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Efficient method for position control of a redundant robot

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Abstract. To begin the work presents some redundancy resolution schemes for robotic arms, i.e., the techniques for exploiting the redundant degrees of freedom in the solution of the inverse kinematics problem. This is obviously an issue of major relevance for motion planning and control purposes. In particular, task-oriented kinematics and the basic methods for its inversion at the velocity (first-order differential) level are first recalled. This paper focuses on modeling and simulations of the inverse kinematics of an anthropomorphic redundant robotic structure with seven degrees of freedom and a workspace similar to human arm. Also the kinematic model of the robotic arm in the MATLAB and Simulink environment is presented. A method of resolving the redundancy of a seven degrees of freedom robotic arm when a degree of freedom has a known variation is presented. The kinematic analysis and virtual simulation share similar results.

Keywords: robot arm, redundancy, inverse kinematics.

1. Introduction

The term robot dates from several decades ago. Man has imagined intelligent mechanized devices that can take a significant amount of physical effort. Thus, humans have built automatic mechanisms and intelligent toys[1],[2].
By introducing robots in industrial automation a clear increase in productivity and product quality resulted. Robots can work day and night without fatigue nor reducing performance. They consistently achieve substantial reductions in the cost

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price, first by reducing the consumption of raw materials and then by processing them automatically [3],[4].

With the rapid development of industry and computers, one can observe the evolution of robots towards intelligent generations with features that allow them to understand the environment in which they work.

After a preliminary analysis of the literature, mainly based on series and anthropomorphic robots, the established aim was to develop a method for solving the kinematic problem of an anthropomorphic serial robot with seven degrees of freedom. The serial anthropomorphic robot is very similar to the structure of the human arm. The robot acquires by association with man seven degrees of freedom, with one degree of freedom more than what is required for positioning and orientating a solid body in Euclidean space [5], [6].

Therefore the robotic arm with seven degrees of freedom is redundant. As a conclusion the inverse kinematics resolution of a seven DOF robotic arm has an infinite number of solutions.

2. The structure of the human arm

There can be considered different parameters that can characterize certain properties of human arm, for example degrees of mobility, the magnitude and dimensions of arm movements. We can consider that the human arm represented in simplified form, acquires three DOF for the shoulder joint, one DOF for the elbow joint and three DOF for the hand wrist[7],[9]. A simplified structure of a human arm joints is shown in Fig.1.

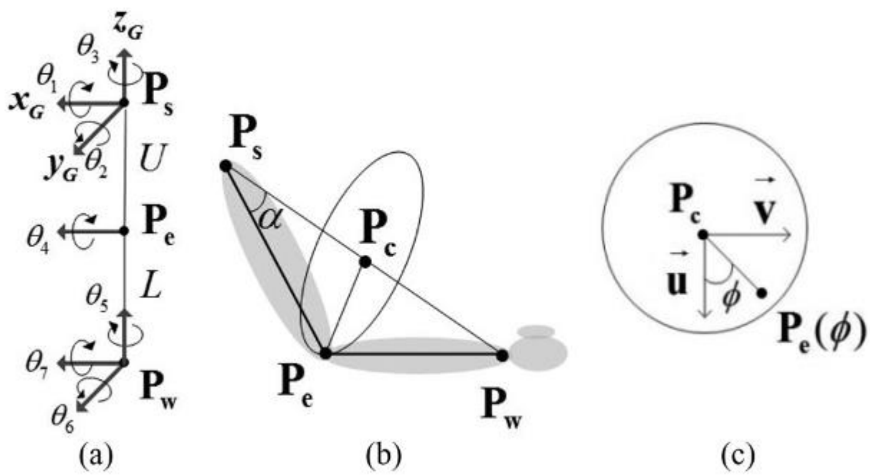

Fig. 1. Human arm structure.

