Technical Sciences
Academy of Romania
www.jesi.astr.ro

Journal of Engineering Sciences and Innovation
Volume 5, Issue 3 / 2020, pp. 205 - 220
http://doi.org/10.56958/jesi.2020.5.3.205

B. Chemical Engineering, Materials Science and Engineering

Received ***19 May 2020*** *Accepted* ***15 September 2020***
Received in revised from ***30 June 2020***

CONTRIBUTIONS TO THE INGOTS' HEATING STUDY IN INDUSTRIAL FURNACES

IOAN ILCA[1*], DIANA BISTRIAN[2]

[1]TECHNICAL SCIENCES ACADEMY OF ROMANIA
[2]University Politehnica Timisoara, Faculty of Engineering Hunedoara

Abstract: Establishing the technology for hot processing of some steels with special destination is connected in the first place to defining the warming conditions, depending on their technological characteristics. The choice of optimal regime is made, most of the times, relying on the practical experience because the analytical calculation methods give relative values.
This paper proposes to compare the data obtained from industrial experiments with theoretical calculation methods for their approximation. The key innovation introduced in this paper is represented by analytical solutions to determine the evolution of the ingot mass temperature. We focused on presenting an analytical solution to determine the evolution of temperature in the ingots'mass and comparing it with the data resulted from experiments.
The proposed analytical procedure successfully substitutes for hard-to-perform experiments, in conditions of the current practice.
The usage of the proposed mathematical technique also assures the shortage of the total heat duration with economic effects generated by the reduction of fuel quantity and by the increase of productivity of the deep furnaces.

Keywords: steel ingot, deep furnaces, internal stresses, structural phases, thermocouples, thermal conductivity optimum regime, deep furnaces, turned flame, corrosion, refractory.

1. Introduction

In industrial practice, heating steel ingots starting from cold state usually lasts 8-12 hours, but in the case of particularly large bullion or steel qualities that require special treatment, heating can take up to 18 hours.

*Correspondence address: ilca@fih.upt.ro

Heating the regular carbon steel qualities in modern deep furnaces, equipped with automatic control, the problem is rather simple, since the operator sets the furnace at the temperature required by the process.
In the first phases of the heating process the furnace is heated using the nominal or maximum fuel flow. Then this flow rate is gradually reduced as the heat penetrates inside the bullion and reaches a minimum level in the final phase when a predetermined time is maintained for equalization. This time can be half an hour or more, depending on the experience of each plant.
When high carbon steel ingots are heated or alloyed steels with special purpose that are in cold condition or have had a high transit time, the furnace is usually cooled to the temperature of the previous loaded charge [1,2].
The ingots are then slowly heated to a predetermined temperature below the plastic state processing temperature. After the ingots have been evenly heated to this temperature, most steel qualities can be heated quickly to the required processing temperature [3].
One of the common methods used for heating some alloyed steels is the so called stage heating. This method consists of heating ingots in several steps up to the processing temperature, each step having a specific duration corresponding to the time necessary to uniformize the temperature in the mass of the metal material.
This process eliminates internal tensions caused by large temperature differences from the interior of the ingots and assures a slow and uniform heating [4]-[7].
If heating for plastic processing starts from cold state, for steels with special qualities, susceptible to solid phase transformations like the ones this work refers to, then setting the heating speed up to the temperature of 700-800 Celsius degrees must be done with special care, this being in fact the most suitable period for the appearance of internal and structural defects [8].
Analyzing the situation in the country and abroad, the heating process for these steels raises difficult problems and differs from one company to another, based mostly on its own experience gained. Although it has been practically proven to increase the heating speed, there is a danger of insufficient heating on the section and the destruction of steel compactness during the initial heating period [9].
Thus, in the preheating period i.e. until the ingot center reaches the temperature of 500-550 Celsius degrees, the heating speed is limited by the temperature difference that forms between the surface and the center of the ingot, which must not exceed a critical value to avoid the destructive effect of thermal and structural tensions [10].

2. Experimental and analytical research

We consider in this work the steel of type Cr-Ni-Mo, who falls into the category of austenitic stainless steels, mark 10TiMoNiCr175, according to STAS 3589-87. Steels of this type raise problems when processing by laming or forging because they are far from the state of thermodynamic balance, almost always having a multiphasic structure [11].

At the steel company from Hunedoara the heating regime for this steel is determined according to a number of factors such as: chemical composition, phasic steel composition, heat deformation resistance, furnace type, weight and ingot size, lamination scheme [12]. The structure of these ingots is unfavorable in terms of deformability by excessive expansion of the columnar crystal area and strong segregation of the alloying elements. It is possible to appear constituents with negative influence on plasticity as well as reduced thermal conductivity. Therefore, heating these ingots for plastic processing up to almost 800 Celsius degrees must be done at a reduced speed so that at higher temperatures a higher heating speed can be adopted [13].

To the best of our knowledge, in the preheating period, i.e. until 500-550 Celsius degrees are obtained in the center of the ingot, the heating speed is limited by the temperature difference between the surface and the center of the ingot. It must not exceed a certain critical value, in order to avoid the destructive effect of thermal and structural tensions [14].

