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Development of precessional transmissions from invention to applications

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Abstract. The paper contains results of the research and development of precessional transmissions on structural synthesis, kinematics and the design of A^B and A^D gears with the gearings A^B_{CX-R}, A^B_{CX-CV}, A^D_{CX-CV}, A^D_{CV-CV}, A^B_{CX-R}. There is presented the analysis of the technical solutions regarding the increase of the bearing capacity of A^B and A^D gears and of the mechanical efficiency of the transmissions by using different geometries of the teeth contact with convex/concave profiles with the small difference of the radii of curvature and with reduced friction slip between the flanks. The concepts of $G^{con}_{r.s}$ and $G^{disc}_{r.s}$ spatial tumbling-rolling processes of generation of teeth with non-standard variable profiles are described, the fields of application of the precessional power and kinematic transmissions are described.
Keywords: Precessional transmissions, bearing capacity and mechanical efficiency, concave-concave contact, convex/concave profiles, teeth generation by spatial tumbling-rolling.

1. Introduction

Statement of the paper:
How to combine in a single mechanical transmission
the kinematic possibilities as wide as possible,
high load-bearing capacity and energy
losses as small as possible?

The development of precessional transmissions from the invention to applications essentially pursued the purpose of identifying the answer to the question: *How to*

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combine in a single mechanical transmission the kinematic possibilities as wide as possible, high load-bearing capacity and energy losses as small as possible?
The accomplishment of these three performances in a single mechanical transmission was tried by proposing and developing:
– a new kinematic principle of motion transformation and transmission of spherospatial interaction of conjugated teeth;
– multipara gearing with up to 100% pairs of teeth in contact simultaneously;
– geometry of the concave-concave contact of the teeth with variable convex/concave flank profiles with minimal difference in the radius of curvature and slip with reduced friction between the flanks.
These three innovative solutions as a whole define the precessional planetary transmission (PPT), to which complex research presented in this paper is devoted.
To realize the major importance of these three performances, it is sufficient to perceive the dimensions of the economic efficiency obtained only from the reduction of the energy losses in the mechanical transmissions. If we consider that 80% of the global energy is transmitted to the drive mechanisms of the machines through mechanical transmissions, then the increase of their mechanical efficiency by only 1% will lead to the saving of 0,8% of the energy produced globally.
Identifying the answer to the fundamental question: *How to combine the three performances in a single mechanical transmission?* dominated in the objectives throughout the research and development of precessional transmissions, summarizing in the following findings:
– *the extension of the kinematic possibilities* resides in the elaborated kinematic structures themselves *K-H-V, 2K-H* and *3K-H,* which, through the range of the realized transmission reports, do not have analogues among the known mechanical transmissions;
– *the increase of the bearing capacity* can be achieved by developing the geometry of the concave-concave contact of the teeth with the small difference of the radii of curvature;
– *the increase of the mechanical efficiency* can be achieved by diminishing the frictional slip between the conjugated flanks, ensuring their rolling from the account of the spherospatial movement of the satellite.

2. Brief history

The first patent for bolt gear A^B were: *Precessional planetary transmission* with multipara gear A^B_{CX-R}, registered on 30.05.1983 (SU 1020667 A) with priority of 11.02.1981, and with multipara gear A^B_{CX-CV}, registered on 07.06.1988 (SU 1401203 A1) with priority of 26.05.1986, and the first invention with toothed gear A^D – *Toothed precessional transmission* with multipara gear A^{D}_{CX-R}, was registered on 30.01.1989 (SU 1455094 A1) with priority of 13.05.1986, author Ion Bostan.

The modification of the geometry of the convex/concave profile of the geared teeth flanks A^{B}_{CX-CV}, and its dependence on the parametric configuration $\left[Z_g - \theta, \pm 1\right]$ were formulated in the patent (US 1563319) of 29.09.1987, with the application of the State Secret protection with the "Service use".

Together with the research and development of the precessional gears A^{B} [3, 4] and A^{D} [4, 5], the technologies of manufacture of the conical wheels with non-standardized flank profiles were developed. Thus, on 05.01.1988 the patent for the invention of the process $G^{con}_{r.s}$ and of the machine for the generation by spatial tumbling-rolling of the conical wheel teeth with convex / concave flank profile (SU 1663857 A1) was registered, with the application of the State Secret protection with the "Service use". The method and the equipment ensure the generation of an infinity of variable convex / concave profiles with the profile generating tool with the same geometric shape, including with longitudinal and profile modification of the teeth flanks according to the invention (SU 1646818 A1) from 07.05.1991, with priority from 27.06.1988.

The development of the researches of the precessional planetary transmissions in the chronological order of the researches included the gear with bolts A^{B} with the gear *tooth - bolt*, with convex-concave contact A^{B}_{CX-CV} [1] (chapters 3 - 6) followed by the toothed gear A^{D} [2] (chapter 7) with the gears with convex-rectilinear A^{D}_{CX-R}, convex-concave A^{D}_{CX-CV} and concave-concave A^{D}_{CV-CV} contact with straight teeth and $A^{D,\beta}_{CV-CV}$ with inclined teeth.

The dissemination of the precessional transmissions with non-standardized flank profiles obviously included the elaboration of the procedures and equipment for generating teeth. Thus, the procedures for generating the central wheel teeth with variable convex/concave flank profiles were developed by spatial tumbling-rolling in two variants: with a "cone trunk" shaped tool $G^{con}_{r.s}$ [3, 4], and with a "peripheral profiled disk" shaped tool $G^{disc}_{r.s}$ [2], including the procedures for the generation with the cylindrical tool of the flanks of the straight teeth $G^{cil}_{m.ax}$ and the inclined teeth $G^{cil,\beta}_{m.ax}$ on numerically controlled multi-axial machine tools [2] (chapter 8).

The process $G^{con}_{r.s}$ also includes the generation of the teeth of the conical center wheels with convex/concave flank profile with longitudinal and profile

modifications, including the generation of the satellite crown teeth with cycloidal and circle arc flank profile [3, 4].
The research and development of precessional transmissions covered the thematic spectrum from structural concepts of precessional gears to theoretical and experimental approaches on physical models, from non-standardized forms of flank profiles of the teeth to the elaboration of their generation processes, from constructive-functional design of different transmissions to their industrial application.

3. Synthesis and kinematics of precessional transmissions

Based on the first patents of invention, between 1981 and 1989 more than 30 kinematic structures of the precessional transmissions of types *K-H-V* (fig. 1) and *2K-H* (fig. 2) and complexes were created [3, 4, 5].
Precessional transmissions have been developed with different kinematic and functional possibilities, such as reducer, multiplier, differential, self-braking, or transmission of motion and torque moment "through the wall" in tight spaces (fig. 1 h). Based on the *K-H-V* and *2K-H* structures, complex structures (fig. 2 g, h) of transmissions and drive mechanisms with very large transmission ratios (fig. 2 g) can be created, including gearboxes and variators [1-5].

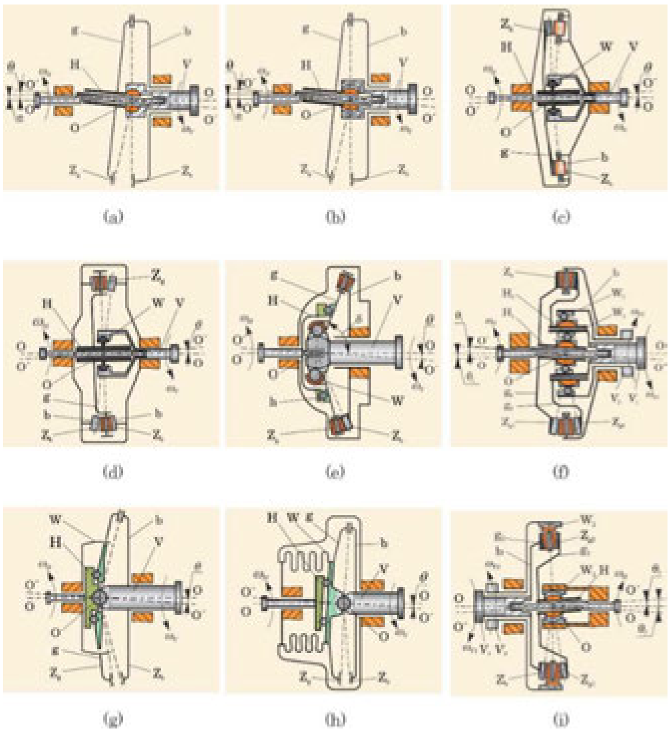

Fig. 1. Kinematic schemes of *K-H-V* type precessional transmissions.

The precessional transmissions developed according to the kinematic schemes shown in figures 1 and 2 can be with gears with bolts A^B, toothed with straight A^D and inclined $A^{D,\beta}$ teeth with the gears A^B_{CX-R}, A^B_{CX-CV}, A^D_{CX-CV}, A^D_{CV-CV}, A^D_{CX-R} and $A^{D,\beta}_{CV-CV}$.

