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Analytic description of teeth profile and justification of precessional gear parameters selection

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Abstract. The most requirements forwarded by the beneficiaries of mechanical transmissions consists in increasing reliability, efficiency and loaded capacity. The target problem can be solved with special effects by developing new types of multipliers based on planetary precessional transmissions with multiple gear, that were developed by the authors. Absolute multiplicity of precessional gear (up to 100% pairs of teeth simultaneously involved in gearing, compared to 5%-7% - in classical gearings) provides increased loaded capacity and small mass and dimensions.
Teeth profiles have an important role in the efficient transformation of motion in the precessional transmissions. Under this aspect the paper presents the analytic description of teeth nonstandard profile.

Key words: precessional transmission, multiple gear, teeth profile.

1. Introduction

Diversity of requirements forwarded by the beneficiaries of mechanical transmissions consists, in particular, in increasing reliability, efficiency and lifting capacity, and in reducing the mass and dimensions. It becomes more and more difficult to satisfy the mentioned demands by partial updating of traditional transmissions. The target problem can be solved with special effects by developing new types of multipliers based on precessional planetary transmissions with multiple gear, that were developed

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by the authors. Absolute multiplicity of precessional gear (up to 100% pairs of teeth simultaneously involved in gearing, compared to 5%-7% - in classical gearings) provides increased lifting capacity and small mass and dimensions. Therefore it was necessary to carry out theoretical research to determine the geometrical parameters of the precessional gear that operates in multiplier mode. Also, it was necessary to develop new conceptual diagrams of precessional transmissions that function under multiplier regime.

Teeth profiles have an important role in the efficient transformation of motion in the precessional transmissions. In this respect the description of the non-standard profile of the teeth has been elaborated. Also, the technology for its industrial manufacturing was proposed and patented [1].

Kinematically, the link between the semi product and the tool, in which one of them (the tool) makes spherical-spatial motion being, at the same time, limited from rotating around the axis of the main shaft of the teething machine tool, is similar to the „*satellite-driven shaft*” link from the precessional planetary transmission of the K-H-V type. The kinematical link between the tool and the stationary part of the device represents a Hooke articulation that generates the variability of transfer function in the kinematical link „*tool-semi product*”. This variation will influence the teeth profile. Thus, the connection of tool with the housing registers a certain diagram error $\Delta\psi_3$ (to understand the deviation of the semi product angle of rotation ψ_3 from the angle of rotation of the semi product itself ψ_3^m at its uniform rotation):

$$u_{31}^m = -\frac{z_2 - z_3}{z_3}; \Delta\psi_3 = \psi_3 - u_{31}^m = \frac{z_2}{z_3}\left(\psi - arctg(\cos\theta \cdot tg\psi)\right). \quad (1)$$

Figure 1 shows the dependence of the tool position diagram error $\Delta\psi_3$ at a revolution of the machine tool main shaft ψ. This error is transmitted to the tool that shapes the function and to improve the performances of precessional transmission under multiplication it is necessary to modify teeth profile with the diagram error value $\Delta\psi_3$ by communicating supplementary motion to the tool. In this case the momentary transmission ratio of the manufactured gear will be constant. Usually, in theoretical mechanics the position of the body making spherical-spatial motion is described by Euler angles. The mobile coordinate system $OX_1Y_1Z_1$ is connected rigidly with the satellite wheel, which origin coincides with the centre of precession 0 (fig. 2) and performs spherical-spatial motion together with the satellite wheel relative to the motionless coordinate system $OXYZ$.

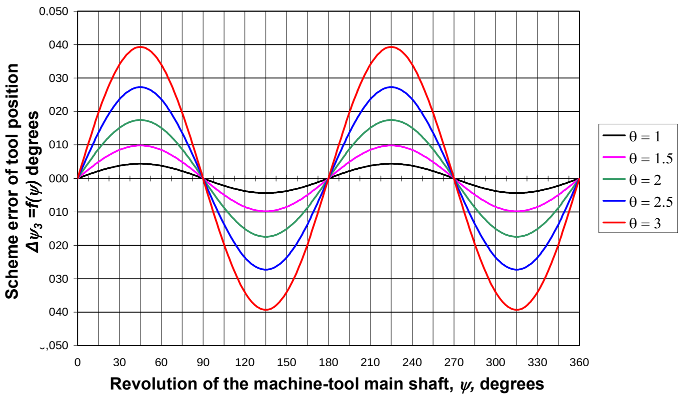


Fig. 1. Dependence of the scheme error of tool position $\Delta\psi_3$ at a revolution of the machine-tool main shaft ψ.