Most of the time, the choice of heating regime is based on practical experience, because analytical calculation methods provide relative values [15].

For the experimental determination of temperature distribution, we considered a randomly selected ingot, of mark 10TiMoNiCr175, with mass of 3,6 tons. It was equipped with temperature measuring thermocouples and it was heated in a deep crucible, along with the other ingots.

The measurement of the temperature evolution on the ingot section was carried out with the Pt-Pt.Rh thermocouples, located as in Figure 1. The evolution of the temperature according to the heating regime industrially applied for this steel quality, is shown in Figure 2. Curve 1 represents the evolution of the temperature of the furnace given by the thermocouple on the panel of appliances. The installation of the two ingot thermocouples were made as shown in Figure 1. Curve 2 and curve 4 represent the values of the experimentally measured temperatures for the ingot surface and for the orifice on the ingot's axis, respectively.

The heating was carried out by starting with the cold furnace, due to thermal insulation mounting difficulties for the connecting wires of the thermocouples, which required protection against temperature and atmosphere in the furnace. Experimentations were carried out in the temperature range 20-860 Celsius degrees (Figure 2), which includes the preheating period for ingots of this steel that are inserted into the furnace in the cold state. After the end of this period, the heating speed can be increased because the danger of thermal tensions disappears due to the large temperature differences on the ingot section.

Obviously, conducting such experiments with ingots in industrial furnaces, under the conditions of current practice are cumbersome, long-lasting and difficult.

For this reason, in this work we resorted to finding analytical solutions to determine the evolution of temperature in the ingots mass and to compare their values with the data experimentally obtained.

Among the existing analytical solutions, the utmost interest stands for mathematical methods based on differential equations of thermal conductivity, so-called physicomathematical methods. These methods offer complete solutions for various heating regimes. Thus, the most important problem of the analytical theory of heat consists is the determination of temperature distribution in the metal ingot according to coordinates and time.

In the paper, the industrial ingot is approximated with a finite cylinder, at finding some analytical solutions [16,17] to determine the temperature's evolution in the ingots' mass and comparing them with the experimental obtained data.

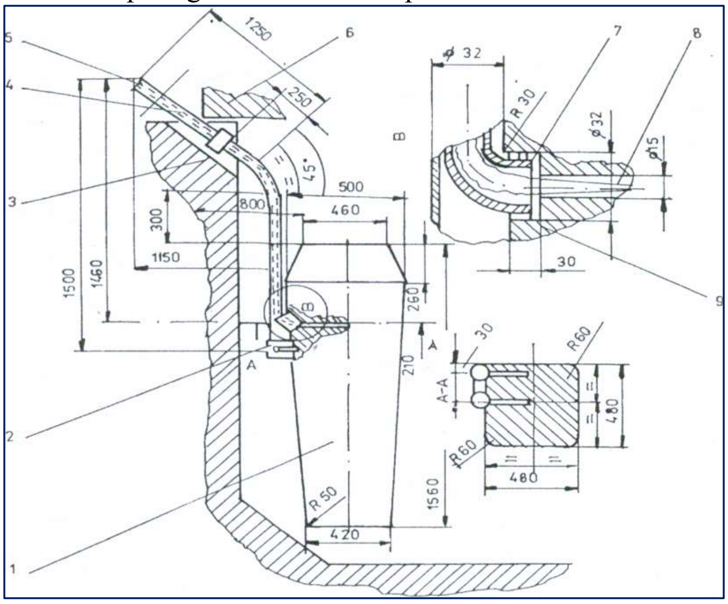


Fig.1 Experimental determination of temperature distribution when heating the cold ingots 1-ingot; 2-cooling coil for thermocouples; 3-furnace's masonry; 4-thermocouples' protection; 5-thermocouples' conductors; 6-lid; 7-thermocouple connection; 8-conductive contact; 9-the ingot's reaming for placing the thermocouples.

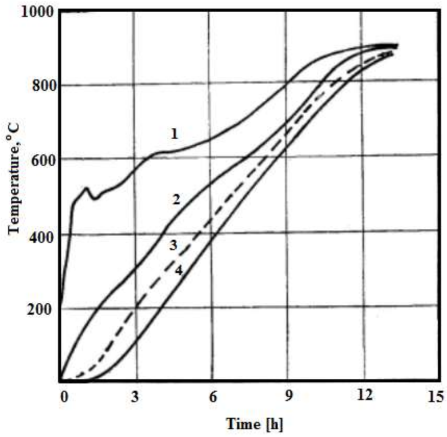


Fig. 2- Variation curves of the ingot's temperature.1-furnace's temperature; 2-the temperature experimentally measured at the surface of the ingot; 3-the resulted temperature from calculus in the center of the ingot; 4- the temperature experimentally measured in the center of the ingot.

Resolving this problem imposes firstly the reduction of the differential equation of the thermal conductibility which settles the link between the temperature variation in space and time.