The transmission ratios of the precessional transmissions (PT) are determined from the relations:

$$i^b_{HV} = -\frac{Z_g}{Z_b - Z_g} \text{ - for } K\text{-}H\text{-}V \text{ PT type} \qquad (1)$$

$$i^b_{HV} = -\frac{Z_{g_1} Z_c}{Z_b - Z_{g_2} - Z_{g_1} Z_c} \text{ - for } 2K\text{-}H \text{ PT type} \qquad (2)$$

$$i^b_{HV_2} = -\frac{Z_{g_1} Z_c Z_{e_1} Z_d}{Z_b Z_{g_2}\left(Z_b Z_{e_2} - Z_{e_1} Z_d\right) - Z_{g_1} Z_c \left(Z_b Z_{e_2} - Z_{e_1} Z_d\right)} \text{ - for } 3K\text{-}2 \text{ PT type (see fig. 2)} \qquad (3)$$

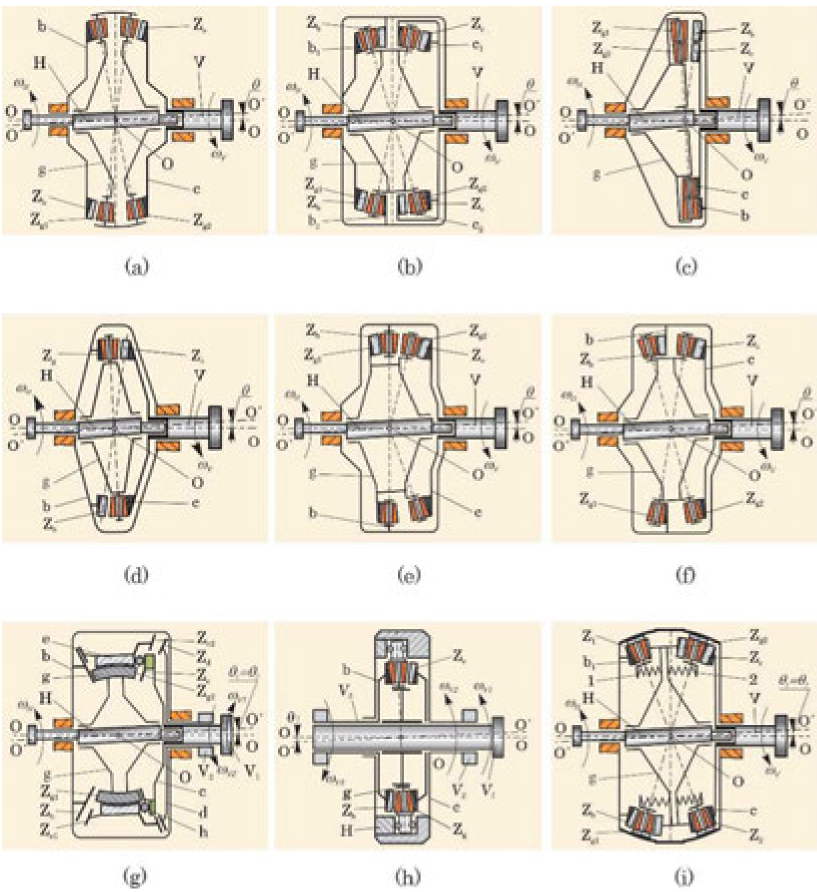

Fig. 2. Kinematic schemes of *2K-H* type precessional transmissions.

The precessional transmissions (figs. 1 and 2) ensure the transmission ratios in the ranges:

$i_{HV}^{b} = \pm 5 \ldots \pm 60 -$ possible in $K{-}H{-}V$ PT type;

$i_{HV}^{b} = \pm 10 \ldots \pm 3600 -$ possible in $2K{-}H$ PT type;

$i_{HV}^{b} = \infty -$ possible in $2K{-}H$ PT, if $Z_{g_1} = Z_{g_2}$, $Z_a = Z_g$ (see 2);

$i_{HV_2}^{b} = \pm 3600 \ldots \pm 10 -$ of possible millions in $3K\text{-}2H$ PT type.

Conclusion 1. *In the precessional transmissions the ratio of the teeth numbers of the conjugated wheels influences on the shape of the teeth profiles, on the geometry and kinematics of the contact point and on the direction of rotation of the shafts.*

In the structural analysis of the precessional transmissions [3-5] it was found that for the $K{-}H{-}V$ transmission there can be two configurations of the teeth numbers, namely $Z_1 = Z_2 - 1$, in which the driven and conductive shafts rotate in different directions, and $Z_1 = Z_2 + 1$, in which the shafts rotate in the same direction. The co-ratio $Z_1 = Z_2 + 1$ is recommended for transmissions with multiplier operating mode, and $Z_1 = Z_2 - 1$ - for reducer operating mode.

For the $2K{-}H$ type precessional transmission, there can be six configurations of the teeth numbers [2], shown in figure 3, namely:

– Configuration I: $Z_1 = Z_2 - 1$, $Z_4 = Z_3 - 1$, $Z_2 = Z_3 \pm 1,2,3...$, in which the driven and conductive shafts rotate in different directions for $Z_2 > Z_3$ (fig. 3 a).

– Configuration II: $Z_1 = Z_2 - 1$, $Z_4 = Z_3 - 1$, $Z_2 = Z_3 \pm 1,2,3...$, in which the driven and conductive shafts rotate in the same direction for $Z_2 < Z_3$ (fig. 3 b).

– Configuration III: $Z_1 = Z_2 - 1$, $Z_4 = Z_3 + 1$, $Z_2 = Z_3 \pm 1,2,3...$, in which the driven and conductive shafts rotate in the same direction for both $Z_2 > Z_3$ and $Z_2 < Z_3$ (fig. 3 c).

– Configuration IV: $Z_1 = Z_2 + 1$, $Z_4 = Z_3 - 1$, $Z_2 = Z_3 \pm 1,2,3...$, in which the driven and conductive shafts rotate in different directions for both $Z_2 > Z_3$ and for $Z_2 < Z_3$ (fig. 3 d).

– Configuration V: $Z_1 = Z_2 + 1$, $Z_4 = Z_3 + 1$, $Z_2 = Z_3 + 1,2,3...$, in which the driven and conductive shafts rotate in the same direction for $Z_2 > Z_3$ (fig. 3 e).

– Configuration VI: $Z_1 = Z_2 + 1$, $Z_4 = Z_3 + 1$, $Z_2 = Z_3 - 1,2,3...$, in which the driven and conductive shafts rotate in different directions for $Z_2 < Z_3$ (fig. 3 f).

Figure 3 shows the kinematics of the $2K{-}H$, precessional gear with bolts A^B, and figure 4 of the toothed precessional gear A^D.

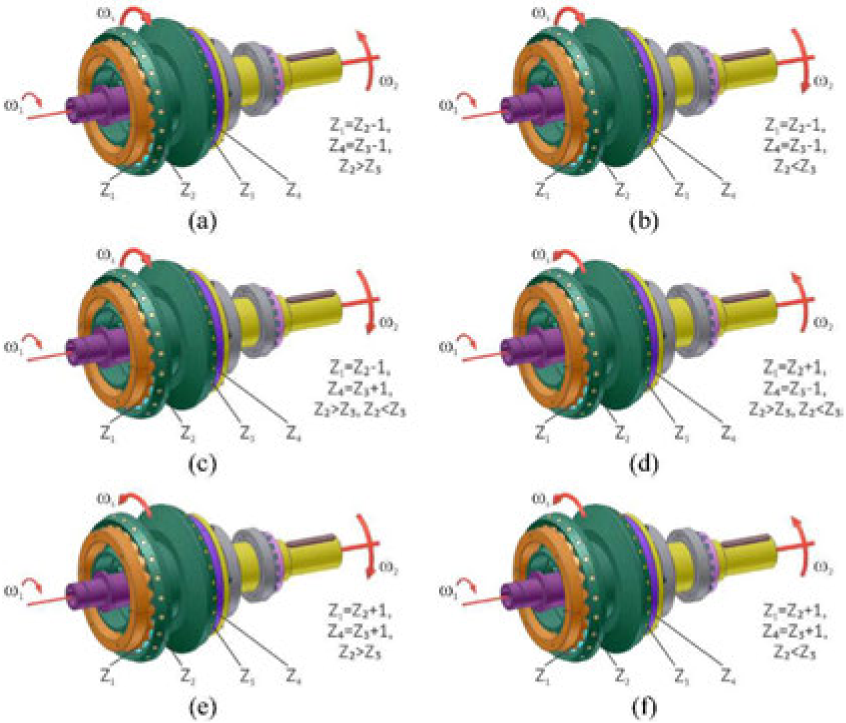

Fig. 3. Kinematics of the 2*K-H* precessional gear with bolts according to the number of teeth.

Based on the analysis of the kinematics of tooth interactions in tooth precessional gear and computer simulations on mathematical models, including the results of experimental research on physical models, we can draw the following conclusions:

1. In the A^D toothed precessional gear of the 2*K−H* type, to ensure the reduction of energy losses when overcoming the frictional forces of slip from the contact of the teeth and, consequently, to increase the mechanical efficiency of the transmission, the correlation of the number of teeth of the conjugated wheels is recommended to be according to configurations I and II. The correlation of the number of teeth (fig. 4 a) $Z_1 = Z_2 - 1$, $Z_4 = Z_3 - 1$ and $Z_2 > Z_3$ ensures the rotation of the driver and driven shafts in different directions, and the correlation $Z_1 = Z_2 - 1$, $Z_4 = Z_3 - 1$ and $Z_2 < Z_3$ (fig. 4 b) ensures the rotation of the shafts in the same direction. The 2*K-H* and *K-H-V* precessional gears with the coratio of Z teeth numbers according to figure 4 (a, b) may be recommended for use in transmissions with a functional gearbox. The configuration of the number of teeth is chosen according to table 2.1 [1]
2. The 2*K−H* type toothed precessional gears with at least one gear with the teeth ratio $Z_1 = Z_2 + 1$ (or $Z_4 = Z_3 + 1$) and *K−H-V* type with $Z_1 = Z_2 + 1$ can be recommended for transmissions with multiplier operation due to the low profile angle of the central wheel teeth.
3. The configurations of teeth numbers III, IV, V and VI are not recommended to

be used in the elaboration of toothed precessional transmissions with reducer regime, because they include at least one gear $(Z_1 - Z_2)$ or $(Z_3 - Z_4)$ in which the ration of the teeth number $Z_1 = Z_2 + 1$ or $Z_4 = Z_3 + 1$ leads to considerable energy losses for defeating slip friction forces.

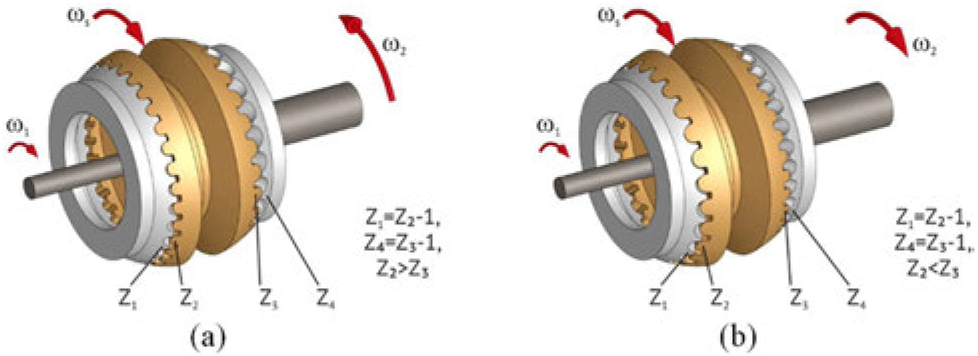

Fig. 4. The kinematics of the $2K\!-\!H$ toothed precessional gear with the ratio of the teeth numbers of the satellite $Z_2 > Z_3$ (a) and $Z_2 < Z_3$ (b).

