Fig.1), where the magnetic field (**B**,**H**) verifies the following boundary conditions:
(α) on disjoint surfaces $S_1, S_2 \subset \partial\Omega$, the tangent component of the magnetic field strength **H** is null;
(β) on the rest of the boundary S_0, the normal component of the magnetic induction **B** is null.
The conducting domain Ω with the boundary conditions (α), (β) is called **magnetic circuit branch**.
Because in Ω we have $\nabla \times \mathbf{H} = 0$, it results the scalar magnetic potential (V_m) theorem is valid. From the boundary condition (α), it results the surfaces S_1, S_2 are magnetically equipotential. They are called magnetic terminals. We denote by V_{m1} and V_{m2} the potentials corresponding to these terminals. The magnetic voltage of the magnetic circuit branch u_m is defined as the magnetic voltage along any curve *C* which connects the two terminals. We have:

$$u_m = \int_C \mathbf{H} \cdot d\mathbf{l} = V_{m1} - V_{m2} \tag{1}$$

From the boundary condition (β), it results that it is well-defined the magnetic flux of the magnetic circuit branch, called the *fascicular flux,* as being the magnetic flux through any transversal section *S* of the resistor.

$$\varphi = \int_S \mathbf{B} \cdot \mathbf{n} dS \tag{2}$$

We assume the constitutive relation **B**=*F*(**H**) is Lipschitz:

$$\left\| F(\mathbf{H'}) - F(\mathbf{H''}) \right\| \le \Lambda \left\| \mathbf{H'} - \mathbf{H''} \right\|, \quad (\forall)\mathbf{H'}, \mathbf{H''} \in \mathrm{L}^2(\Omega) \tag{3}$$

and coercive:

$$\left\langle F(\mathbf{H'}) - F(\mathbf{H''}), \mathbf{H'} - \mathbf{H''} \right\rangle \le \lambda \left\| \mathbf{H'} - \mathbf{H''} \right\|^2, \quad (\forall)\mathbf{H'}, \mathbf{H''} \in \mathrm{L}^2(\Omega) \tag{4}$$

where $\Lambda < \infty$ and $\lambda > 0$ and the scalar product is:

$$\left\langle \mathbf{H'}, \mathbf{H''} \right\rangle = \int_\Omega \mathbf{H'} \cdot \mathbf{H''} d\Omega \tag{5}$$

where $d\Omega$ is the volume element. If in almost all points P from Ω the local constitutive relation $\mathbf{B}(P) = f(P, \mathbf{H}(P))$ is Lipschitz:

$$\left| f(P, \mathbf{H'}(P)) - f(P, \mathbf{H''}(P)) \right| \le \Lambda(P) \left| \mathbf{H'}(P) - \mathbf{H''}(P) \right|, \quad (\forall)\mathbf{H'}(P), \mathbf{H''}(P) \in R^3 \tag{6}$$

and coercive:

$$[f(P,\mathbf{H}'(P)) - f(P,\mathbf{H}''(P)), \mathbf{H}'(P) - \mathbf{H}''(P)] \le \lambda(P)|\mathbf{H}'(P) - \mathbf{H}''(P)|^2, \quad (\forall)\mathbf{H}'(P), \mathbf{H}''(P) \in R^3 \tag{7}$$

where $\Lambda(P)<\Lambda<\infty$ and $\lambda(P)>\lambda>0$, then the relations (3), (4) are valid.

According to the uniqueness theorem of the stationary fields [6], if the magnetic voltage u_m is given, then the magnetic field (**B,H**) is uniquely determined and so the fascicular magnetic field φ is uniquely determined. The following function it is therefore well-defined:

$$u_m \xrightarrow{\gamma} \varphi = \gamma(u_m) \tag{8}$$

For linear media, the function f is linear and relation (3) becomes: $\varphi = \Lambda\, u_m$, where Λ is the MCB permeance.
Also, if the fascicular flux is given, it uniquely results the magnetic voltage.

3. Nonlinearity treatment

We admit the magnetic voltage u_m is given, so the magnetic voltage V_{m1} is known, considering $V_{m2}=0$. The MCB nonlinearity is treated using the Newton-Raphson method (NRM) or the polarization method (PM).
The second method (PM) is chosen and, because the magnetic potential is given, from which results **H**, the correction in **H** is used [1].