To derive the mathematical model, we consider a circular cylinder with length ℓ and radius $R = \ell_0\sqrt{\pi}$, ℓ_0 being the ingot side.

Used notations:

T - The temperature at a current point of the ingot [K];

τ - Time [h];

λ - The coefficient of thermal conductivity [W/mK];

c - The specific heat [kJ/kg K];

ρ - Density [kg/m^3];

a - Heat dissipation [m^2/h];

α - The overall heat exchange [W/m^2 K];

h - The coefficient of relative heat transfer [m^{-1}];

T_0 - The initial temperature of the ingot [K];

T_g - The temperature of burned gases from the furnace [K].

Temperature distribution in the cylinder, using Cylindrical coordinates, is given by the equation of thermal conduction:

$$\frac{\partial T}{\partial \tau} = a\left(\frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\right) \tag{1}$$

with initial condition:

$$T(r,z,0) = T_0 \tag{2}$$

and boundary conditions:

$$T(r,0,\tau) = T_0 \tag{3}$$

$$\frac{\partial T}{\partial z}(r,\ell,\tau) = h\left[T_g - T(r,\ell,\tau)\right] \tag{4}$$

$$\frac{\partial T}{\partial r}(R,z,\tau) = h\left[T_g - T(R,z,\tau)\right] \tag{5}$$

A variety of analytic techniques have been applied to the nonlinear heat conduction problems. Marshak [18] was the first to obtain a self-similar solution to the nonlinear radiation heat conduction equation. Parlange et al. [19] obtained approximate analytical solution of the nonlinear diffusion equation for arbitrary boundary conditions. Recently, Barbaro et al. [20] applied the Kirchhoff transform to the enthalpy formulation of the heat conduction equation to obtain approximate solutions for temperature-dependent thermal properties.

We adopt in this study a decoupling method for solving the equation of thermal conduction in Cylindrical coordinates, previously used for Navier-Stokes equations

[21]. By dividing the temperature into a steady-state part $T_1(r,z)$ and a time dependent part $T_2(r,z,\tau)$:

$$T(r,z,\tau)=T_1(r,z)+T_2(r,z,\tau)+T_g \tag{6}$$

the functions T_1 and T_2 are satisfying, respectively, the following Cauchy problems:

$$\frac{\partial^2 T_1}{\partial z^2}+\frac{1}{r}\frac{\partial T_1}{\partial r}+\frac{\partial^2 T_1}{\partial r^2}=0 \tag{7}$$

$$T_1(r,0)=T_0-T_g \tag{8}$$

$$\frac{\partial T_1}{\partial z}(r,\ell)+hT_1(r,\ell)=0 \tag{9}$$

$$\frac{\partial T_1}{\partial r}(R,z)+hT_1(R,z)=0 \tag{10}$$

and

$$\frac{1}{a}\frac{\partial T_2}{\partial \tau}=\frac{\partial^2 T_2}{\partial z^2}+\frac{1}{r}\frac{\partial T_2}{\partial r}+\frac{\partial^2 T_2}{\partial r^2} \tag{11}$$

$$T_2(r,z,0)=T_0-T_g-T_1(r,z) \tag{12}$$

$$T_2(r,0,\tau)=0 \tag{13}$$

$$\frac{\partial T_2}{\partial z}(r,\ell,\tau)+hT_2(r,\ell,\tau)=0 \tag{14}$$

$$\frac{\partial T_2}{\partial r}(R,z,\tau)+hT_2(R,z,\tau)=0 \tag{15}$$

Solving the Cauchy problem (7)-(10)

To solve the problem (7)-(10) we are searching the solution of the form:

$$T_1(r,z)=U(r)V(z) \tag{16}$$

which substituted into the Eq. (7) leads to the following two differential equations:

$$\frac{1}{U}\frac{d^2U}{dr^2}+\frac{1}{rU}\frac{dU}{dr}=-\frac{1}{V}\frac{d^2V}{dz^2}\left(=const=-\frac{c^2}{R^2}\right) \tag{17}$$

Differential equation:

$$r\frac{d^2U}{dr^2}+\frac{dU}{dr}+\frac{c^2}{R^2}rU=0 \tag{18}$$

with the boundary condition:

$$\frac{dU}{dr}+hU=0, \quad for \;\; r=R \tag{19}$$

has the solution:

$$U(r) = K_1 J_0\left(\frac{c}{R}r\right) \tag{20}$$

where c represent the roots of the equation:

$$hRJ_0(c) = cJ_1(c) \tag{21}$$

and J_0, J_1 are respectively, the Bessel functions.