The elaboration of the mathematic model of the modified teeth profile is based integrally on the mathematic model of teeth profile, previously developed by the authors. With this purpose it is necessary to present the detailed description of teeth profile without modification and, then, to present of the description of modified profile peculiarities.

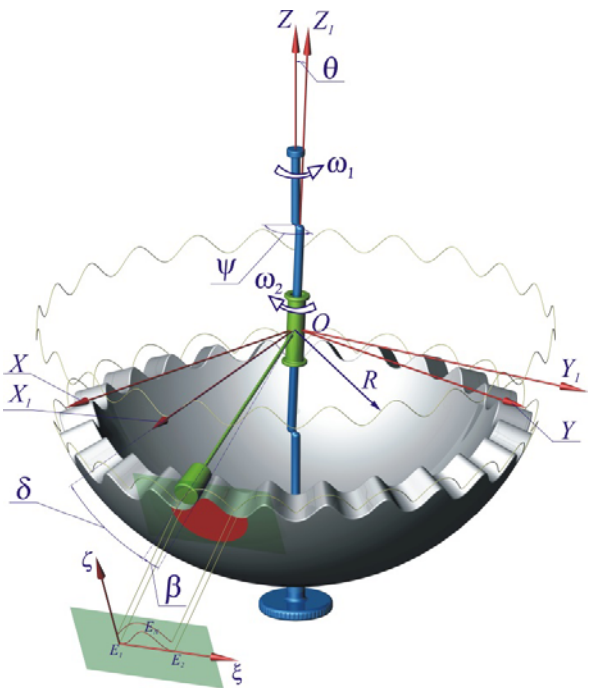


Fig. 2. Tooth profile in normal section.

2. Description of teeth profile designed on sphere

An arbitrary point ***D*** of the tool axis describes a trajectory relative to the fixed system according to the equations [2,3]:

$$\begin{aligned} X_D^m &= -\sin\delta \sin\left[Y_C^m \sin\theta + Z_C^m\left(1-\cos\theta\right)\cos\psi\right];\\ Y_D^m &= -Y_C^m \cos\delta + Z_C^m \sin\delta\left[\cos^2\psi + \cos\theta\sin^2\psi\right];\\ Z_D^m &= -Y_C^m \sin\delta\left(\cos^2\psi + \cos\theta\sin^2\psi\right) - Z_C^m\cos\delta. \end{aligned} \tag{2}$$

Index ***m*** means „*modified*".

The motion of point $\boldsymbol{D^m}$ compared to the movable system connected rigidly to the semi product is described by formulas:

$$\begin{aligned} X_{1D}^m &= X_D^m \cos\frac{\psi}{Z_1} - Y_D^m \sin\frac{\psi}{Z_1};\\ Y_{1D}^m &= X_D^m \sin\frac{\psi}{Z_1} + Y_D^m \cos\frac{\psi}{Z_1};\\ Z_{1D}^m &= Z_D^m. \end{aligned} \tag{3}$$

The projections of point $\boldsymbol{D^m}$ velocities is expressed by formulas:

$$\begin{aligned} \dot{X}_D^m &= -\sin\delta\cos\psi\left[Y_C^m\sin\theta + Z_C^m\left(1-\cos\theta\right)\cos\psi\right]\dot{\psi} -\\ &- \sin\delta\sin\psi\left[\dot{Y}_C^m\sin\theta + \dot{Z}_C^m\left(1-\cos\theta\right)\cos\psi - Z_C^m\left(1-\cos\theta\right)\sin\psi\cdot\dot{\psi}\right];\\ \dot{Y}_D^m &= -\dot{Y}_C^m\cos\delta + \dot{Z}_C^m\sin\delta\left[\cos^2\psi + \cos\theta\sin^2\psi\right] +\\ &+ Z_C^m\sin\delta\left[-2\cos\psi\sin\psi + 2\cos\theta\sin\psi\cos\psi\right]\dot{\psi};\\ \dot{X}_{1D}^m &= \dot{X}_D^m\cos\frac{\psi}{Z_1} - \frac{\dot{\psi}}{Z_1}X_D^m\sin\frac{\psi}{Z_1} - \dot{Y}_D^m\sin\frac{\psi}{Z_1} - \frac{\dot{\psi}}{Z_1}Y_D^m\cos\frac{\psi}{Z_1};\\ \dot{Y}_{1D}^m &= \dot{X}_D^m\sin\frac{\psi}{Z_1} + \frac{\dot{\psi}}{Z_1}X_D^m\cos\frac{\psi}{Z_1} + \dot{Y}_D^m\cos\frac{\psi}{Z_1} - \frac{\dot{\psi}}{Z_1}Y_D^m\sin\frac{\psi}{Z_1}. \end{aligned} \tag{4}$$