The constitutive relation **B**=F(**H**) is replaced by:

$$\mathbf{B} = \mu(\mathbf{H} + \mathbf{M}) \tag{9}$$

the computation magnetization **M** being corrected function of **H:**

$$\mathbf{M} = \frac{1}{\mu}\left(F(\mathbf{H}) - \mathbf{H}\right) = G(\mathbf{H}) \tag{10}$$

Locally:

$$\mathbf{M}(P) = \frac{1}{\mu}\left[f(P,\mathbf{H}(P)) - \mathbf{H}(P)\right] = g(P,\mathbf{H}(P)) \tag{11}$$

If, in each point P, the computation permeability $\mu(P)$ it is chosen with the restriction:

$$\mu(P) > \frac{1}{2} \underset{\mathbf{H}',\mathbf{H}'' \in R^3}{Sup} \frac{|f(P,\mathbf{H}') - f(P,\mathbf{H}'')|}{|\mathbf{H}' - \mathbf{H}''|}, \tag{12}$$

the function g is a contraction [1].

$$\mu(P)\left|g\left(P,\mathbf{H'}(P)\right)-g\left(P,\mathbf{H''}(P)\right)\right|\le\theta(P)(P)\left|\mathbf{H'}(P)-\mathbf{H''}(P)\right|,(\forall)\mathbf{H'}(P),\mathbf{H''}(P)\in R^3,$$
$$\text{with } \theta(P)<1 \quad (13)$$

It results also the global function G is a contraction:

$$\left\|G(\mathbf{H'})-G(\mathbf{H''})\right\|_\mu\le\theta\left\|\mathbf{H'}-\mathbf{H''}\right\|_\mu,(\forall)\mathbf{H'},\mathbf{H''}\in \mathrm{L}^2_\mu(\Omega)\text{, cu } \underset{P\in\Omega}{Sup}\left(\theta(P)\right)=\theta<1 \quad (14)$$

where:

$$\left\langle\mathbf{H'},\mathbf{H''}\right\rangle_\mu=\int_\Omega\mu\mathbf{H'}\cdot\mathbf{H''}\,d\Omega \quad (15)$$

For any of two computation magnetizations $\mathbf{M'},\mathbf{M''}\in\mathbf{L}^2(\Omega)$, there are the unique magnetic fields (**B**’,**H**’), (**B**”,**H**”), which verify the relation (9) and they have the same boundary conditions [8] (the function $\mathbf{M}\xrightarrow{W}\mathbf{H}=W(\mathbf{M})$ is well-defined). The difference magnetic field $(\mathbf{B}_\mathrm{d},\mathbf{H}_\mathrm{d})$ =(**B**’,**H**’)-(**B**”,**H**”) verifies the relation:

$$\left\langle\mathbf{B}_\mathrm{d},\mathbf{H}_\mathrm{d}\right\rangle=0 \quad (16)$$

and, from relation (9) it results:

$$\left\langle\mu\mathbf{M}_\mathrm{d},\mathbf{H}_\mathrm{d}\right\rangle+\left\langle\mu\mathbf{H}_\mathrm{d},\mathbf{H}_\mathrm{d}\right\rangle=0 \quad (17)$$

from where:

$$\left\|\mathbf{H}_\mathrm{d}\right\|^2_\mu=-\left\langle\mathbf{M}_\mathrm{d},\mathbf{H}_\mathrm{d}\right\rangle_\mu\le\left\|\mathbf{H}_\mathrm{d}\right\|_\mu\left\|\mathbf{M}_\mathrm{d}\right\|_\mu \quad (18).$$

So the function W is non-expansive:

$$\left\|\mathbf{M'}-\mathbf{M''}\right\|_\mu=\left\|W(\mathbf{M'})-W(\mathbf{M''})\right\|_\mu\le\left\|\mathbf{M'}-\mathbf{M''}\right\|_\mu,(\forall)\mathbf{M'},\mathbf{M''}\in\mathrm{L}^2_\mu(\Omega) \quad (19)$$

For an arbitrary initial value $\mathbf{M}^{(0)}$, the polarization method builds the sequences $\left(\mathbf{M}^{(k)}\right)_{k\ge1}$, $\left(\mathbf{H}^{(k)}\right)_{k\ge1}$ with the relations:

$$\mathbf{H}^{(k)}=W(\mathbf{M}^{(k-1)}),\ \mathbf{M}^{(k)}=G(\mathbf{H}^{(k)}),\ \ k\ge1 \quad (20)$$

Taking into account that W is non-expansive and G is a contraction, the composed functions $W\circ G$ and $G\circ W$ are contractions and $\left(\mathbf{M}^{(k)}\right)_{k\ge1}$, $\left(\mathbf{H}^{(k)}\right)_{k\ge1}$ are Picard-Banach convergent sequences.