Function $V(z)$ must satisfy the differential equation:

$$\frac{d^2V}{dz^2} - \frac{c^2}{R^2}V = 0 \tag{22}$$

with the boundary conditions:

$$U(r)V(0) = T_0 - T_g \tag{23}$$

$$\frac{dV}{dz} + hV = 0, \quad for \;\; z = \ell \tag{24}$$

Equation (22) together with the boundary condition (24) has the solution:

$$V(z) = K_2\left[ch\left(\frac{c}{R}z\right) - Qsh\left(\frac{c}{R}z\right)\right] \tag{25}$$

where

$$Q = \frac{c\, sh\frac{c\ell}{R} + hR\, ch\frac{c\ell}{R}}{c\, ch\frac{c\ell}{R} + hR\, sh\frac{c\ell}{R}} \tag{26}$$

Having the product $U(r)V(z)$ and imposing condition (23) leads to solution of $T_1(r,z)$ in the form:

$$T_1(r,z) = (T_0 - T_g)\sum_{\nu=1}^{\infty}\frac{2J_1(c_\nu)}{c_\nu\left[J_0^2(c_\nu) + J_1^2(c_\nu)\right]}\cdot J_0\left(\frac{c_\nu}{R}r\right)\cdot$$
$$\cdot\left[ch\left(\frac{c_\nu}{R}z\right) - \frac{c_\nu\, sh\left(\frac{c_\nu}{R}\ell\right) + hR\, ch\left(\frac{c_\nu}{R}\ell\right)}{c_\nu\, ch\left(\frac{c_\nu}{R}\ell\right) + hR\, sh\left(\frac{c_\nu}{R}\ell\right)}\cdot sh\left(\frac{c_\nu}{R}z\right)\right] \tag{27}$$

where c_ν represent the roots of Eq. (21).

Solving the Cauchy problem (11)-(15)

To solve the problem (11)-(15) we are searching the solution of the form:

$$T_2(r,z,\tau)=X(r)W(z)e^{-\delta a\tau} \tag{28}$$

This leads to:

$$\frac{1}{X}\frac{d^2X}{dr^2}+\frac{1}{rX}\frac{dX}{dr}=-\left(\frac{1}{W}\frac{d^2W}{dz^2}+\delta\right)\left(=const=-\frac{c^2}{R^2}\right) \tag{29}$$

It is obviously that function $X(r)$ has the form of Eq. (20).

Function $W(z)$ satisfies:

$$\frac{d^2W}{dz^2}+\frac{\mu^2}{\ell^2}W=0 \tag{30}$$

where:

$$\frac{\mu^2}{\ell^2}=\delta^2-\frac{c^2}{R^2} \tag{31}$$

and the boundary conditions:

$$W(0)=0 \tag{32}$$

$$\frac{dW}{dz}+hW=0 \quad for \;\; z=\ell \tag{33}$$

and its calculated solution is of form:

$$W(z)=K_3\sin\left(\frac{\mu}{\ell}z\right) \tag{34}$$

where μ represent the roots of the equation:

$$tg\ \mu=-\frac{\mu}{h\,\ell} \tag{35}$$

The constant K_1K_3 which occurs in Eq. (28) is calculated based on Eq. (12).

The final form of the distribution of the temperature is given by:

$$T(r,z,\tau)=T_g-(T_g-T_0)\sum_{v=1}^{2}A_vJ_0\left(\frac{c_v}{R}r\right)\left[ch\left(\frac{c_v}{R}z\right)-Qsh\left(\frac{c_v}{R}z\right)\right]$$
$$-(T_g-T_0)\sum_{v=1}^{2}\sum_{k=1}^{2}B_{vk}J_0\left(\frac{c_v}{R}r\right)\sin\left(\frac{\mu_k}{l}z\right)e^{-\left(\frac{c_v{}^2}{R^2}+\frac{\mu_k^2}{l^2}\right)a\tau} \tag{36}$$

where:

$$A_v=\frac{2J_0(c_v)}{c_v\left[J_0^2(c_v)+J_1^2(c_v)\right]} \tag{37}$$

$$Q=\frac{c_v sh\left(\frac{c_v}{R}\ell\right)+hRch\left(\frac{c_v}{R}\ell\right)}{c_v ch\left(\frac{c_v}{R}\ell\right)+hRsh\left(\frac{c_v}{R}\ell\right)} \tag{38}$$

$$B_{vk}=\frac{8J_0(c_v)\left[\frac{c_v^2}{R^2}-\cos\mu_k\left(\frac{c_v^2}{R^2}+\frac{\mu_k^2}{\ell^2}\right)\right]}{c_v\left[J_0^2(c_v)+J_1^2(c_v)\right]\left(\frac{c_v^2}{R^2}+\frac{\mu_k^2}{\ell^2}\right)(2\mu_k-\sin 2\mu_k)} \tag{39}$$

and c_v, μ_k represent respectively, the positive roots of the following equations:

$$\frac{J_0(c_v)}{J_1(c_v)}=\frac{c_v}{hR}\ , \tag{40}$$

$$tg\,\mu_k=-\frac{\mu_k}{h\ell} \tag{41}$$

On the basis of Eqs. (36)-(41) we calculate the curve of the evolution of the temperature in the center of the ingot (Figure 2, curve 3).
Four intervals of three hours each were considered for drawing the curved. Thermal parameters of the originating ingot material are presented in Table 1. Solutions of Eqs. (40) - (41) are listed in Table 2. The theoretical drawn curve represents the interpolation values from the beginning of the periods of heating.
By analyzing the curves shown in Figure 2 (3 and 4) it should be noted that experimental values of the temperature does not varies greatly from the calculated ones, justifying the usefulness of the method for practical situations. The analytical method presented here can safely substitute difficult industrial experiments.