The coordinates of point $\boldsymbol{E^m}$ on the sphere is calculated by formulas:

$$\begin{aligned}
&X^m_{1E} = k^m_2 Z^m_{1E} + d^m_2;\\
&Y^m_{1E} = k^m_1 Z^m_{1E} - d^m_1;\\
&Z^m_{1E} = \frac{(k^m_1 d^m_1 - k^m_2 d^m_2) - \sqrt{(k^m_1 d^m_1 - k^m_2 d^m_2)^2 + (k^{m2}_1 + k^{m2}_2 + 1)\cdot(R^2_D - d^{m2}_1 - d^{m2}_2)}}{k^{m2}_1 + k^{m2}_2 + 1},
\end{aligned} \qquad (5)$$

where: $k^m_1 = \dfrac{X^m_{1D}\left(X^m_{1D}\dot{X}^m_{1D} + Y^m_{1D}\dot{Y}^m_{1D}\right) + Z^{m\,2}_{1D}\dot{X}^m_{1D}}{Z^m_{1D}\left(X^m_{1D}\dot{Y}^m_{1D} - Y^m_{1D}\dot{X}^m_{1D}\right)}$; $k^m_2 = -\dfrac{\left(k^m_1 Y^m_{1D} + Z^m_{1D}\right)}{X^m_{1D}}$;

$$d^m_1 = \frac{R^2_D \cos\beta\, \dot{X}^m_{1D}}{\left(X^m_{1D}\dot{Y}^m_{1D} - \dot{X}^m_{1D} Y^m_{1D}\right)}; \quad d^m_2 = \frac{\left(R^2_D \cos\beta + d^m_1 Y^m_{1D}\right)}{X^m_{1D}}.$$

According to the obtained analytical relations a soft for the calculation and generation of teeth was developed in CATIA V5R7 modelling system that allowed obtaining the modified trajectories of points $E^m{}_e$ and $E^m{}_i$ on the spherical front surfaces, both exterior and interior ones, by which the teeth surface was generated (figure 3).

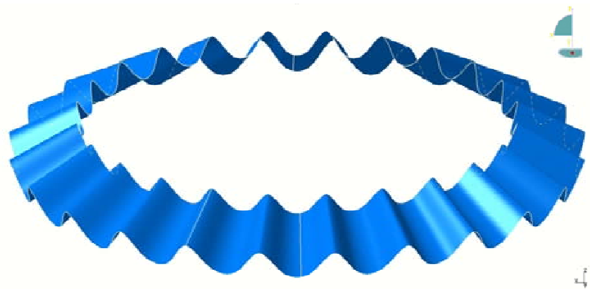


Fig. 3. Teeth generating surface.

3. Description of modified teeth profile projected on a transversal surface

Projection of point E^m on the tooth transversal plane has the following coordinates:

$$X''^m_E = \varepsilon^m \cdot X^m_{1E}, \quad Y''^m_E = \varepsilon^m \cdot Y''^m_{1E}, \quad Z''^m_E = \varepsilon^m \cdot Z''^m_{1E}, \qquad (6)$$

where $\varepsilon^m = -\dfrac{D}{AX^m_{1E} + BY^m_{1E} + CZ^m_{1E}}$.

