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On the acoustic invisibility

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Abstract. Over the past decade a great attention has been paid to the modeling and simulation of making objects invisible or deaf to noise. The recent results have enlightened the mode of control and handling the invisibility and cloaking to noise using the metamaterials engineered at the subwavelength scale in order to interact with acoustic field in a way that traditional materials do not. The acoustic invisibility is based on the property of acoustic equations to be invariant under a spatial compression that means a manipulation of the material parameters. In this paper, the sound invisibility performance is discussed for spherical cloaks. The original domain consists of an alternation of layers made from piezoelectric ceramics and epoxy resin, following a triadic Cantor sequence. The spatial compression, obtained by applying the concave-down transformation, leads to a metamaterial layer with an inhomogeneous and anisotropic distribution of the parameters.

Keywords: metamaterials, acoustic invisibility, cantor sequence, acoustic metamaterial.

1 Introduction

The idea of invisibility reminds us the Greek legend of Perseus versus Medusa to Well's Invisible Man and continue to fascinated us. Newton saw the colour as a physical problem, and Goethe as a mechanics of human vision and information. The acoustic invisibility refers to the “melting” of the sound or its disappearance, with benefits to many applications. The acoustic equation is invariant under a coordinate transformation and this is the fundamental idea to design the acoustic metamaterials who does not hear the sound [1-6]. In other world, the acoustic cloaking is a coating region in which the external noise is hidden. We can imagine a region invisible to sound as a box where the sound cannot penetrate.

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Cummers *et al*. [2] studied a spherical shell that cancels the propagation of noise from an arbitrary source situated within the shell. The simulation confirmed that the waves are bent excluded from the shell. Cummer and Schurig [3] demonstrate that 3D acoustic equations in a fluid are identical as form to Maxwell equations with the same type of boundary conditions. Recent works show that acoustic metamaterials could cloak regions of space, making them invisible to sound [7-15]. The principle of a cloak region which is invisible or transparent to waves was discussed in [16-19].

As an alternative, the sonic composites exhibit the full band-gaps where the sound is not allowed to propagate due to complete reflections [20-27]. Another alternative is the antinoise when the acoustic field is mimicked and reduced by using another source [28- 32]. This is in contrast to 3D electromagnetic cloak [33] for which the analysis does not need reasons to show that the scattering is canceled [34]. This is due to the acoustic waves that involve solutions of the Helmholtz equation expressed by spherical Bessel functions that do not all tend to zero as their argument approaches zero. By using this result, a class of rectangular cylindrical devices for noise shielding was investigated [35]. The correspondence between the electromagnetic and acoustic cloaks is analyzed by exploiting the nature of the partial differential equations which are reducing to Helmholtz equations which are the key for sound invisibility [21-28].

In this paper, we apply the concave-down transformation to obtain a spherical cloak which surrounds a noisy source (Fig. 1). The original domain is a sphere of radius R_2 consisting of layers of piezoelectric ceramics and epoxy resin following a triadic Cantor sequence. After the transformation, the cloak is a region $r < R_1$ filled with air and containing the noisy source and the layer $R_1 < r < R_2$ is filled by a metamaterial.

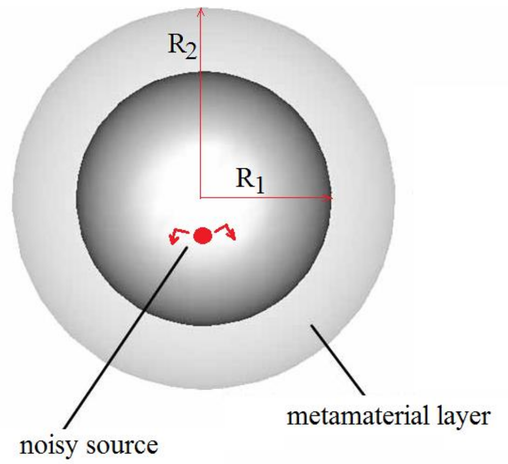

Fig. 1. The spherical cloak surrounding a noisy machine.