Table 1

Temperature of burned gases [°C]	450	610	705	860
λ [W/mK]	17.3	19.8	21.2	22.1
a [m^2/h]	0.0116	0.0115	0.0113	0.0110
c [J/kg K]	586	723	814	887

Table 2

Temperature of burned gases [°C]	450	610	705	860
c_1	1.9	1.8	1.7	1.7
c_2	4.6	4.5	4.4	4.3
μ_1	3	3	2.9	2.9
μ_2	6	5.9	5.9	5.9

The theory of heating of steel ingots susceptible to phase transformations in solid state, can be concluded by solutions of the following form:

Equation of thermal conduction:

$$\frac{\partial T}{\partial \tau} = a\left(\frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\right) \quad (1)$$

with initial condition:

$$T(r,z,0) = T_0 \quad (2)$$

and boundary conditions:

$$T(r,0,\tau) = T_0 \quad (3)$$

$$\frac{\partial T}{\partial z}(r,\ell,\tau) = h\left[T_g - T(r,\ell,\tau)\right] \quad (4)$$

$$\frac{\partial T}{\partial r}(R,z,\tau) = h\left[T_g - T(R,z,\tau)\right] \quad (5)$$

By dividing the temperature into a steady-state part and a time dependent part

$$T(r,z,\tau) = T_1(r,z) + T_2(r,z,\tau) + T_g \quad (6)$$

the functions T_1 and T_2 are satisfying, respectively, the following problems:

Equation with partial derivatives:

$$\frac{\partial^2 T_1}{\partial z^2} + \frac{1}{r}\frac{\partial T_1}{\partial r} + \frac{\partial^2 T_1}{\partial r^2} = 0 \quad (7)$$

with boundary conditions:

$$T_1(r,0) = T_0 - T_g \quad (8)$$

$$\frac{\partial T_1}{\partial z}(r,\ell) + hT_1(r,\ell) = 0 \quad (9)$$

$$\frac{\partial T_1}{\partial r}(R,z) + hT_1(R,z) = 0 \quad (10)$$

Equation with partial derivatives:

$$\frac{1}{a}\frac{\partial T_2}{\partial \tau} = \frac{\partial^2 T_2}{\partial z^2} + \frac{1}{r}\frac{\partial T_2}{\partial r} + \frac{\partial^2 T_2}{\partial r^2} \quad (20)$$

with initial condition:

$$T_2(r,z,0) = T_0 - T_g - T_1(r,z) \quad (21)$$

and boundary conditions:

$$T_2(r,0,\tau) = 0 \quad (22)$$

$$\frac{\partial T_2}{\partial z}(r,\ell,\tau) + hT_2(r,\ell,\tau) = 0 \quad (23)$$

$$\frac{\partial T_2}{\partial r}(R,z,\tau) + hT_2(R,z,\tau) = 0 \quad (24)$$

The solution of the problem (7)-(10) has the form:

$$T_1(r,z)=U(r)V(z) \quad (11)$$

Differential equation:

$$r\frac{d^2U}{dr^2}+\frac{dU}{dr}+\frac{c^2}{R^2}rU=0 \quad (12)$$

with the boundary condition:

$$\frac{dU}{dr}+hU=0,\ for\ \ r=R \quad (13)$$

has the solution:

$$U(r)=K_1J_0\left(\frac{c}{R}r\right) \quad (14)$$

Differential equation:

$$\frac{d^2V}{dz^2}-\frac{c^2}{R^2}V=0 \quad (15)$$

with the boundary conditions:

$$U(r)V(0)=T_0-T_g \quad (16)$$

$$\frac{dV}{dz}+hV=0,\ for\ \ z=\ell \quad (17)$$

has the solution:

$$V(z)=K_2\left[ch\left(\frac{c}{R}z\right)-Qsh\left(\frac{c}{R}z\right)\right]$$

(18)

The solution of the problem (20)-(24) has the form:

$$T_2(r,z,\tau)=X(r)W(z)e^{-\delta a\tau} \quad (25)$$

Differential equation:

$$r\frac{d^2X}{dr^2}+\frac{dX}{dr}+\frac{c^2}{R^2}rX=0 \quad (26)$$

with the boundary condition:

$$\frac{dX}{dr}+hX=0,\ for\ \ r=R \quad (27)$$

has the solution:

$$X(r)=K_1J_0\left(\frac{c}{R}r\right) \quad (28)$$

Differential equation:

$$\frac{d^2W}{dz^2}+\frac{\mu^2}{\ell^2}W=0 \quad (29)$$

where:

$$\frac{\mu^2}{\ell^2}=\delta-\frac{c^2}{R^2} \quad (30)$$

with the boundary conditions:

$$W(0)=0 \quad (31)$$

$$\frac{dW}{dz}+hW=0 \quad (32)$$

has the solution:

$$W(z)=K_3\sin\left(\frac{\mu}{\ell}z\right) \quad (33)$$

Having the product $U(r)V(z)$ and imposing condition (16) leads to solution of $T_1(r,z)$ in the form:

$$T_1(r,z)=(T_0-T_g)\sum_{\nu=1}^{\infty}\frac{2J_1(c_\nu)}{c_\nu\left[J_0^2(c_\nu)+J_1^2(c_\nu)\right]}\cdot J_0\left(\frac{c_\nu}{R}r\right)\cdot \left[ch\left(\frac{c_\nu}{R}z\right)-\frac{c_\nu\, sh\left(\frac{c_\nu}{R}\ell\right)+hR\,ch\left(\frac{c_\nu}{R}\ell\right)}{c_\nu\, ch\left(\frac{c_\nu}{R}\ell\right)+hR\,sh\left(\frac{c_\nu}{R}\ell\right)}\cdot sh\left(\frac{c_\nu}{R}z\right)\right] \quad (19)$$

The solution of $T_2(r,z,\tau)$ has the form:

$$T_2(r,z,\tau)=K_1K_3J_0\left(\frac{c}{R}r\right)\sin\left(\frac{\mu}{\ell}z\right)e^{-\delta a\tau} \quad (34)$$

The constant K_1K_3 is calculated based on Eq.(21).

The final form of the distribution of the temperature is given by:

$$T(r,z,\tau) = T_1(r,z) + T_2(r,z,\tau) + T_g \quad \Leftrightarrow$$

$$T(r,z,\tau) = T_g - (T_g - T_0)\sum_{\nu=1}^{\infty} A_\nu J_0\left(\frac{c_\nu}{R} r\right)\left[ch\left(\frac{c_\nu}{R} z\right) - Q sh\left(\frac{c_\nu}{R} z\right)\right] -$$

$$-(T_g - T_0)\sum_{\nu=1}^{\infty}\sum_{k=1}^{\infty} B_{\nu k} J_0\left(\frac{c_\nu}{R} r\right)\sin\left(\frac{\mu_k}{\ell} z\right) e^{-\left(\frac{c_\nu^2}{R^2}+\frac{\mu_k^2}{\ell^2}\right)a\tau} \tag{35}$$

where:

$$A_\nu = \frac{2J_0(c_\nu)}{c_\nu\left[J_0^2(c_\nu) + J_1^2(c_\nu)\right]}, \quad Q = \frac{c_\nu sh\left(\frac{c_\nu}{R}\ell\right) + hR ch\left(\frac{c_\nu}{R}\ell\right)}{c_\nu ch\left(\frac{c_\nu}{R}\ell\right) + hR sh\left(\frac{c_\nu}{R}\ell\right)}$$

$$B_{\nu k} = \frac{8J_0(c_\nu)\left[\frac{c_\nu^2}{R^2} - \cos\mu_k\left(\frac{c_\nu^2}{R^2} + \frac{\mu_k^2}{\ell^2}\right)\right]}{c_\nu\left[J_0^2(c_\nu) + J_1^2(c_\nu)\right]\left(\frac{c_\nu^2}{R^2} + \frac{\mu_k^2}{\ell^2}\right)(2\mu_k - \sin 2\mu_k)}$$

and c_ν, μ_k represent respectively, the positive roots of the following equations:

$$\frac{J_0(c_\nu)}{J_1(c_\nu)} = \frac{c_\nu}{hR}, \quad tg\,\mu_k = -\frac{\mu_k}{h\ell}. \tag{36}$$

3. Verifying the study

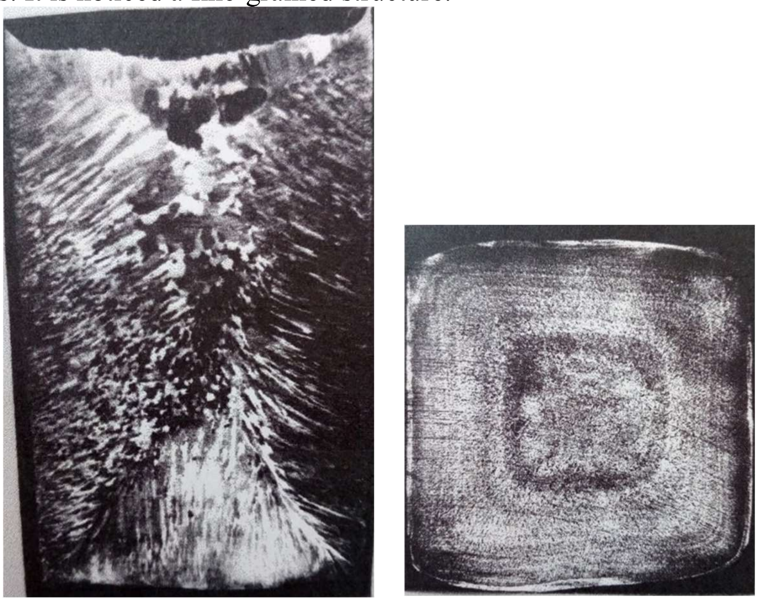
Generally, in the metallurgical industry from Hunedoara half-finished products are made from these steels destined to further processing (pipe extrusion or lamination, forging etc.).