Conclusion 2. *The precessional transmissions have no analogues among the transmissions known worldwide on the wide range of transmission ratios, including the kinematic possibilities to operate in reducer, multiplier or differential regime.*

4. Design of tooth contact in A^D precessional gear

The purpose of the research and development of the A^D precessional gears with convex-concave K_{CX-CV} or concave-concave K_{CV-CV} contact described in [2] is to increase the bearing capacity [7] and to reduce the energy losses from the contact of the teeth, including the exclusion of the size constraints for kinematic transmissions.

The efficiency of these two interdependent objectives determines the functional performances of a mechanical transmission and can be achieved by:

1. Designing the geometry of the concave-concave or convex-concave contact of the teeth with the small difference of the curves of the conjugated flanks.
2. Reduction or exclusion of friction with relative slip between the active flanks of the conjugated teeth.

In the context of the aforementioned, the research-development of the convex-concave K_{CX-CV} *or concave-concave* K_{CV-CV} *contact of the teeth is proposed by assigning a profile in a circle arc to the teeth of the crowns of the precessional satellite, and to the teeth of the central wheels - a concave profile, determined by* *the wrapper of the circle arcs family with the radius r in their spherospatial motion and determined by the parametric configuration* $\left[Z_g-\theta,\pm 1\right]$ *[1, 2].*

The kinematic relations of the satellite interaction with spherospatial motion with the central wheel of the precessional transmission are expressed by the Euler equations, set out in [3-5] “The fundamental theory of the precessional gear”.

Theoretical elaborations presented in [3-5] for *tooth - bolt* gearing, the models and methods applied are also fully valid for *tooth - tooth* gearing presented in [5, 2].

In order to create the convex-concave contact of the gear teeth with spherospatial motion, we admit that the profile of the satellite teeth is designated by the *LEM* curve, for example, in a circle arc of radius r originating in point G (fig. 5), called the *reference point* of satellite teeth [2, 7].

From the Euler kinematic equations, taking into account the kinematic relation between the angles φ and ψ expressed by $\varphi=-Z_1\psi/Z_2$, we obtain the coordinates of the origin G of the radius of the circle arcs X_G, Y_G, Z_G according to the rotation angle of the crankshaft ψ :

$$\begin{aligned}
X_G &= R\cos\delta\left[-\cos\psi\sin\left(Z_1\psi/Z_2\right)+\sin\psi\cos\left(Z_1\psi/Z_2\right)\cos\theta\right]-R\sin\delta\sin\psi\sin\theta,\\
Y_G &= -R\cos\delta\left[\sin\psi\sin\left(Z_1\psi/Z_2\right)+\cos\psi\cos\left(Z_1\psi/Z_2\right)\cos\theta\right]+R\sin\delta\sin\psi\sin\theta,\\
Z_G &= -R\cos\delta\cos\left(Z_1\psi/Z_2\right)\sin\theta-R\sin\delta\cos\theta.
\end{aligned} \tag{4}$$

The origin G of the radius of the circle arc, with which the satellite teeth are arbitrarily described, moves on the surface of the sphere with radius R with the origin in the precession center *O*, describing the trajectory $\zeta_1=f\left(\xi_1\right)$, expressed by the coordinates X_G, Y_G, Z_G (fig. 5).

The trajectory of the motion of the origin G of the *LEM* circle arc on the sphere with radius *R* is projected on the P_1 plane using the rules of spherical trigonometry [4]. Thus we obtain the trajectory T_G of the movement of the origin of the radius of the circle arc G on the plane P_1, expressed by the dependence $\zeta_1=f\left(\xi_1\right)$.

Knowing the trajectory $\zeta_1=f\left(\xi_1\right)$ of the movement of the origin of the radius of the circle arc G, expressed in the coordinates X_G, Y_G, Z_G (fig. 6), we determine the position of the contact point E of the satellite tooth profile in the circle arc with the profile of the central wheel teeth.

The contact point E of the teeth of the satellite in a circle arc and of the central wheel, for any angular position ψ of the crankshaft, is located at the distance of the radius *r* of the circle arc, on the normal at the speed vector V_G of its origin in point G (fig. 6).

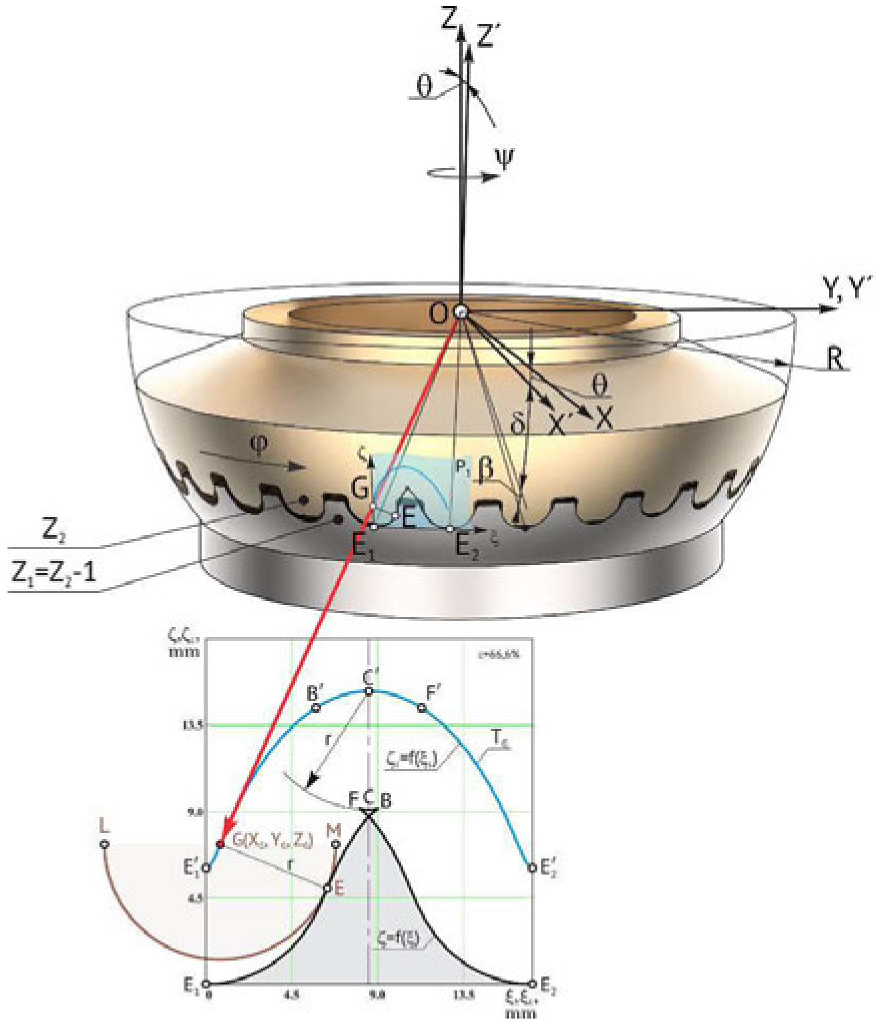

Fig. 5. Description of the trajectory of the motion of the origin *G* of the radius of the circle arc in coordinates $\zeta - \xi$, projected on the plane P_1 .

The family of contact points E obtained within a precession cycle $0 \le \psi \le 2\pi Z_2 / Z_1$ represents the profile of the central wheel teeth.

Next, to describe the profile of the central wheel teeth we determine the projections of the $\boldsymbol{V}_G$ speed vector on the coordinate axes of the $OX_1Y_1Z_1$ mobile system.

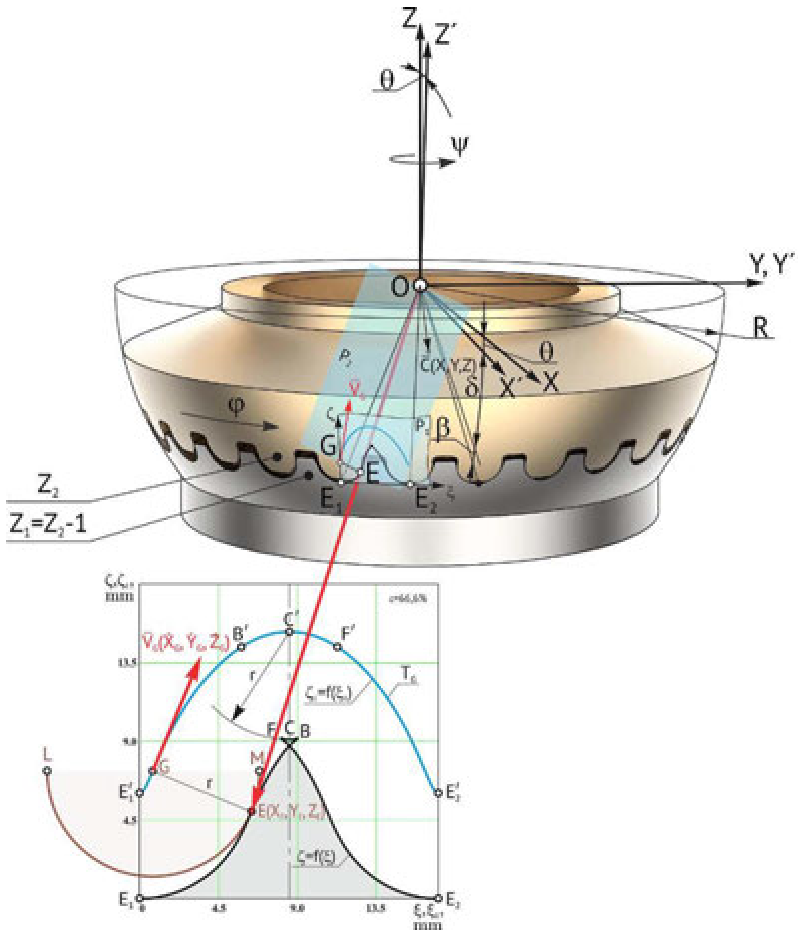

Fig. 6. Description of the profile of the central wheel teeth through the speed vector V_G of the origin G of the circle arc.