2. The coordinate transformation

The idea of the invisibility is related to the acoustic path $c^{-1}\mathrm{d}s$, $c^{-1}=\sqrt{\rho/\kappa}$, where ρ is the fluid density and κ the compression modulus of the fluid in the region where the sound travels [4, 5, 36-38].

The acoustic equation of the pressure waves in the initial domain of a bounded fluid region $\Omega\subset \mathrm{R}^3$ is

$$\nabla\cdot(\underline{\underline{\rho}}^{-1}\nabla p)+\frac{\omega^2}{\kappa}p=0, \tag{1}$$

where p is the pressure, $\underline{\underline{\rho}}$ is the tensor of the fluid density, κ is the compression modulus of the fluid, and ω is the wave frequency.

The coordinate transformation changes the coordinate system (x',y',z') of the compressed space to the original coordinate system (x,y,z) being characterized by the Jacobian $J_{xx'}$

$$\begin{pmatrix}\mathrm{d}x\\ \mathrm{d}y\\ \mathrm{d}z\end{pmatrix}=J_{xx'}\begin{pmatrix}\mathrm{d}x'\\ \mathrm{d}y'\\ \mathrm{d}z'\end{pmatrix},\qquad J_{xx'}=\frac{\partial(x,y,z)}{\partial(x',y',z')}. \tag{2}$$

The associated metric tensor is given by [39, 40]

$$T=\frac{J_{xx'}^{\mathrm{T}}\,J_{xx'}}{\det(J_{xx'})}. \tag{3}$$

By applying (2), the homogeneous and isotropic material from the original region is replaced by another anisotropic and inhomogeneous material whose properties are expressed by

$$\underline{\underline{\rho}}'=J_{x'x}^{-\mathrm{T}}\cdot\rho\cdot J_{x'x}^{-1}\cdot\det(J_{x'x}),\quad \kappa'=\kappa\det(J_{x'x}), \tag{4}$$

or

$$\underline{\underline{\rho}}'=\frac{J_{xx'}^{\mathrm{T}}\cdot\rho\cdot J_{x'x}}{\det(J_{xx'})},\ \kappa'=\frac{\kappa}{\det(J_{xx'})}, \tag{5}$$

where $\underline{\underline{\rho}}'$ is a second order tensor. Multiplying (1) by a test function φ and integrating by parts, we obtain [4, 5]

$$-\int_{\Omega}\left(\nabla_{(x,y,z)}\varphi\cdot\underline{\underline{\rho}}^{-1}\nabla_{(x,y,z)}p\right)\mathrm{d}V+\int\left(\omega^2\kappa^{-1}p\varphi\right)\mathrm{d}V=0. \tag{6}$$

By applying the coordinate transformation $(x,y,z)\to(x',y',z')$ to (6), we obtain from (2)

$$-\int_{\Omega}\left(J_{x'x}^{\mathrm{T}}\nabla_{(x',y',z')}\varphi\cdot\underline{\underline{\rho}}^{-1}J_{x'x}^{\mathrm{T}}\nabla_{(x,y,z)}p\right)\det(J_{xx'})\mathrm{d}V'+\int\left(\det(J_{xx'})\omega^2\kappa^{-1}p\varphi\right)\mathrm{d}V'=0, \tag{7}$$

or

$$-\int_{\Omega}\left(\left(\nabla_{(x',y',z')}\varphi\right)^{\mathrm{T}}\frac{J_{x'x}\underline{\underline{\rho}}^{-1}J_{x'x}^{\mathrm{T}}}{\det(J_{x'x})}\nabla_{(x',y',z')}p\right)\mathrm{d}V'+\int\left(\frac{\kappa^{-1}}{\det(J_{x'x})}\omega^{2}p\varphi\right)\mathrm{d}V'=0\,. \quad (8)$$