The guarantee of the usage level can be made by sending a laminate with as constant as possible physical-chemical properties on the length and section of the laminate.

From the technological process point of view to the ingot phase, the reached performances are normal in the world (electrical furnace flow –VAD-VOD); as a result the attention is focused on the deforming part, meaning the most sensitive period, of passing from the solidifying structure of the ingot (non-homogenous, segregated, loose) to the relatively finished structure of the laminate.

The guarantee of the stability of the process, knowing the deforming capacity in specific industrial condition; gas level in steel, pouring temperature, thickness and the temperature of the wall in the ingot, train refractory, dust ointment, period of maintaining at the pouring stand, stripping conditions, heating regime etc. can assure a product (in our case a half-finished product) which allows at further processing an insurance of the mechanical qualities for corrosion resistance and corresponding refractory to the final usage of the product.

The steel used for experiments (10TiMoNiCr175 steel grade), was elaborated in an electrical furnace of 20 tones, and poured in ingots of 3,6 tones. Figure 3 a shows the internal structure resulting from the longitudinal sectioning of one of the obtained ingots. It is noticed a pronounced segregation. Figure 3 b highlights the structure obtained after rolling, on another ingot obtained under the same technological conditions. It is noticed a fine-grained structure.



a - ingot structure b - laminate structure

Fig.3. Passing from the solidification structure (a), to the laminate structure (b).

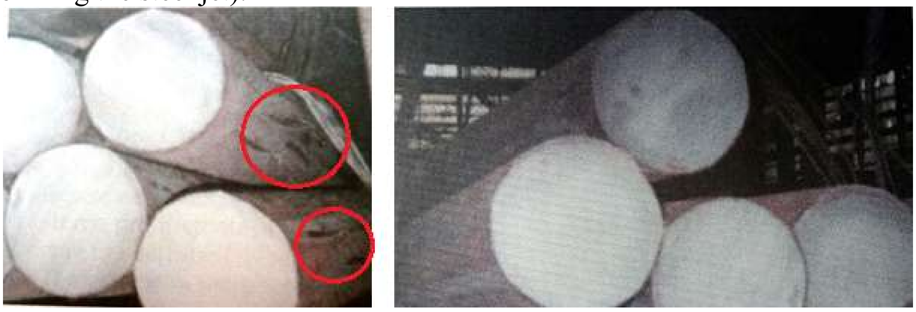
From the elaborated steel 6 ingots were poured. Three of these (first lot) were heated according to the technology used in the plant, and the other three (second lot) according to the diagram from Fig.2, which took into account the evolution of the ingot temperature during the preheating period (20-860°C).

The interrupted curve 3 from figure 2 was analytically calculated after the mathematical study accomplished in the paper.

In both of the cases, according to the actual technology, half-finished round profile products were laminated with a diameter of 180mm.

At receiving the first lot, on the surface of the laminates flaws have appeared with variable depth and distribution, highlighted in red in figure 4a. These flaws are caused by an insufficient mixing (chemical-by diffusion and thermal) of the ingots during the heating.

Receiving the second lot, the surface of the material was corresponding on its entire length (figure 4b). On a single laminated bar, two flaws had appeared under the aspect of some prints, due to the drops from the initial pouring phase (before stabilizing the steel jet).



a) b)
Fig. 4. The surface of obtained laminates (a-first lot; b-second lot)

The proposed alternative (heating according to figure 2) assures the shortening of the heating time with over 5 hours, with economic effects generated by reducing the quantity of the fuel and increasing the productivity of the furnaces.

4. Conclusions

In this paper, we have proposed a framework for definition and knowledge of the evolution of the thermal field in industrial steel ingots of mark 10TiMoNiCr175, during the preheating period. Experiments of this type in current production conditions are particularly difficult. Difficulties are caused by the need for thermal insulation of the connecting wires of the thermocouples against the temperature and atmosphere in the furnace.
For this reason, the key innovation introduced in this paper is represented by analytical solutions to determine the evolution of the ingot mass temperature. The paper's credit consists in presenting an analytical methodology to determine the evolution of temperature from the center of some massive steel ingots, as opposed to the situation when this evolution is emphasized in an experimental way.
Figure 2 (curves 3 and 4) illustrates that the experimental and analytically calculated values of the ingot temperature in the center differ insignificantly, validating the analytical method and justifying the usefulness of the presented method for practical situations.
Calculating the evolution of the temperature from the center of the ingots, the proposed mathematical technique substitutes with high accuracy some difficult experiments, almost impossible to perform in conditions of the current practice. The use of the proposed analytical procedure assures the shortage of the total duration of the heating, with economic effects generated by reducing the fuel quantity and increasing the productivity of the deep furnace.