For this purpose, we derive in time equations (4) and obtain:

$$\dot{X}_G = R\dot{\psi}\cos\delta\left[\left(1-Z_1/Z_2\cos\theta\right)\sin\psi\sin\left(\psi Z_1/Z_2\right)+\left(\cos\theta-Z_1/Z_2\right)\cos\psi\cos\left(\psi Z_1/Z_2\right)\right] - R\dot{\psi}\sin\delta\sin\theta\cos\psi,$$

$$\dot{Y}_G = -R\dot{\psi}\cos\delta\left[\left(1-Z_1/Z_2\cos\theta\right)\cos\psi\sin\left(\psi Z_1/Z_2\right)-\left(\cos\theta-Z_1/Z_2\right)\sin\psi\cos\left(\psi Z_1/Z_2\right)\right] - R\dot{\psi}\sin\delta\sin\theta\sin\psi, \quad (5)$$

$$\dot{Z}_G = -R\dot{\psi}\left(Z_1/Z_2\right)\cos\delta\sin\theta\sin\left(\psi Z_1/Z_2\right),$$

where $\dot{\psi}$ is the angular speed of the crankshaft.

To determine the position of the contact point E of the teeth on the spherical surface, we identify the equation of a plane P_2 drawn perpendicularly on the speed vector V_G, which passes through the precession center O and the origin of the radius of the circle arc G. The equation of the plane P_2 can be written by the expression:

$$\left[\boldsymbol{OG}\times\boldsymbol{OC}\right]\times\boldsymbol{V_G}=\mathbf{0} \tag{6}$$

where $\boldsymbol{OG}$ and $\boldsymbol{OC}$ are vectors that determine the position of the reference point of the satellite tooth G in a circle arc and, respectively, of an arbitrary point C of the P_2 plane with respect to the origin of the immobile $OXYZ$ coordinate system (fig. 6).

The vector product $\left[\boldsymbol{OG}\times\boldsymbol{OC}\right]$ is expressed in the form of a determinant of the third order and, opening it after the elements of the first row, we obtain:

$$\left[OG\times OC\right]=\begin{vmatrix} i & j & k \\ X_G & Y_G & Z_G \\ X & Y & Z \end{vmatrix} \tag{7}$$
$$=i\left(Y_G Z - Z_G Y\right)+j\left(Z_G X - X_G Z\right)+k\left(X_G Y - Y_G X\right),$$

where X_G, Y_G, Z_G are the coordinates of the reference point G of the teeth of the satellite in a circle arc; X, Y, Z - the coordinates of the arbitrary point C on the plane P_2.

Analogously, we describe the vector equation $\left[\boldsymbol{OG}\times\boldsymbol{OC}\right]\times\boldsymbol{V_G}=\mathbf{0}$, using (7).

The contact point of the teeth E belongs at the same time to the sphere with radius R, that is, its coordinates satisfy its equation:

$$X_E^2+Y_E^2+Z_E^2-R^2=0. \tag{8}$$

From figure 5 we observe that the angle between the position vectors of the reference point of the tooth in the circle arc $\boldsymbol{OG}$ of the satellite and the vector of the position of the contact point E of the teeth $\boldsymbol{OE}$ represents the conicity angle β of the teeth of the satellite in the circle arc, which results in:

$$\boldsymbol{OG}\cdot\boldsymbol{OC}=R^2\cos\beta. \tag{9}$$

or

$$X_E Z_G + X_E Y_G + Z_E Z_G - R^2\cos\beta=0. \tag{10}$$

From equation (10) we determine:

$$X_E=\left(R^2\cos\beta - Y_E Y_G - Z_E Z_G\right)/\,X_G. \tag{11}$$

To determine the Y_E coordinate of the contact point of the teeth E, we substitute (11) in (8) and obtain:

$$Y_E=k_1 Z_E - d_1, \tag{12}$$

and by replacing (12) in (11), we obtain the expression of the X_E coordinate of the contact point:

$$X_E = k_2 Z_E + d_2, \tag{13}$$

where

$$k_1 = \left[X_G \left(X_G \cdot \dot{X}_G + Y_G \dot{Y}_G \right) + Z_G^2 \dot{X}_G \right] \Big/ \left(X_G \dot{Y}_G - Y_G \dot{X}_G \right) Z_G$$

$$d_1 = R^2 \cos\beta \dot{X}_G \Big/ \left(X_G \dot{Y}_G - Y_G \dot{X}_G \right) \tag{14}$$

$$k_2 = -\left(k_1 Y_G + Z_G \right) / X_G$$

$$d_2 = \left(R^2 cos\beta + d_1 Y_G \right) / X_G$$

By substituting (12) and (13) in (8) and solving the equation obtained in relation to the Z_E coordinate of the contact point E, we obtain:

$$Z_E = \frac{(\mathrm{k}_1 d_1 - k_2 d_2) \pm [(k_1 d_1 - k_2 d_2)^2 + (k_1^2 + k_2^2 + 1)(\mathrm{R}^2 - d_1^2 - d_2^2)]^{1/2}}{(k_1^2 + k_2^2 + 1)}. \tag{15}$$

It is worth mentioning that the curve of the profile of the central wheel teeth is equidistant from the trajectory of the movement of origin G of the radius of the circle arc, and for any rotation angle ψ of the crankshaft the condition $Z_E < Z_G$ must be met.

After some transformations of the expression (15), the Z_E coordinate can be determined by the relation:

$$Z_E = \frac{(\mathrm{k}_1 d_1 - k_2 d_2) - [(k_1 d_1 - k_2 d_2)^2 + (k_1^2 + k_2^2 + 1)(\mathrm{R}^2 - d_1^2 - d_2^2)]^{1/2}}{(k_1^2 + k_2^2 + 1)}. \tag{16}$$

The relations (12), (13) and (16) determine the X_E, Y_E and Z_E coordinates of the contact point E of the teeth, the set of which in a precession cycle represents the profile of the central wheel teeth, placed on the sphere.

The precessional gear being conical, with the extensions of the generators intersected in the precession center, it is appropriate to examine the profile of the teeth in the normal cross-section, for example, in the plane P_1 drawn through points E_1 and E_2 perpendicular to the plane OE_1E_2 (fig. 6).

The X_E, Y_E, Z_E coordinates of points E_1 and E_2 on the teeth profile on the sphere are determined from the relations (12), (13) and (16) for the precession angles $\psi = 0$ and $\psi = 2\pi Z_2 / Z_1$, corresponding to a precession cycle.

Using the rules of spherical trigonometry, we design the profile of the teeth on the sphere with radius R on the P_1 plane.

To design the profile of the central wheel teeth in two coordinates ζ and ξ in the plane P_1 we draw the coordinate system $E_1\xi\zeta$ with the origin in point E_1, whose axis $E_1\xi$ passes through point E_2 (fig. 6). From the X_N, Y_N and Z_N coordinates we move to the ξ and ζ coordinates using the relations:

$$\xi = \frac{\left[(E_1E_2)^2 + v_1^2 - v_2^2\right]}{2(E_1E_2)}, \quad \zeta = \sqrt{v_1^2 - \xi^2}. \tag{17}$$

Conclusion 3. *The expressions (17) represent the coordinates of the points of the curve, the family of which constitutes the profile of the flanks of the central wheel teeth, designed on the P_1 plane, expressed in parametric form by varying the precession angle from $\psi = 0$ to $\psi = 2\pi Z_2/Z_1^2$.*

To design the trajectory of the movement of origin G of the circle arcs in 2D, we move from the X_n, Y_E and Z_n coordinates to the Cartesian coordinates ξ_1, ζ_1 using the relations:

$$\xi_1 = \frac{\left[(E_1E_2)^2 - S_1^2 - S_2^2\right]}{2(E_1E_2)}, \quad \zeta_1 = \sqrt{S_1^2 - \xi_1^2}. \tag{18}$$

The function ξ_1 by ζ_1 (18) represents the projection of the movement trajectory of the origin G of the circle arcs on the plane P_1, and the function ξ by ζ (17) represents the profile of the central wheel teeth projected on the plane P_1.

Figure 7 shows the profilograms of the teeth of the toothed gear $Z_1 - Z_2$ with the parametric configuration $Z_1 = 21$, $Z_2 = 22$, $r = 5{,}5\ mm$, $\theta = 3{,}5^{\circ}$, $\delta = 22{,}5^{\circ}$ and $R = 75\,mm$ (a) and of the toothed gear $Z_3 - Z_4$ with the parametric configuration $Z_3 = 31$, $Z_4 = 32$, $r = 4{,}5\ mm$, $\theta = 3{,}5^{\circ}$, $\delta = 22{,}5^{\circ}$ and $R = 75\,mm$ of the $2K - H$ precessional reducer with the transmission ratio $i = -67{,}2$.

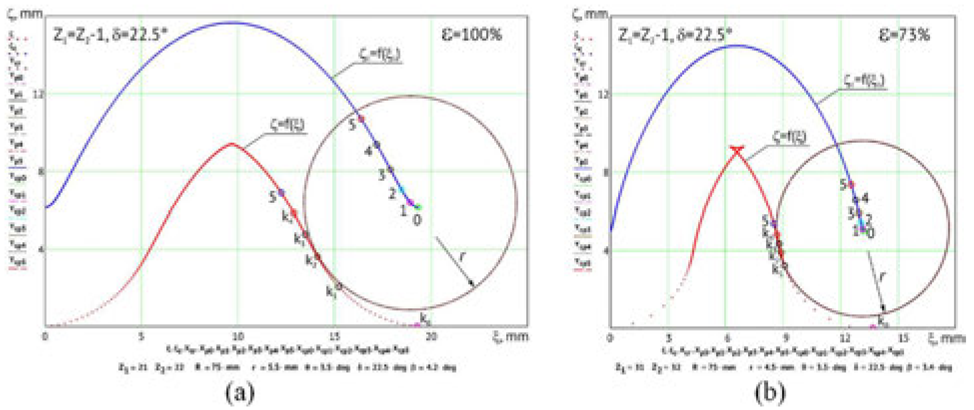

Fig. 7. The reference profilograms of the flanks of the central wheel teeth $\zeta = f(\xi)$ for the toothed precessional gear $2K - H$, $i = -67{,}2$ with the reference multiplicity $\varepsilon = 100\%$ (a) $\varepsilon = 73\%$ (b).

The profiles of the teeth of the central wheels are presented by the functions $\zeta = f(\xi)$ constructed after the relations (17), and of the teeth of the satellite are prescribed in a circle arc with radius r .