The coordinate transformations are concave-up or concave-down depending on the sign of the second order derivative of the transformation function. The concave-down transformation compresses a sphere of radius R_2 in Ω into a shell $R_1 < r' < R_2$ in the compressed space Ω' as

$$r(\beta)=\frac{R_2^{\beta+1}}{R_2^{\beta}-R_1^{\beta}}\left(1-\left(\frac{R_1}{r'}\right)^{\beta}\right), \quad (9)$$

where β the degree of the nonlinearity of the transformation. For $\beta \to 0$, we obtain the linear case

$$r(\beta)=\frac{R_2\mathrm{Ln}(r'/R_1)}{\mathrm{Ln}(R_2/R_1)}\,. \quad (10)$$

All curves (9) have negative second order derivative with respect to the physical space r'. This class of transformations is the *concave-down* transformation. The transformation function (9) depends on the radial component r' in the spherical coordinate system (r',θ',ϕ') [41].

The concave-up nonlinear transformation compresses a sphere of the radius R_2 in the original space Ω into a shell region $R_1 < r' < R_2$ in the compressed space Ω' as

$$r(\beta)=\frac{R_2R_1^{\beta}}{R_2^{\beta}-R_1^{\beta}}\left(\left(\frac{r'}{R_1}\right)^{\beta}-1\right). \quad (11)$$

For $\beta \to 0$,we obtain (10). This class of transformations is the *concave-up* transformation because (11) has positive second order derivatives. The cloak properties in the both transformed coordinates are given by (4) and (5) where $J_{r'r}=\partial r'/\partial r$.

The equations of propagation of the elastic waves with a time harmonic dependence are written as [2, 6]

$$\nabla\cdot C:\nabla u+\rho\omega^2 u=0\,, \quad (12)$$

where ρ is the scalar density of an isotropic heterogeneous elastic medium, C is the fourth-order elasticity tensor, ω is the wave angular frequency, and $u(x_1,x_2,x_3,t)=u(x_1,x_2,x_3)\exp(-\mathrm{i}\omega t)$ is the displacement vector. Under a change of coordinates (x',y',z') to (x,y,z) with $u'(x')=J_{x'x}^{-\mathrm{T}}u(x)$, $J_{x'x}=\dfrac{\partial(x',y',z')}{\partial(x,y,z)}$, we obtain from (12)

$$\nabla''\cdot(C'+S'):\nabla'u'+\underline{\underline{\rho}}'\omega^2u'=D':\nabla'u'\,, \quad (13)$$

which preserves the symmetry of the new elasticity tensor $C' + S'$ [5]. Equation (13) contains two third-order symmetric tensors S' and D' with $D'_{pqr} = S'_{qrp}$ and a second-order tensor ρ'_{pq}.

3. The spherical acoustic cloak

The initial domain is made from concentric homogeneous and isotropic layers situated in the sphere Ω of radius R_2. After the coordinate transformation, an equivalent compressed inhomogeneous anisotropic material described by (3) is obtained.

The sphere Ω consists of an alternation of layers made from piezoelectric ceramics and epoxy resin, following a triadic Cantor sequence up to 31 elements (Fig. 2). The dashed regions are occupied by piezoelectric ceramics and the white regions are occupied by epoxy-resins.

The experimental and theoretical evidence of extremely low thresholds for the subharmonic generation of ultrasonic waves in artificial piezoelectric plates with Cantor-like structure as compared to the corresponding homogeneous and periodical plates were performed in [43-46]. A nonlinear interaction between the extended-vibration (phonon) and the localized-mode (fracton) regimes explained this behavior. The equations which govern the subharmonic ultrasonic wave phenomenon were solved by using the cnoidal method, which employed the cnoidal wave as the fundamental basis function [37, 47, 48].