References

[1] Samoilă C., Drugă L., Stan L., *Furnaces and heating installations*, Technical Publishing House, Bucureşti, 1983.

[2] Dima A., Caţarschi V., Fodor V., *Efficient heat transfer to the semi-finished products in the work room, by optimizing the design and operation of the heating furnaces*, Section works, "FORTUS" 85, Iaşi, 17-18 Oct. 1985, p. 15-18.
[3] Vida-Simiti I., Jumate N., Thalmaier G., Sechel N., Moldovan V., *Metallic membranes obtained with graded structure for microfiltration*, Environmental Engineering and Management Journal, **10**, 10, 2011, p. 1425-1608.
[4] Bhadeshia N.K.D.H., Honeycombe R., *Steel Microstructure and Properties*, Edition III, Ed.Butterworth – Elsevier Ltd., USA, 2006.
[5] Yanushkevich Z., Lugovskaya A., Belyakov A., Kaibyshev R., *Deformation microstructures and tensile properties of an austenitic stainless steel subjected to multiple warm rolling*, Materials Science & Engineering, **A 667**, 2016, p. 279–285.
[6] Huang J., Ye X., Xu Z., *Effect of Cold Rolling on Microstructure and Mechanical Properties of AISI 301LN Metastable Austenitic Stainless Steels*, Journal of Iron and Steel Research, International, **19**, 10, 2012, p. 59-63.
[7] Sun G. et. al., *Plastic flow behavior and its relationship to tensile mechanical properties of high nitrogen nickel-free austenitic stainless steel,* Materials Science and Engineering, **A 662,** 2016, p. 432–442.
[8] Ilca I., *Thermal stresses at ingots heating of quality steels in industrial furnaces,* "Advanced materials, thermal treatments and quality management", Academy of Technical Sciences, Timișoara, 2001, p.144.
[9] Ilca I., Gaita D., Miloştean D., *Heating optimization of high grade steels for plastic processing*, XVI International - multidisciplinary conference "Professor Dorin Pavel - the founder of Romanian hydropower", Sebeş, Science and Engineering, 29-30, 2016, p. 99-106
[10] Ilca I., Kiss I., Alexa V., Rațiu S.A., *Optimisation of the thermal treatment technologies for the cast hipereutectoid steel rolls,* Annals of Faculty Engineering Hunedoara – International Journal of Engineering, Tom XIV/3, 2016, p. 201-206.
[11] Xi T., Yang C., Shahzad M.B., Yang K., *Study of the processing map and hot deformation behavior of a Cu-bearing 317LN austenitic stainless steel*, Materials & Design, **87**, 2015, p. 303–312.
[12] Ilca I., Kiss I., Miloștean D., *Study on the quality of Boron micro–alloyed steels destined to applications in the automotive sector*, International Conference on Applied Sciences, ICAS 2018, Banja Luca 10-11 May 2018, IOP Conf. Series: Materials Science and Engineering, **477**, 2019.
[13] Catarschi V., Cucos I., Ioniță I., Comaneci R*., Researches on the Heating Speed Influence on the Gradient and on Thermal Tension at the Alloy Steel Ingot Heating*, Proceedings of the first International Congress of Materials Science and Engineering, Iași, 1995, Bulletin of the Technical University of Iași, Tom XL/II, 1995, p 980-985.
[14] Ilca I., *The theory and practice of heating quality steel ingots*, Journal of Engineering Science Innovation, **1**, 1, 2016, p 120-130.
[15] Ilca I., *The determination of the gradient and thermal stresses for the heating of type 18-10 stainless steel ingots,* Journal of Engineering Science Innovation, **2**, 2, 2017, p. 58-74.
[16] Taiţ N., *Tehnologia nagreva stali*, Metalurgiydat, 1962, Moscva.
[17] Ilca I., Magaon J., *Structural research on semi–industrial high temperature stainless steel, thermostable*, VII International Congress, „Machinery, Technology, Materials" Sofia, Bulgaria, 2010.
[18] Marshak R. E., *Effect of radiation on shock wave behavior*, Physics of Fluids, **1**, 1958, p. 24–29.
[19] Parlange J. Y., Hogarth W. L., Parlange M. B., Haverkamp R., Barry D. A., Ross P. J., Steenhuis T. S., *Approximate analytical solution of the nonlinear diffusion equation for arbitrary boundary conditions*, Transport in Porous Media, **30**, 1, 1998, p. 45–55.
[20] Barbaro G., Bianco N. and Manca O., *One-dimensional approximate analytical solutions of heat conduction in semi-infinite solids with temperature-dependent properties*, Hybrid Methods in Engineering, Volume ¾, 2001, p. 345–380.
[21] Bistrian D. A., *Mathematical Models and Numerical Algorithms for Stability Investigation of Swirling Hydrodynamic Systems*, Politehnica Publishing House, ISBN: 978-606-554-264-8, 2011.