3. The kinematic analysis of an anthropomorphic robotic arm

In the Figure 2 is presented the structural scheme of the seven DOF robotic arm. If we attach to each element "i", (i = 0…7) of the structure, one fixed coordinate system $k_i(O_i, x_i, y_i, z_i)$, then we can express the homogeneous transfer matrices A_i which characterize the relative movements between each element of the mechanic structure. If we know the relative parameters θ_i (i = 1 …7) and the homogeneous transfer matrix form between two elements or the homogeneous transfer matrix between the coordinate systems attached to each element, we can determine the total transfer matrix between the system $k_7(O_7, x_7, y_7, z_7)$ and system $k_0(O_0, x_0, y_0, z_0)$

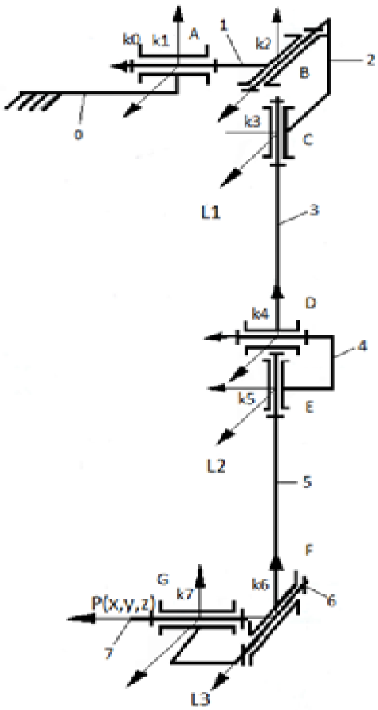

Fig. 2. Kinematic structure of the robotic arm.

$$H_{07} = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6 \cdot A_7 = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \quad (1)$$

were:

$$A_1 = R_y(\theta_1) = \begin{pmatrix} cos(\theta_1) & 0 & sin(\theta_1) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(\theta_1) & 0 & cos(\theta_1) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

$$A_2 = R_x(\theta_2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(\theta_2) & -sin(\theta_2) & 0 \\ 0 & sin(\theta_2) & cos(\theta_2) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

$$A_3 = R_z(\theta_3) \cdot T_z(L1) = \begin{pmatrix} \cos(\theta_3) & \sin(\theta_3) & 0 & 0 \\ -\sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & L1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{4}$$

$$A_4 = R_y(\theta_4) \cdot T_z(L2) = \begin{pmatrix} \cos(\theta_4) & 0 & \sin(\theta_4) & \sin(\theta_4) \cdot L2 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta_4) & 0 & \cos(\theta_4) & \cos(\theta_4) \cdot L2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{5}$$

$$A_5 = R_z(\theta_5) = \begin{pmatrix} \cos(\theta_5) & \sin(\theta_5) & 0 & 0 \\ -\sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{6}$$

$$A_6 = R_x(\theta_6) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_6) & -\sin(\theta_6) & 0 \\ 0 & \sin(\theta_6) & \cos(\theta_6) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{7}$$

$$A_7 = R_y(\theta_7) \cdot T_y(L3) = \begin{pmatrix} \cos(\theta_7) & 0 & \sin(\theta_7) & 0 \\ 0 & 1 & 0 & L3 \\ -\sin(\theta_7) & 0 & \cos(\theta_7) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{8}$$

To resolve the inverse kinematics problem we need to start from the fact that we know the position and the orientation of the end effector $x, y, z, \varphi_x, \varphi_y, \varphi_z$ in reference to the fixed coordinate system and we need to determine the relative positions between robot's elements. Therefore we wish to determine the relative

parameters θ_i between elements which represent the rotation in the kinematic couplings. In this case (i = 1 .. 7). Since this type of robot has a degree of mobility in excess of six possible in the 3D space we impose an additional condition to solve the system of equations [8], [10]. With this constraint one of the angles θ_i is considered known. The steps for solving the inverse kinematic problem are:
The transfer matrix which characterize the end effector position and orientation in reference to a fixed coordinate system $k_0(O_0, x_0, y_0, z_0)$ has to be created, based on the absolute parameters $x, y, z, \varphi_x, \varphi_y, \varphi_z$; The wrist position x_{im}, y_{im}, z_{im} has to be determined; Knowing the wrist position and orientation we will determine the relative parameters θ_i, *(i=1...7)*; We will consider these notations:
P (x, y, z) is the end effector position in reference to the fixed coordinate system, this section has attached the coordinate system $k_7(O_7, x_7, y_7, z_7)$, L1 - is the arm length; L2 - the length of the forearm; L3 - the length from P (x, y, z) to the wrist ;
The transfer matrix which expresses the position and orientation of the end effector in reference to a fixed coordinate system $k_0(O_0, x_0, y_0, z_0)$ is composed. The transfer matrix is denoted H_{07} and consists of the product of all homogeneous transfer matrices that characterize the end effector position and orientation in reference to a fixed system of coordinates,