The quasistatic motion equations and constitutive laws of initial material are

$$\rho \ddot{u}_i = t_{ij,j}\,, \tag{14}$$

$$D_{i,i} = 0\,, \quad E_i + \varphi_{e,i} = 0\,, \tag{15}$$

$$t_{ij} = \lambda^p \varepsilon_{kk} \delta_{ij} + 2\mu^p \varepsilon_{ij} - e_k^p E_k \delta_{ij}\,, \tag{16}$$

$$t_{ij} = \lambda^e \varepsilon_{kk} \delta_{ij} + 2\mu^e \varepsilon_{ij}\,, \tag{17}$$

$$D_i = \overline{\varepsilon}^p E_i - e_i^p \varepsilon_{kk}\,, \tag{18}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})\,. \tag{19}$$

Indices p and e denote the piezoelectric (PZ) and non-piezoelectric (ER) materials, respectively, ρ is the density, u_i, $i = 1,2,3$, are the components of the displacement vector, t_{ij}, $i = j = 1,2,3$, are the components of the stress tensor, D_i, $i = 1,2,3$, are the components of the electric induction vector, E_i, $i = 1,2,3$, are the components of the electric field and φ_e is the electric potential, ε_{ij}, $i = j = 1,2,3$, are the components of the strain tensor, λ, μ are the Lamé constants, $\bar{\varepsilon}^p$ is the dielectric constant and e_i^p ($e_3^p = e_2^p = e_1^p$) are the piezoelectricity coefficients. The coordinate x_1 is directed along the radial direction, x_3 is directed along the circumferential direction, while x_2 is located within the layer.

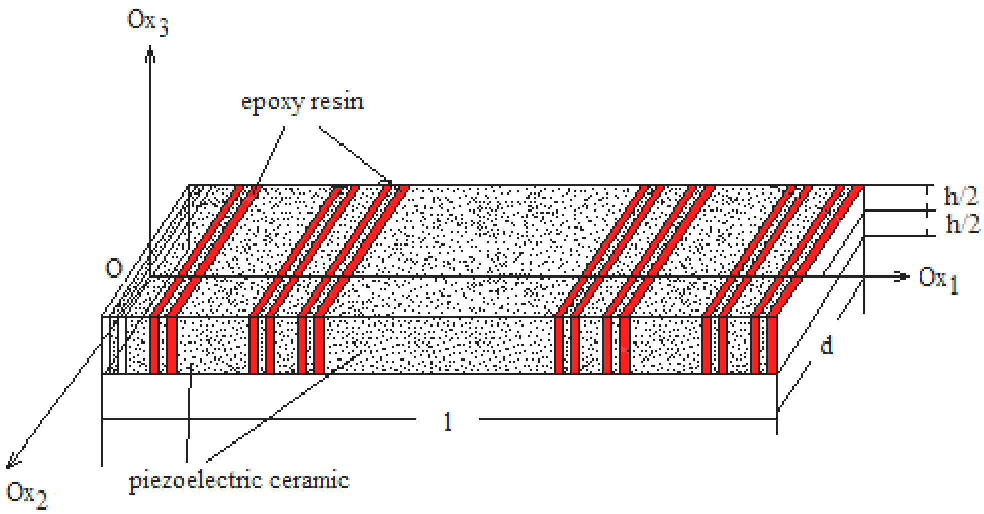

Fig. 2. The Cantor-like structure [5].

The solution of (1-14) is the theta function Θ which verifies the equation [5, 47]

$$\nabla \cdot \zeta_{p,e}^{-1} \nabla(\Delta\nabla \cdot \zeta_{p,e}^{-1} \nabla\Theta) - \Lambda^{-1}\gamma_0^4\Theta = 0, \tag{20}$$

where $\zeta = E^{-1/2}$, E is the effective Young modulus of the composite, $\gamma_0^4 = \omega^2\rho h / D_0$, D_0 is the flexural rigidity of the plate, ρ its effective density, h its thickness, $\Lambda = \rho^{-1}$ and ω the frequency. Eq. (20) can be factorized as a Helmholtz operator and an anti-Helmholtz operator

$$(\nabla^2 + \gamma_0^2)((\nabla^2 - \gamma_0^2)\Theta = 0, \tag{21}$$

where for simplicity $\zeta = \Lambda = 1$. We write the Helmholtz equation in the coordinate system (x_1, x_2, x_3) as

$$\nabla \cdot (\zeta^{-1}\nabla\Theta) + \omega^2\Lambda^{-1}\Theta = 0. \tag{22}$$