5. Geometry and kinematics of concave-concave contact in precessional gear

In the classic transmissions with cylindrical or conical wheels, the *geometry of the teeth contact* is predefined by the curve (curves) that describes the flanks of the conjugated teeth, among which the evolvent used in over 90% of gears, cycloid, epihipocycloid, trohoid, circle arc, etc.

Unlike the conventional ones, in the precessional transmission the profile of the central wheel teeth is variable depending on the parameters of the configuration $\left[Z_g - \theta, \pm 1\right]$ [1], (ch. 3). This results in the *variation of the geometry of the teeth contact* in the same gear (see fig. 8 a, b), passing from one form to another, namely from the convex-concave at the foot of the central wheel tooth to the convex-rectilinear to the middle of the tooth and convex-convex toward the tip of the tooth [2, 7] (see subsections 1, 3).

In A^D toothed precessional gears, unlike those with A^B bolts, the transformation and transmission of movement and load occur with the presence of the relative frictional slip between the flanks of the teeth, depending on the same parametric configuration $\left[Z_g - \theta, \pm 1\right]$ [1, 2, 3–5].

Therefore, the calculation and design of the toothed precessional gears, as opposed to the conventional gears, including the precessional with bolts, include a separate algorithm for designing the geometry of the teeth contact, which generally defines the bearing capacity and the mechanical efficiency of the transmission.

The design of the geometry of the teeth contact in the toothed precessional gear A^D summarizes in the identification of the contact form and its geometry parameters, determination of the load distribution in the contact and of the kinematics of the contact point of the flanks considered as tribosystem - all subject to the purpose of increasing the bearing and mechanical efficiency of the teeth contact.

In [2] (subsection 7.7.1) there were mentioned the possibilities of increasing the bearing capacity and the mechanical efficiency of the toothed precessional gear by transforming the *contact geometry*, namely changing the shape of the profile of the teeth of the conjugated wheels. It was found that, despite the reduction of the gear multiplication $\varepsilon(\%)$, in order to reduce the relative slip between the flanks, the load capacity can be increased *due to the convex-concave geometry of the contact*, with the small difference of the curvature radii of the conjugated flanks.

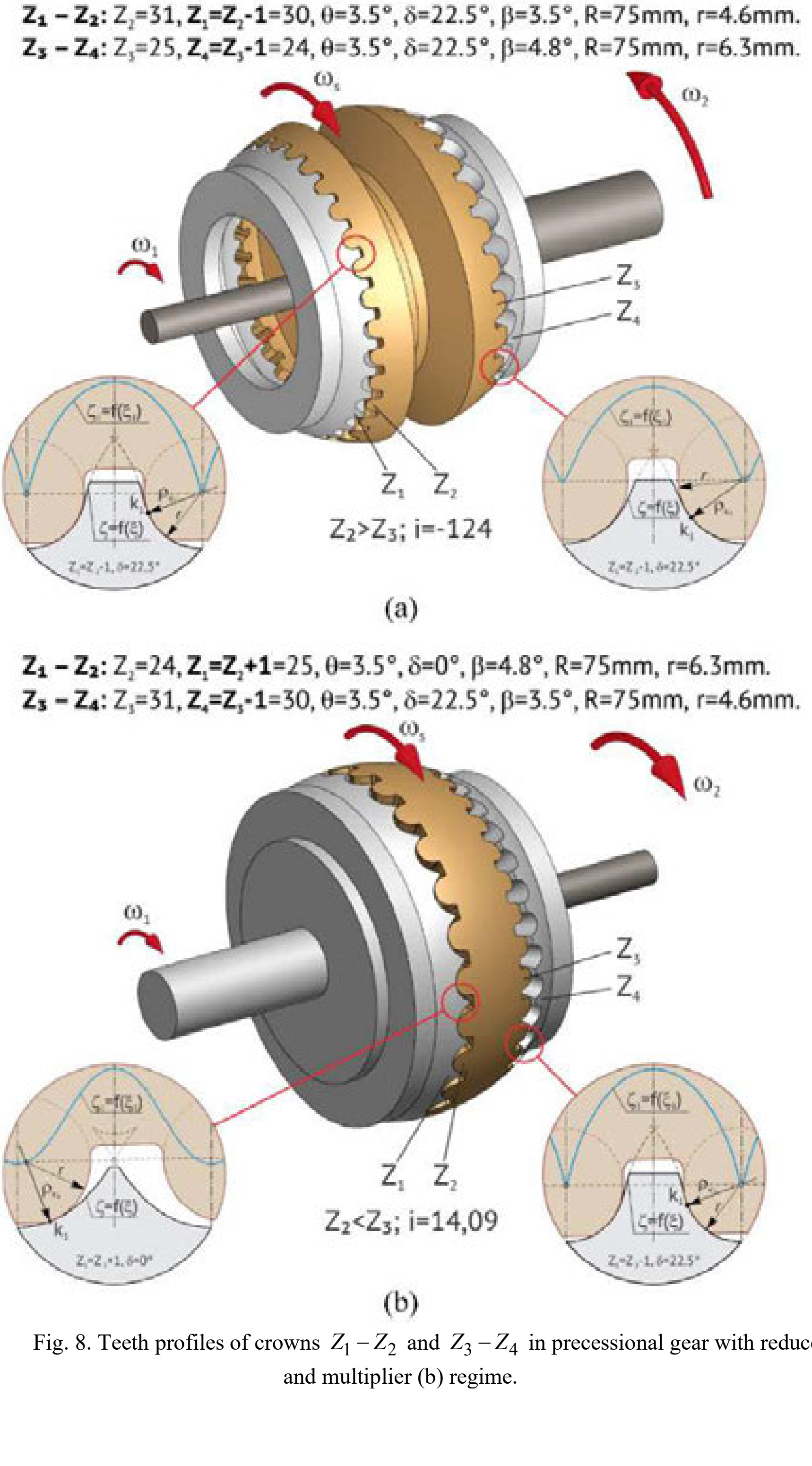

Fig. 8. Teeth profiles of crowns $Z_1 - Z_2$ and $Z_3 - Z_4$ in precessional gear with reducer (a) and multiplier (b) regime.

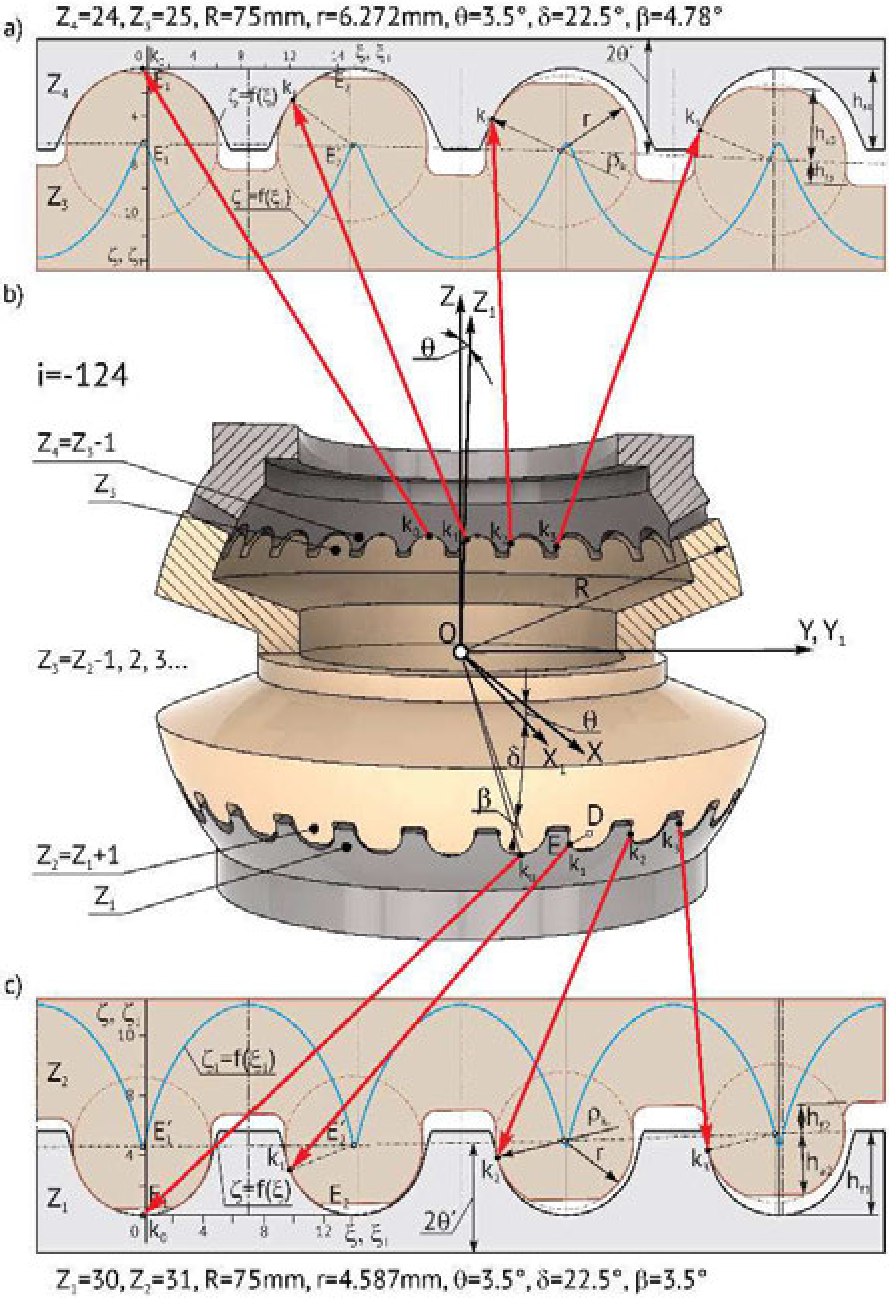

Fig. 9. Kinematics and geometry of tooth contact in the toothed precessional gear $2K-H$ (b): gear Z_3-Z_4 with $Z_4=Z_3-1$ and $\delta=22{,}5^\circ$ (a) and gear Z_1-Z_2 with $Z_1=Z_2-1$ and $\delta=22{,}5^\circ$ (c).