$$H_{07} = T_x \cdot T_y \cdot T_z \cdot R_x \cdot R_y \cdot R_z \tag{9}$$

The matrix that determines the position of the hand wrist is shown below:

$$H_{im} = H_{07} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \tag{10}$$

We will note:

$$(a_{14} \cdot a_{24} \cdot a_{34} \cdot a_{44})^T = H_{im} \cdot (0 \quad 0 \quad 0 \quad 1)^T \tag{11}$$

The distance between the hand wrist position and $k_0(O_0, x_0, y_0, z_0)$ is:

$$p = \sqrt{a^2{}_{24} + a^2{}_{24} + a^2{}_{34}} \tag{12}$$

The angle θ_4 is determined using the expression:

$$\theta_4 = \pi \pm \arccos\left(\frac{L1^2 + L2^2 - p^2}{2 \cdot L1 \cdot L2}\right) \tag{13}$$

To determine the other angles θ_1 angle is considered as known, or has a known law of variation over time. This is the easiest method of solving the inverse kinematics problem for this serial and redundant robot. We have the following system of equations:

$$A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a_{14} \\ a_{24} \\ a_{34} \\ 1 \end{pmatrix}. \tag{14}$$

If you multiply to the right with A_1^{-1} results the following system of equations:

$$A_2 \cdot A_3 \cdot A_4 \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b_{14} \\ b_{24} \\ b_{34} \\ 1 \end{pmatrix}. \tag{15}$$

We also note:

$$\begin{pmatrix} b_{14} \\ b_{24} \\ b_{34} \\ 1 \end{pmatrix} = A_1^{-1} \cdot \begin{pmatrix} a_{14} \\ a_{24} \\ a_{34} \\ 1 \end{pmatrix}. \tag{16}$$

Because θ_4 is known it results from the first equation of the system (16)

$$\theta_3 = \arccos\left(\frac{b_{14}}{L2 \cdot \sin(\theta_4)} \right). \tag{17}$$

To determine the angles $\theta_5, \theta_6, \theta_7$ we consider the following system :

$$A_5 \cdot A_6 \cdot A_7 = A_4^{-1} \cdot A_3^{-1} \cdot A_2^{-1} \cdot A_1^{-1} \cdot H_{im} \tag{18}$$

Also we will note:

$$A_4^{-1} \cdot A_3^{-1} \cdot A_2^{-1} \cdot A_1^{-1} \cdot H_{im} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \tag{19}$$

From the equality of the two matrices results the following:

$$\begin{cases} \theta_6 = \arcsin(m_{32}) \\ \theta_7 = -\arctan 2(m_{31}, m_{33}) \\ \theta_5 = -\arctan 2(m_{12}, m_{22}) \end{cases} \tag{20}$$

4. Inverse kinematics simulation

In order to obtain numerical results for the kinematic analysis, the mathematical equations presented above, were written in MATLAB ® as cod.m then implemented in Simulink ®.The implementation was done in an Embedded MATLAB Function block. Numerical results for inverse kinematics of the positions were obtained for the following values of absolute parameters:

$$X = 428.2 \text{ mm}, Y = 19.79 \text{ mm}, Z = 272.4 \text{ mm},$$
$$\varphi_x = -51.85^\circ, \varphi_y = 60.57^\circ, \varphi_z = 40.86^\circ.$$

The input values were chosen in such way to carry out a verification of the inverse kinematics equations. The results for angles q_i are listed below:

$$q_1 = 50^\circ, q_2 = 30^\circ, q_3 = 75.01^\circ, q_4 = 60^\circ,$$
$$q_5 = -80.01^\circ, q_6 = 5.002^\circ, q_7 = 6.009^\circ$$

In order to simulate the kinematics of the robot arm virtual models were needed to be generated. After the inverse kinematics simulation, for the same values of end effector position and orientation, used in numerical analysis we have reached the same values of angles q_i see the Fig. 3.