Let us apply the concave-down transformation (9) to (20), which compresses the original domain Ω occupied by a sphere of radius R_2 into a shell region $R_1 < r' < R_2$ in the compressed space Ω' , characterized by

$$\underline{\underline{\zeta}}'^{-1}_{p,e}(r') = J^{\mathrm{T}}_{rr'}\zeta^{-1}_{p,r}(r)J_{rr'} / \det(J_{rr'}),\ \underline{\underline{\Lambda}}'^{-1}(r') = J^{\mathrm{T}}_{rr'}\Lambda^{-1}(r)J_{rr'} / \det(J_{rr'}),$$

$$J_{rr'} = \partial r / \partial r', \tag{23}$$

In the new coordinates, the transformed equation (20) now reads as

$$\nabla \cdot \underline{\underline{\zeta}}^{-1}_{p,e}\nabla(\Delta_{33}\nabla \cdot \underline{\underline{\zeta}}^{-1}_{p,e}\nabla\Theta') - \Lambda^{-1}_{33}\gamma_0^4\Theta' = 0, \tag{24}$$

where $\underline{\underline{\zeta}}^{-1}_{p,e}$ is the upper diagonal part of the inverse of $\underline{\underline{\zeta}}$ and Λ^{-1}_{33} is the third diagonal entry of $\underline{\underline{\Lambda}}^{-1}$ [40]. The cloak has the inner radius $R_1 = 0.5\mathrm{m}$ and outer radius $R_2 = 1\mathrm{m}$. The concave-down transformation presents an overlapping for all curves for $\beta < 0.1$, which means the same results in applications. The effect of β on the amplitude of displacements inside the cloak $r \le R_1$ is shown in Fig. 3. When β increases, the amplitude also increases due to the fact that more energy is guided towards the inner boundary $r = R_1$, which in turn makes the cloaked object more *acoustically visible* to external waves. For $\beta =$ 0.1 and 0.4, the acoustically invisibility is good. The effect of β on the amplitude of displacements in the shell region $R_1 < r < R_2$ is shown in Fig. 4. When β increases, the amplitude also increases in the shell region of the cloak.

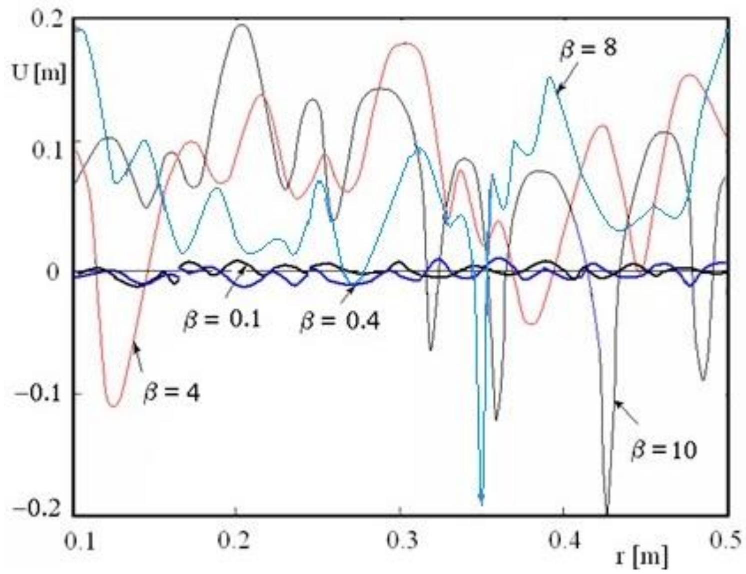

Fig. 3. Variation of the displacement amplitude with respect to β in the region $r \le R_1$.