In the toothed precessional gear A^D shown in figure 9 with three pairs of teeth simultaneously geared, by changing the shape of the central wheel teeth, the gear multiplicity was reduced from $100(\%)$ pairs of teeth simultaneously geared up to $20,0(\%)$ pairs, or $10,0(\%)$ teeth pairs with contact on the active profile of the central wheel teeth. Thus, the contacts of the flanks with relatively high slip were excluded, creating favorable conditions for variation of the parametric configuration $\left[Z_g - \theta, \pm 1\right]$ regarding the creation of the geometry of the convex-concave contact with the small difference of the radius of curvature of the conjugated profiles.

In classical mechanical transmissions, in order to ensure the transformation of the movement with a constant transmission ratio, it is necessary that at the exit of a pair of teeth from the gear, the preceeding pair is already in gear, thus ensuring the coverage degree $\varepsilon > 1$.

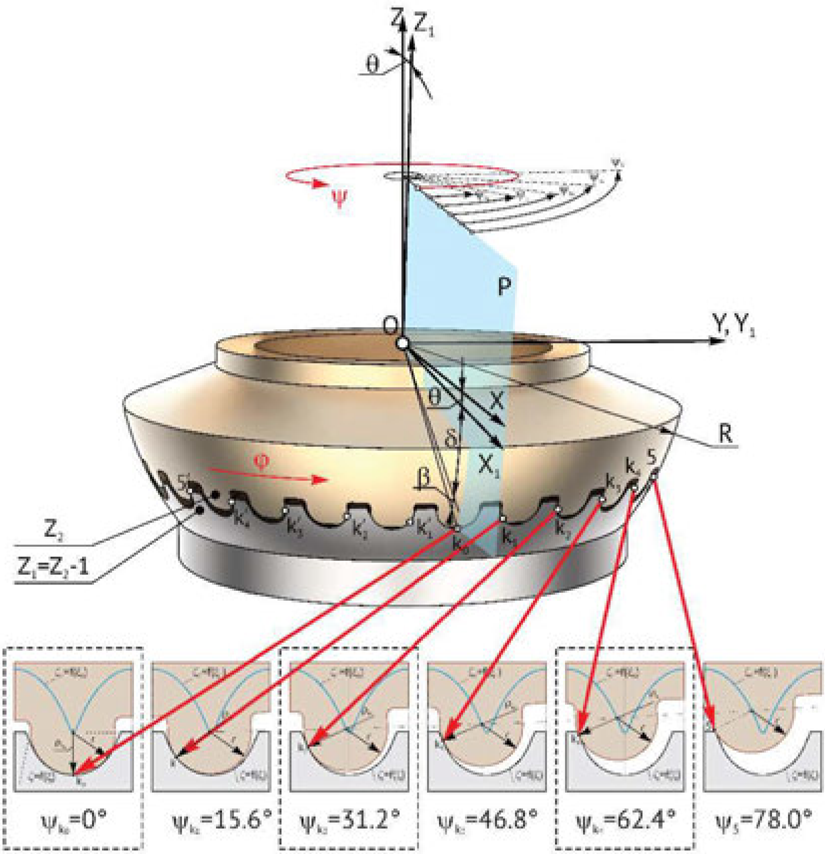

Fig. 10. Evolution of the variation of the teeth contact geometry with four pairs of teeth simultaneously engaged.

In the toothed precessional gear shown in figure 10, four pairs of teeth transmitting the load and four pairs of passive teeth (not transmitting the load), located on both sides of the contact k_0, are in gear. When rotating the crankshaft, the teeth pair conjugated in contact k_0 comes out of gear, and the pair with position 5 forms a new load bearing contact k_4, thus constantly maintaining four pairs of teeth carrying load.

According to figure 10, each of the four pairs of teeth simultaneously engaged have angular coordinates expressed by positioning the crankshaft according to the angles at the center $\psi k_1 \ldots \psi k_4$ increasing from contact to contact with step $\psi = 360 Z_2 / Z_1$. All four pairs of teeth required with the load rotate around the Z axis with angular speed ψ and the starting coordinate located in the plane P passing through the contact k_0.

Figure 10 shows the positions of the contacts $k_0 \ldots k_4$ and point 5 on the satellite tooth profile corresponding to the positioning angles $\psi_{k_0} = 0^\circ$, $\psi_{k_1} = 15{,}6^\circ$, $\psi_{k_2} = 31{,}2^\circ$, $\psi_{k_3} = 46{,}8^\circ$, $\psi_{k_4} = 62{,}4^\circ$ and $\psi_{k_5} = 78{,}0^\circ$, determined from the relation $\psi_{k_i} = 360 i Z_2 / Z_1^2$, where $i = 0,1,2,3,4\ldots$ is the order number of the contact. The radii of curvature of the profile of the central wheel teeth ρ_i in the contacts $k_0 \ldots k_4$ according to [2], chapter 7. Within the design of the contact geometry, the difference of the radii of curvature $(\rho_1 - r)$ of the geared flanks is calculated by alternating, varying the geometrical parameters of the configuration $\left[Z_g - \theta, \pm 1\right]$.

It is worth mentioning that analogous to the toothed precessional gear with four pairs of teeth simultaneously engaged, shown in figure 10, three, two and a pair of gear teeth can be designed, correspondingly changing the shape of the teeth profile of the central wheel and satellite by shortening of the height of the teeth of both gear wheels respectively.

6. Relative slip between the flanks of the teeth in gear

The kinematics of the contact point of the teeth in precessional gear and the geometrical shape of the conjugated flanks represent two determining characteristics of the mechanical efficiency and the bearing capacity of the contact.

The mechanical efficiency of the gear is the expression of the energy losses generated by the frictional forces with slip between the conjugated flanks, and the bearing capacity of the convex-concave contact results from the size of the difference of their radius of curvature.

For these reasons, the kinematics and the geometry of the contact in the A^{D}_{CX-CV} gear (fig. 11) are examined for gears with parametric configurations $\left[Z_g - \theta, \pm 1\right]$

different between them only by the correlation of the teeth numbers $Z_1 = Z_2 \pm 1$ and the angle of the conical axoid $\delta \geq 0^\circ$. From the aforementioned, the generalized configuration can be expressed by the parameters $Z_1 = 24(25)$, $Z_2 = 25(24)$, $\theta = 3{,}5^\circ$, $\delta = 22{,}5^\circ(0^\circ)$, $r = 6{,}27\,mm$ and $R = 75\,mm$.

The kinematics analysis of the contact points $k_0, k_1, k_2 \ldots k_i$ corresponding to the positioning angles of the crankshaft, notifying $\psi = 0$ for k_0, occurs by varying the linear speeds of the contact points E_1 on the profile of the central wheel teeth and E_2 on the profile of the satellite teeth and the relative speed of slip between the $V_{al_{k_i}}$ flanks, and the geometry of the teeth contact is presented by the radii of curvature ρ_{k_i} of the profile of the teeth of the central wheel and of the profile of the teeth of the satellite r and their difference $(\rho_1 - r)$. The kinematic analysis of the contact of the teeth is performed for the frequency of crankshaft revs $n_1 = 300\ \min^{-1}$.

Thus, in the gear corresponding to the configuration $[Z_g - \theta, -1]$ with the correlation of the teeth numbers $Z_1 = Z_2 - 1$ and the angle of the conical axoid $\delta = 22{,}5^\circ$, shown in figure 11 (a), in the contact of the teeth k_0 linear speed $V_{E_1} = 9{,}83\,m/s$, $V_{E_2} = 9{,}69\,m/s$, $V_{al_{k0}} = 0{,}14\,m/s$, and the radius of curvature of the profile of the central wheel teeth $\rho_{k_0} = 6{,}43\,mm$ of the teeth of the satellite $r = 6{,}27\,mm$ and their difference $(\rho_{k_0} - r) = 0{,}16\,mm$ (fig. 11 b).

With the increase of the angular coordinate from one conjugated pair to another with the step $\psi = 360i\,Z_2/Z_1$, for example from the angular coordinate $\psi_{k_0} = 0^\circ$ to $\psi = 15{,}6^\circ$ assigned to the contact k_1, the linear speeds V_{E_1} and V_{E_2} diminish, recording in contact k_1 the difference $V_{al_{k_i}} = V_{E_1 k_1} - V_{E_2 k_2} = 0{,}34\,m/s$, and the difference of the radii of curvature of the conjugated flanks in k_1 $(\rho_{k_2} - r) = 1{,}17\,mm$; in the k_2 contact corresponding to $\psi = 31{,}2^\circ$ $V_{al} = 0{,}67\,m/s$ and the difference of the radii of curvature $(\rho_{k_2} - r) = 9{,}55\,mm$; in the contact k_3 corresponding to $\psi = 46{,}8^\circ\ m/s$, and the contact geometry of the teeth passes from convex-concave to convex-convex, with the external curvature radius of the profile of the central wheel teeth $\rho_{k_3} = 57{,}66\,mm$. Figure 11 (c) shows the evolution of the geometry from contact k_0 to contact k_4.