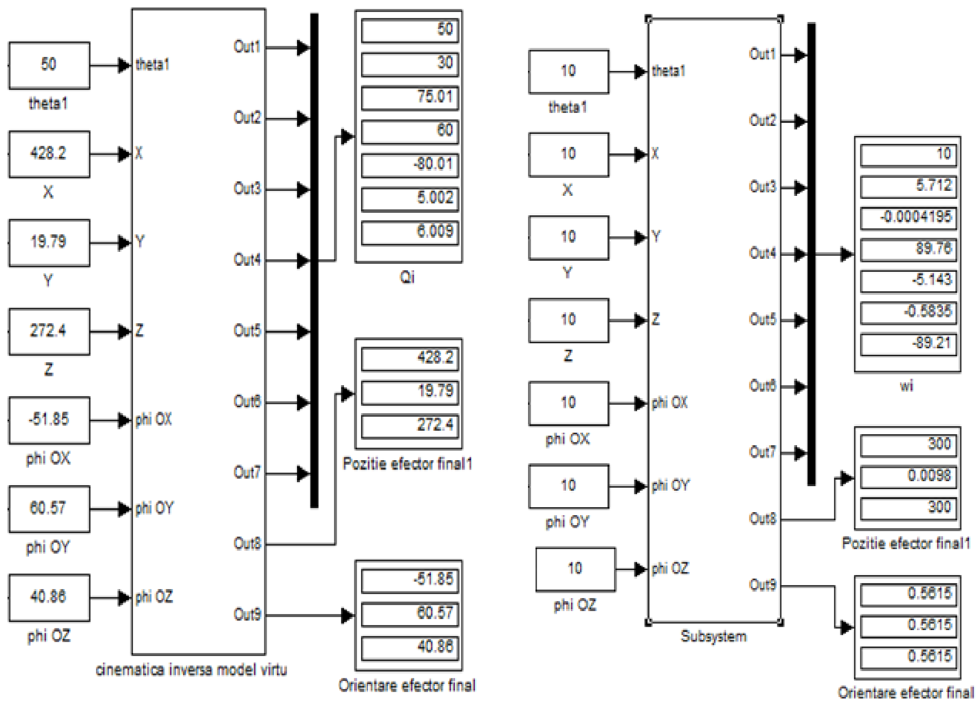

Fig. 3. Simulation results of inverse kinematic positions.

5. Conclusions

A kinematically redundant manipulator possesses more joints than those strictly required to execute its task. This provides the robot with an increased level of dexterity that may be used to avoid singularities, joint limits, and workspace obstacles, but also to minimize joint torque, energy or, in general, to optimize suitable performance indexes. The biological archetype of kinematically redundant manipulator is the human arm, which, not surprisingly, also inspires the terminology used to characterize the structure of serial-chain manipulators.
In fact, the human arm has three degrees of freedom at the shoulder, one degree of freedom at the elbow and three degrees of freedom at the wrist. The available redundancy can be easily verified by locking one's wrist, e.g., on a table and moving the elbow while keeping the shoulder still. Mathematical equations were developed based on the kinematic structure but also after consultation of the scientific literature. The chosen analytical method for solving inverse kinematics problem was effective, accurate and easy to implement. It is based on the fact that a DOF is considered known, as from the seven DOF only six remain unknown. Euclidean three-dimensional space has six DOF so the equations for the kinematic system is determined. The fact that a DOF is considered known is not an ideal solution to explore the full redundancy of the robot. This doesn't pose as a seriously problem. Mathematical equations underlying the robot kinematics have been implemented in MATLAB which led to their effective resolution.

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