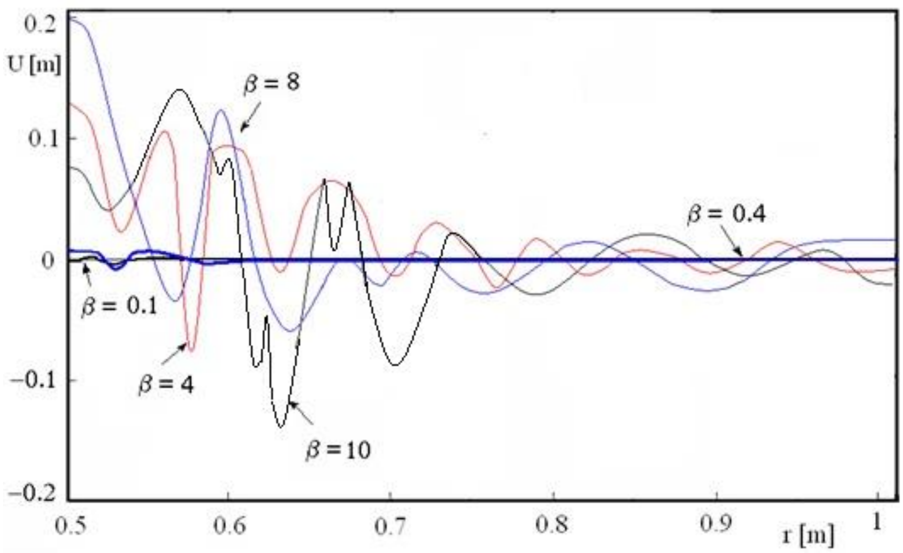

Fig. 4. Variation of the displacement amplitude
with respect to β in the region $R_1 < r < R_2$.

The absence of the scattering of waves generated by an external source outside the cloak $R_1 < r < R_2$ is observed in Fig. 5 for $\beta = 0.1$. The waves are smoothly bent around the central region inside the cloak. The results show that the wave field inside the cloak, i.e. the inner region of radius R_1 which surrounds the noisy machine, is completely isolated from the region situated outside the cloak. The waves generated by a noisy source are smoothly confined inside the inner region of the cloak, and the sound invisibility detected from the observer is proportional to β.

The inner region is acoustically isolated and the sound is not detectable by an exterior observer. The domain $r < R_1$ is an acoustic invisible domain for exterior observers. The waves generated by the exterior source outside the cloak do not interact with the interior field of waves. Any interaction between the internal and external wave fields is cancelled out by the presence of the shell region $R_1 < r < R_2$ filled with metamaterial.

The things change if β grows. For β = 4 for example, the waves generated by the noisy machine interact with the waves generated by an exterior source outside the cloak, and waves are complete visible in the cloak $R_1 < r < R_2$ (Fig.7).

Conclusions are that for the concave-down spherical cloaks, smaller values for β lead to a smaller disturbance in the acoustic fields in both the inner and the outer spaces $r < R_2$ and $r > R_2$, respectively, and higher values for β lead to a significant disturbance in the acoustic fields in both the inner and the outer spaces.

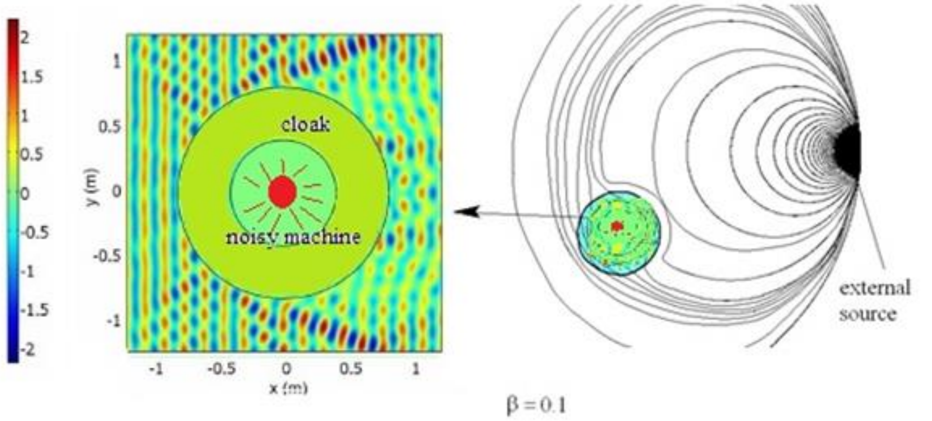

Fig. 5. The wave fields inside and outside the cloak for $\beta = 0.1$.