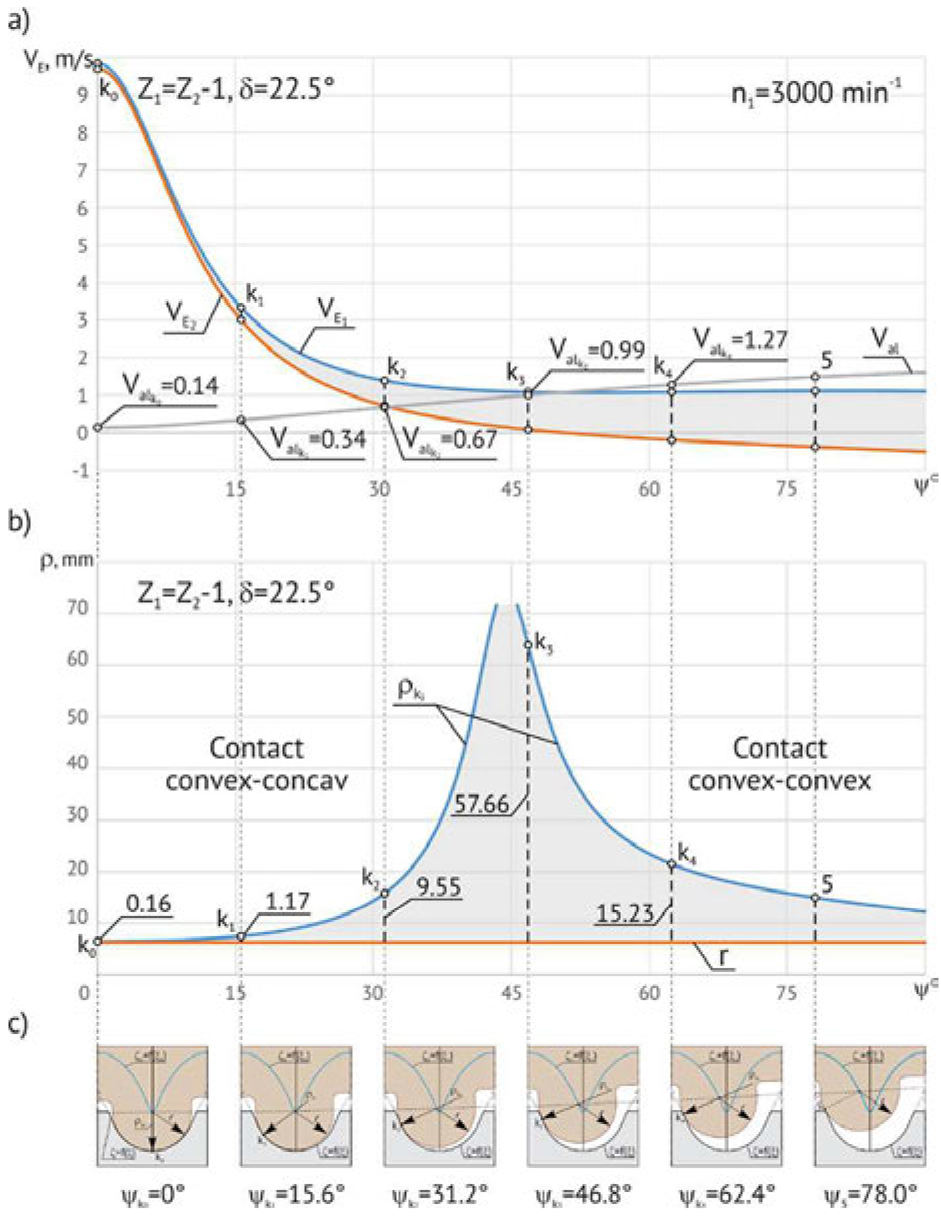

Fig. 11. The linear speeds at the contact point V_{E_1}, V_{E_2}, V_{al} (a) and the difference of the radii of curvature $(\rho_1 - r)$ (b) of the conjugated profiles in the contact k_i (c) as a function of ψ for $Z_1 = Z_2 - 1$ and $\delta = 22{,}5^\circ$ $\left(Z_1 = 24,\ Z_2 = 25,\ \theta = 3{,}5^\circ,\ \delta = 22{,}5^\circ,\ r = 6{,}27\,mm,\ R = 75\,mm\right)$.

Conclusion 4. *The defining characteristics of the bearing capacity and of the mechanical efficiency of the precessional transmission, among which: the* *reference multiplicity of the gear* $\varepsilon(\%)$*; the difference of the radii of curvature of the gear teeth profiles* $\rho_1 - r(mm)$*; the geometric parameters of the teeth contact* $V_{al}(m/s)$ *and* $S_E(mm)$ *depending on the precession angle* ψ *are identified by the preventive design of the contact geometry of the teeth according to the configuration parameters* $\left[Z_g - \theta, \pm 1\right]$.

The difference of the distances traveled by the points E_1 and E_2 between their common contact - for example, in k_0 corresponding to the precession angle $\psi = 0$ and their position when $\psi = \psi_i$, represents the relative slip between the flanks of the teeth of the conjugated wheels, so

$$V_{al} = \Delta S = S_1\psi - S_2\psi. \tag{19}$$

Taking into account the above mentioned, the distance traveled by the contact point E_1 on the flank of the central wheel teeth is determined by the formula:

$$S_1(\psi) = \int_0^{\frac{Z_2}{Z_1}\psi} \sqrt{\left(\frac{dxE_1}{d\psi}\right)^2 + \left(\frac{dyE_1}{d\psi}\right)^2 + \left(\frac{dzE_1}{d\psi}\right)^2}\, d\psi = \int_0^t \sqrt{\dot{x}_{E_1}^2 + \dot{y}_{E_1}^2 + \dot{z}_{E_1}^2}\, dt, \tag{20}$$

where $\dot{x}_{E_1}$, $\dot{y}_{E_1}$, $\dot{z}_{E_1}$ and are the speed vector projections $\boldsymbol{V}_{E_1}$ of point E_1 on the axes X, Y and Z.

The distance traveled by the contact point E_2 on the teeth flank profile of the wheel-satellite in a circle arc, for the same values of the precession angle ψ, is determined by the formula:

$$S_2(\psi) = \int_0^{\frac{Z_2}{Z_1}\psi} \sqrt{\left(\frac{dx_1E_2}{d\psi}\right)^2 + \left(\frac{dy_1E_2}{d\psi}\right)^2 + \left(\frac{dzE_2}{d\psi}\right)^2}\, d\psi$$
$$= \int_0^t \sqrt{\dot{x}_{1E_2}^2 + \dot{y}_{1E_2}^2 + \dot{z}_{1E_2}^2}\, dt, \tag{21}$$

where $\dot{x}_{1E_2}$, $\dot{y}_{1E_2}$, $\dot{z}_{1E_2}$ are the speed vector projections of point E_2 on the coordinate axes x_1, y_1, z_1.

Figure 12 shows the variation of the distances S_1 and S_2 traveled by points E_1 and E_2 between the positions defined with the angles ψ_{k_0} and ψ_{k_i}, corresponding to the contacts $k_0 \ldots k_i$ of the teeth pairs simultaneously geared, and their difference ΔS for the toothed precessional gear with the parameters $Z_1 = 24$, $Z_2 = 25$, $\theta = 3{,}5^\circ$, $\delta = 22{,}5^\circ$, $r = 6{,}27\,mm$ and $R = 75\,mm$.

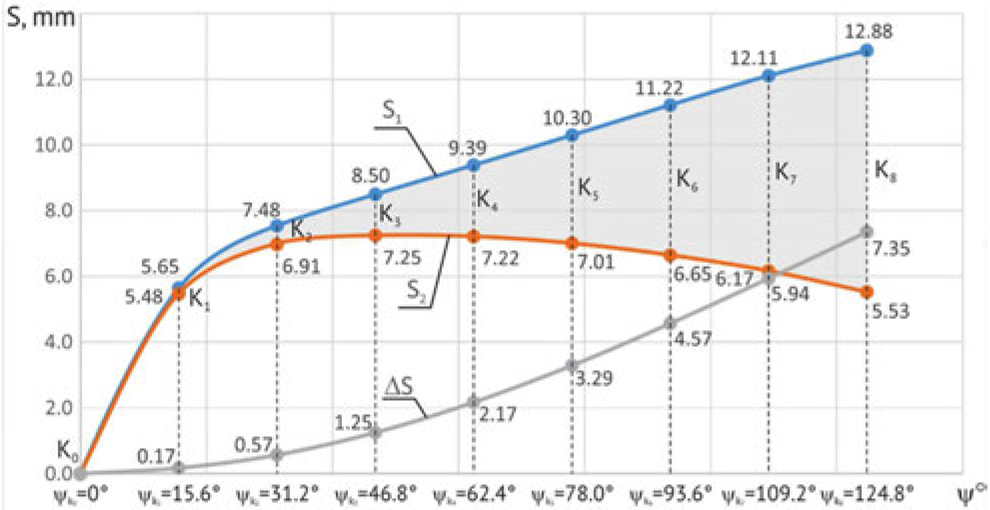

Fig. 12. The distances S_1 and S_2 traveled by the contact points E_1 and E_2 between the positions with ψ_{k_0} and ψ_{k_i} and their difference ΔS (precessional gear $Z_1 = 24$, $Z_2 = 25$, $\theta = 3{,}5^\circ$, $\delta = 22{,}5^\circ$, $\beta = 4{,}78^\circ$, $r = 6{,}27\,mm$ and $R = 75\,mm$).

Conclusion 5. *The difference of the* S_{E_1} *and* S_{E_2} *distances traveled by the contact points on the teeth profiles of the* E_1 *central wheel and of the* E_2 *satellite is of interest for the elaboration of the tribological model of the convex-concave* K_{CX-CV} *and concave-concave* K_{CV-CV} *contact of the teeth in the toothed precessional gear* A^D, *as well as for the study of lubrication of the contact surfaces of the teeth in hydrostatic, hydrodynamic, elastohydrodynamic or squeeze-film regimes*.

7. Generation of teeth with variable non-standard profiles by spatial tumbling-rolling

The creation of a mechanical transmission with gear with non-standardized profile of the teeth requires complex theoretical and experimental research, especially in the field manufacturing technologies of toothed wheels.

The novelty of the precessional transmission and its functional advantages derive from two aspects: 1 - from the geometry of the convex-concave teeth contact with multipara gear or concave-concave with the small difference of the radii of curvature of the flank profiles of the teeth, and 2 - from the transformation of the movement and of load based on the interaction of teeth with spherospatial motion.

The A^{D}_{CX-CV} and A^{B}_{CX-CV} convex-concave geometry of the teeth contact with multipara gear and concave-concave with small difference of A^{D}_{CV-CV} curvature radii assures the gear with a high bearing capacity, and the transformation and transmission of movement and load with spherospatial interaction of the conjugated wheels ensures the transmission unique kinematic possibilities, with transmission ratios of millions.

In [1] chapter 2 it was found that in the precessional gear, both the convex-concave contact of the conjugated teeth, as well as the convex / concave shape of the profile of the central wheel teeth vary depending on the parametric configuration $\left[Z_g - \theta, \pm 1\right]$.

Precessional transmissions with non-standardized teeth profiles and spherospatial interaction can be used only in the case of the elaboration of manufacturing technologies based on new principles of teeth generation, which would ensure low costs and high accuracy of gears.

It is worth mentioning that the manufacture of the wheels of the precessional gears A^D and A^B with variable convex/concave teeth profile cannot be achieved by the existing generation procedures, which requires the development of a new technology.

Conclusion 6. *The principle of forming the coverage of the circle arcs family described with equations (17), with the placement of their ray origins on the curve described with equations (18) (see figure 7), represents the kinematic model of the process of generating by spatial rolling of the central wheel teeth, with the reproduction of the geometry and kinematics of the interaction of the teeth from the real precessional transmission*.