4. Magnetic field computation at each iteration

The magnetic potential is determined by solving the equation:

$$-\nabla\cdot\mu\nabla V_m=-\nabla\cdot\mu\mathbf{M} \quad (21)$$

using the finite element method. On the boundaries S_1 and S_2, $V_m=V_{m1}$ and, respectively $V_m=0$ and on S_0, $B_n=\mu H_n+\mu M_n$=0.

We impose $\mu H_n = -\mu \dfrac{\partial V}{\partial n}$ and it results $\mu M_n = 0$.

Dividing the domain Ω in a tetrahedrons mesh, we approximate:

$$V_m = V_0 + \sum_{p=1}^{N} \alpha_p v_p \tag{22}$$

where v_p are order 1 nodal elements, N is the number of nodes inside the domain Ω and surface S_0 and V_0 verifies the boundary condition on surface S_1:

$$V_0 = \sum_{p_1=1}^{N_1} V_{m1} v_{p_1} \tag{23}$$

where p_1 is a node index from the surface S_1. The relation (21) is projected on functions v_q, then it is integrated by parts and the following algebraic equations system is obtained:

$$\sum_{p=1}^{N} a_{q,p} \alpha_p = b_q, \qquad q=1,2,\ldots,N \tag{24}$$

where:

$$a_{q,p} = \int_{\Omega} \mu \nabla v_q \cdot \nabla v_p d\Omega \tag{25}$$

and taking into account that $\mu M_n = 0$,

$$b_q = \int_{\Omega} \nabla v_q \cdot \mu \mathbf{M} d\Omega - V_{m1} \sum_{p_1=1}^{N_1} \int_{\Omega} \mu \nabla v_{p_1} \cdot \nabla v_q d\Omega \tag{26}$$

By solving the system from (24) we obtain the weights α_p of the nodal elements v_p and the approximate shape (22) of the scalar magnetic potential.

Remarks:

a) ∇v_p, $\mu\mathbf{M}$ and the integrands from relations (25) and (26) are constant in each tetrahedron. The expressions (25) are not zero only when the nodes *p* and *q* edge the same branch or coincide. It results the system (24) (symmetric) has sparse matrix.

b) The approximation using the finite element of the magnetic filed problem computation is non-expansive: $\|\mathbf{H}\|_{\mu,aprox} \le \|\mathbf{H}\|_{\mu}$ [9], such that the PM convergence is preserved.

5. **Illustrative example**

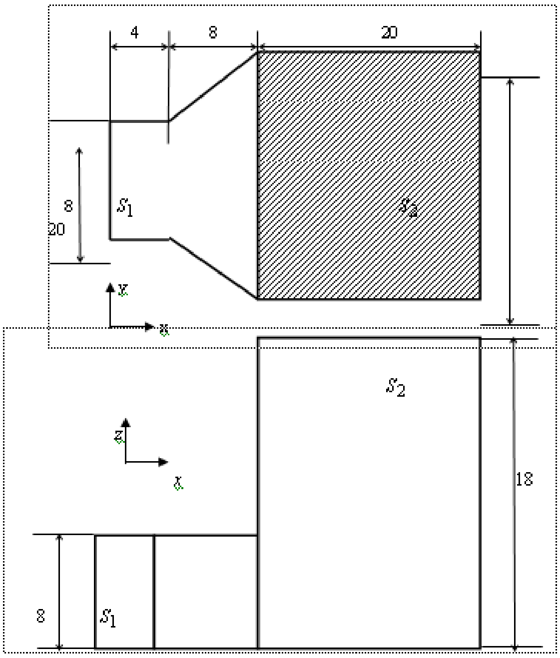

Fig. 2.The magnetic circuit branch.