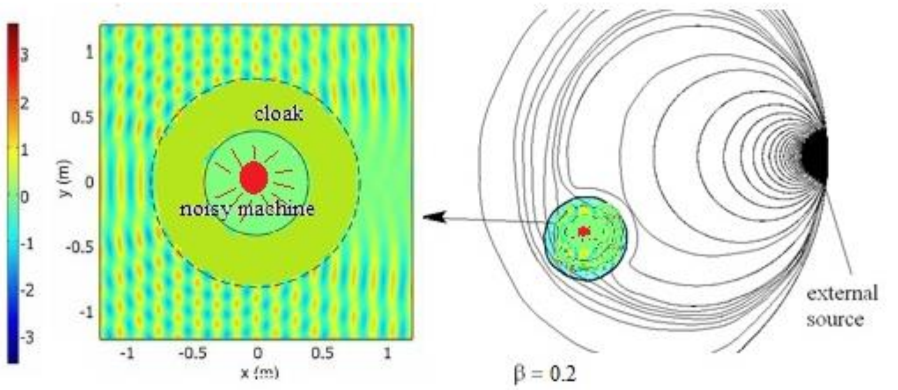

Fig. 6. The wave fields inside and outside the cloak for $\beta = 0.2$.

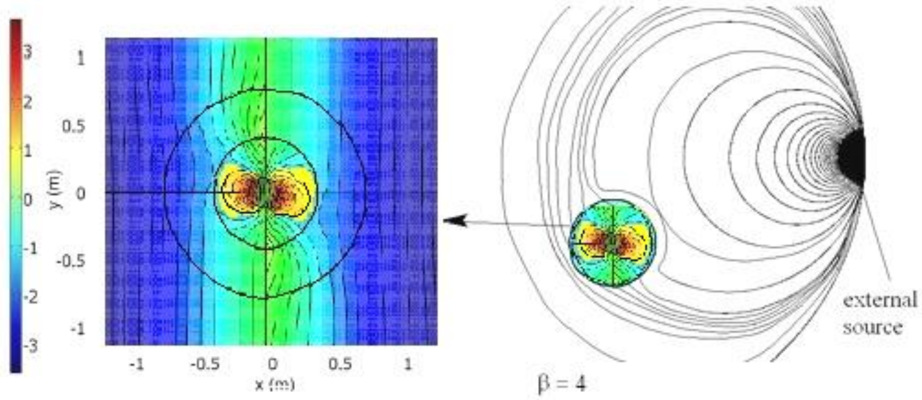

Fig.7. The wave fields inside and outside the cloak for $\beta = 4$.

4. Conclusions

In this paper, we shown the feasibility of designing of the “sound invisibility cloaks” using a layered metamaterial obtained by a coordinate concave-down transformation. The original domain consists of an alternation of concentric layers made from piezoelectric ceramics and epoxy resin, following a triadic Cantor sequence. After transformation, an equivalent domain with an inhomogeneous and anisotropic distribution of the material parameters are obtained. The simulations show that the noise generated by a noisy machine it transparent, or nearly transparent to acoustic waves in the cloak. The same principle is used in acoustics as in hiding objects from electromagnetic waves, i.e. the property of Helmholtz wave equations to be invariant under a coordinate transformation which can be interpreted as a spatial compression. The example of the spherical acoustic cloak looks at sound scattering from a noisy machine and an external noisy source. The waves generated by both, internal and external sources, do not interact between them, and in the cloak is silence, nothing is heard. The versatility of coordinate transformations is useful for bridging the wave phenomena ranging from electromagnetic, elastic, water, to acoustic waves, to the invisibility of such waves by cloaks made from metamaterials. These waves are all governed by Helmholtz scalar partial differential equation invariant under coordinate transformations.

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