The new technological concepts for generating the teeth of the wheels from the precessional transmission are based on the reproduction of the contact geometry and the relative movement of the teeth from the real transmission. Thus, with the $G^{con}_{r.s}$ procedure (see figure 13) it is proposed that the profile generating tool reproduce the shape of the cone trunk with the dimensions of the roller in the real gear, and its movement relative to the semi-manufactured one reproduces the interaction of the teeth with spherospatial movement from the real transmission. In the elaboration of the technology for generating the precessional wheel teeth, it was also taken into account the observance of the principle of the fundamental gear law regarding the continuity of the movement transformation ratio.

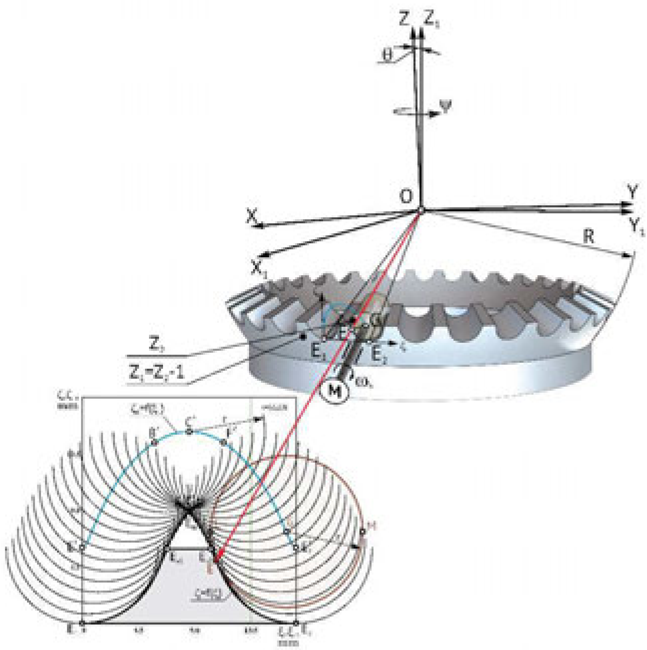

Fig. 13. Procedure for generating teeth with variable convex/concave and concave-concave profiles, by spatial tumbling-rolling with the tool of “cone trunk” shape.

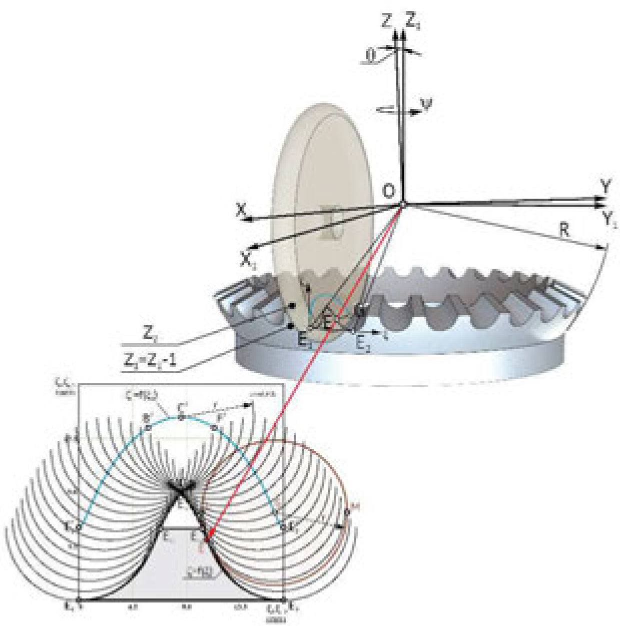

Fig. 14. $G_{r.s}^{disc}$ generation process of teeth with convex/concave profile by spatial tumbling-rolling with the tool of “peripheral profiled disk” shape.

In compliance with these conditions, there were developed new procedures for the generation of teeth by spatial tumbling-rolling with the tool of cone trunk shaped $G_{r.s}^{con}$ (fig. 13) and $G_{r.s}^{disc}$ with the tool - peripheral profiled disc with spherospatial motion (fig. 14), with a fixed point in relation to the teeth of the semi-manufactured wheel.

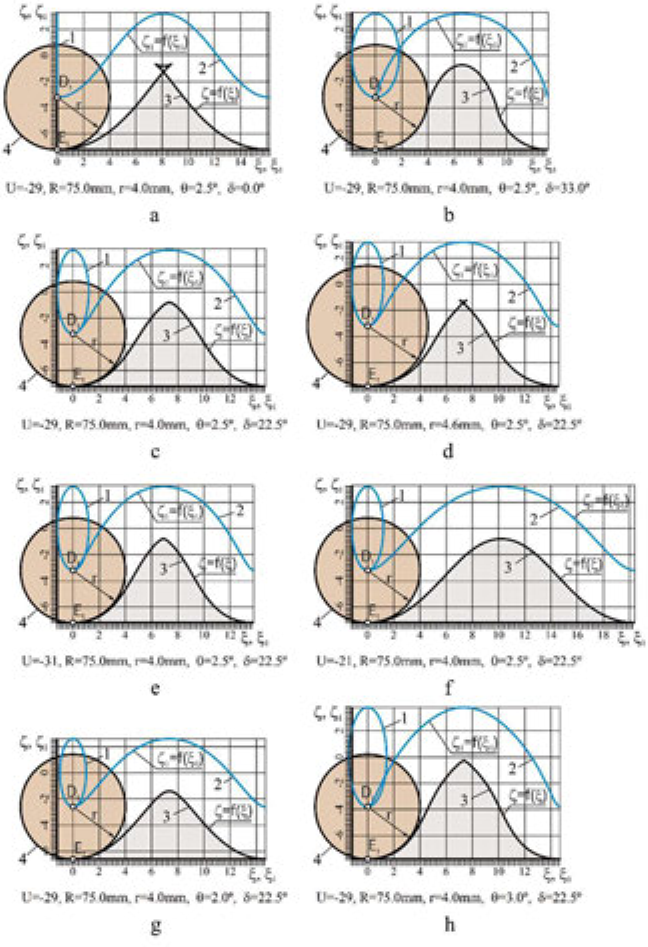

Fig. 15. Profilograms of the $G_{r.s}^{con}$ generation procedure of the teeth profile with the precessional tool of the "cone trunk" shape: 1, 2 - the trajectories of the p. D movement of the tool in the fixed $OXYZ$, and, respectively, mobile $O\overline{XYZ}$ coordinate system; 3 - tooth profile; 4 - the generating contour of the tool.

Figure 15 shows the variable profiles of the teeth of the central wheels generated by spatial tumbling-rolling with the tool of cone trunk shape [3, 4]. When elaborating the theoretical basis of the $G_{r.s}^{con}$ and $G_{r.s}^{disc}$ generation procedures of the teeth profile by tumbling-rolling with the tool with spherospatial movement, there was described the trajectory of the contact point movement of the generating contour of the tool (of the cutting edge) with that of the profile of the tooth of semi-manufactured wheel, by reproduction of the interaction of the teeth from the real gear at a precessional cycle. Thus, the mathematical model of the teeth generation process by spatial tumbling-rolling with the precessional tool was developed, which fully reflects the contact geometry and the interaction movements of the teeth from the real precessional transmission.

8. Applications of precessional transmissions

Precessional transmissions, due to their advantages, can cover the needs of consumers of drive mechanisms for the transformation and transmission of movement and load, with medium and large transmission ratios - from simple general purpose drive mechanisms to special purpose machines, exploited under extreme terrestrial conditions, in vacuumed or submersible spaces, including in the automotive industry, robotics, high precision mechanical systems, fine mechanics, technological equipment, etc. [3, 4, 6].

The areas of use of the precessional transmissions are defined by their constructive-kinematic and functional advantages, due to the specific geometry of the convex-concave contact of the teeth, the multiplicity of gear and the spherospatial movement with a fixed point of the wheel-satellite.

It is worth mentioning that for the manufacture and implementation of a new product, such as precessional transmissions, the designer, as a rule, needs adequate methods of resistance calculation and guidance material for the design, taking into account their constructive-kinematic particularities, the manufacturer - of the technical-technological documentation of assurance of the manufacturing process and of methods of evaluation/control of the precision of the processing of the component parts. The user of the transmissions, in turn, requests the functional characteristics in the format of the technical passport of the product, the permissible operating conditions, the technical requirements of quality assurance of the product as a whole etc.

Precessional planetary transmissions are essentially different from the classical transmissions, both by the unique constructive-kinematic features of the precessional gear and the contact geometry of the teeth, as well as by the new principle of motion transformation and transmission of the load. Therefore, during the *research-design-manufacture-implementation* process, at each stage complex problematic approaches were required, followed by conclusions, findings, constraints and recommendations, subsequently verified in their design practice, either as distinct products or within the design of the various drive mechanisms, based on the precessional transmissions [3, 5, 6].

In this context, based on the fundamental theory of precessional gear and the theory of teeth generation with non-standardized convex / concave and variable teeth profile, there were:

– designed precessional gears with bolts A^B and toothed with straight teeth A^D

and inclined teeth $A^{D,\beta}$ with gears with convex-concave contact A^{D}_{CX-CV}, concave-concave contact with straight A^{D}_{CV-CV} and inclined $A^{D,\beta}_{CV-CV}$ teeth;

– developed new technologies for generating teeth by spatial tumbling-rolling of non-standardized profiles with the tool of "cone trunk" shape $G^{con}_{r.s}$, peripheral profiled disk $G^{disc}_{r.s}$ and cylindrical $G^{cil}_{m.ax}$ on multi-axial machine tools with numerical control;

– developed the methods of engineering calculation and design of the gears A^{B}, A^{D} and $A^{D,\beta}$ taking into account the constructive and kinematic specificity of the gear of teeth with convex-rectilinear contact K_{CX-R}, convex-concave K_{CX-CV}, concave-concave K_{CV-CV} contact with straight teeth and K^{β}_{CV-CV} with inclined teeth;

– synthesized over 30 $2K-H$, $K-H-V$ kinematic and combined structures;

– elaborated the method of evaluation and control of the precision of manufacturing the precessional gear;

– designed, manufactured and experimentally researched precessional reducers and actuation mechanisms based on precessional transmissions for various application areas.

Most applications based on precessional transmissions have been protected by over 200 patents and have been disseminated in national projects and international grants in the field of research - development of precessional transmissions.

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