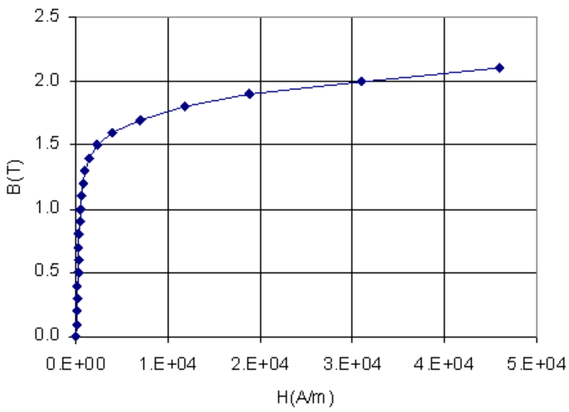

Fig.3. The B-H characteristic.

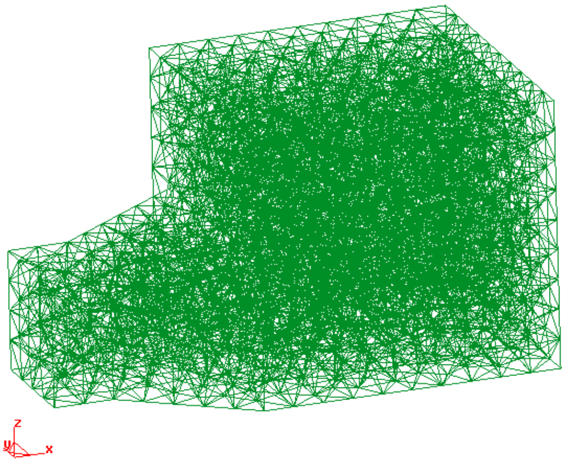

Fig. 4. The discretization mesh.

Let's consider the MCB from Fig. 2, having the medium with the **B-H** characteristic from Fig. 3. The discretization mesh is depicted in Fig. 4, having 2073 nodes and 10037 tetrahedral elements. The characteristic $\varphi - u_m$ from Fig. 5 is obtained.

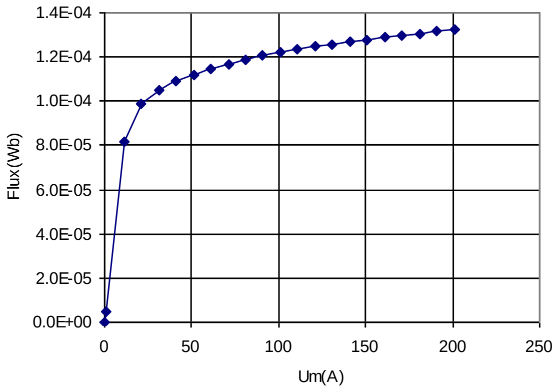

Fig. 5. $\varphi - u_m$ characteristic.

6. Conclusions

As results from Figs. 2 and 4, the MCB structure cannot be analyzed employing a 2D model. In this paper we presented a procedure for obtaining the $\varphi - u_m$ characteristic, in which the nonlinearity is treated by using the polarization method, and the magnetic field computation takes place, at each iteration, by using the

scalar magnetic potential. The correction of the fictitious magnetization takes place in **H**, ensuring therefore the convergence of the approximate iterative procedure. Because, to increase the scalar magnetic potential V_{m1}, a constant step has been chosen, the variation of the fascicular magnetic flux is big for small values of V_{m1} and it decreases when this potential increases (see Fig.5). Obviously, we can choose a variable step for increasing V_{m1} and the differences of increasing φ are diminished.
To raise the $\varphi - u_m$ characteristic, the magnetic potential vector **A** can be also used, therefore resulting the fascicular magnetic flux on any closed curve from the surface S_0 (curl **A**). The use of the nodal and edge elements (for all edges) leads to a system of equations with big dimensions, which contains the gauge condition, beside the differential equation of **A**. The system's dimensions can be reduced if we use the edge elements of the vertexes from the edges co-tree [10]. The boundary condition for $\mathbf{A}_t$ is imposed on the vertexes which close the loops from the boundary. Because the MCB medium is ferromagnetic, the use of the scalar magnetic potential does not lead to instability on the separation surfaces between air and iron.

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