The modified teeth profile in plane is described by the equations:

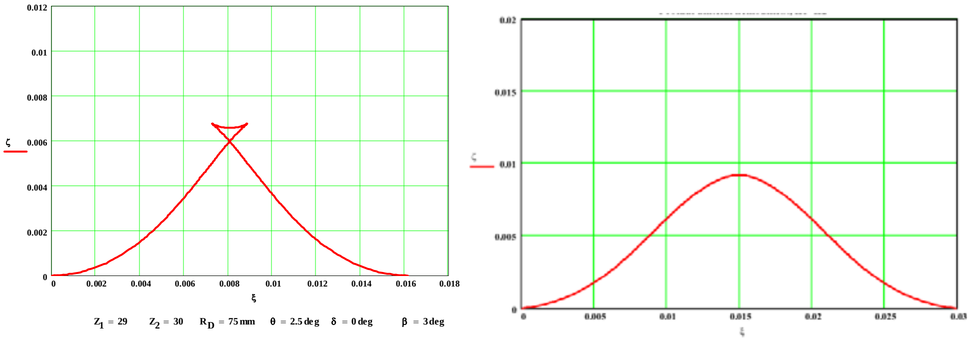


Fig. 4. Teeth profiles for multipliers.

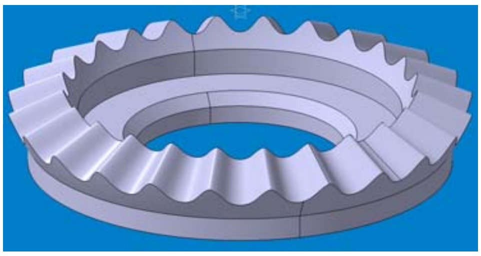


Fig. 5. Computerised model of the sun gear.

$$\xi^{m} = X_{E}^{\prime\prime m} \cos\frac{\pi}{Z_{1}} + \left[R_{D} \cos\left(\delta + \theta + \beta\right) + Y_{E}^{\prime\prime m} \right] \sin\frac{\pi}{Z_{1}};$$

$$\zeta^{m} = X_{E}^{\prime\prime m} \sin\gamma \sin\frac{\pi}{Z_{1}} - \left[R_{D} \cos\left(\delta + \theta + \beta\right) + Y_{E}^{\prime\prime m} \right] \sin\gamma \cos\frac{\pi}{Z_{1}} + \tag{7}$$

$$+ \left[R_{D} \sin\left(\delta + \theta + \beta\right) + Z_{E}^{\prime\prime m} \right] \cos\gamma .$$

A wide range of modified teeth profiles with different geometrical parameters were generated in MathCAD 2001 Professional software (figure 4). The solid model of a gear wheel is shown on figure 5.

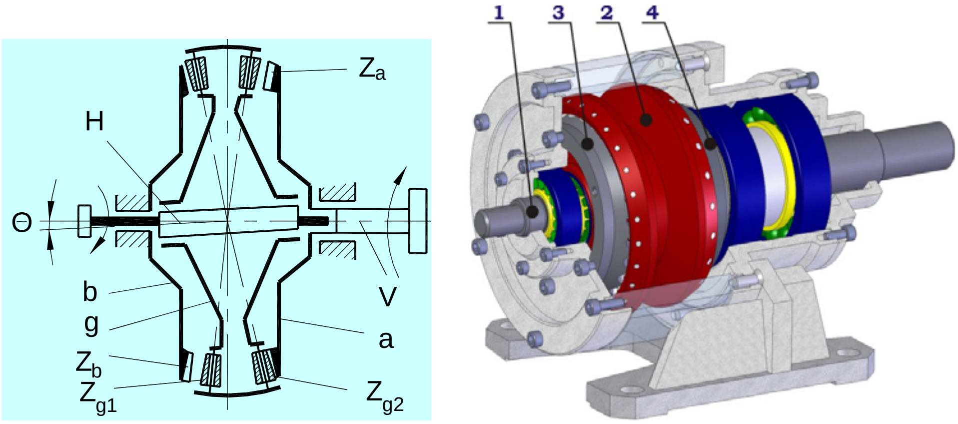
Based on the carried out research it was established that from the point of view of decreasing energy losses in gearing, in the multiplication mode of operation, the gearing angle should be $\alpha > 45^{0}$, and the nutation angle (the pitch angle of the crank shaft) should be $-\ \theta \leq 2{,}5^{0}$. This is dictated by the reverse principle of movement in the multipliers compared to the reducers: the axial component of the normal force in gear must be maximal to drive the crank shaft in the rotation movement through the satellite wheel.

4. Design of planetary precessional reducer

On the basis of the undertaken study, diagram 2K-H (figure 5,a) was selected for the development of universal destination planetary precessional reducer. The transfer ratio of the planetary precessional transmission is defined by relation [1]:

$$i = -\frac{Z_{g_1} Z_a}{Z_b Z_{g_2} - Z_{g_1} Z_a} \tag{8}$$

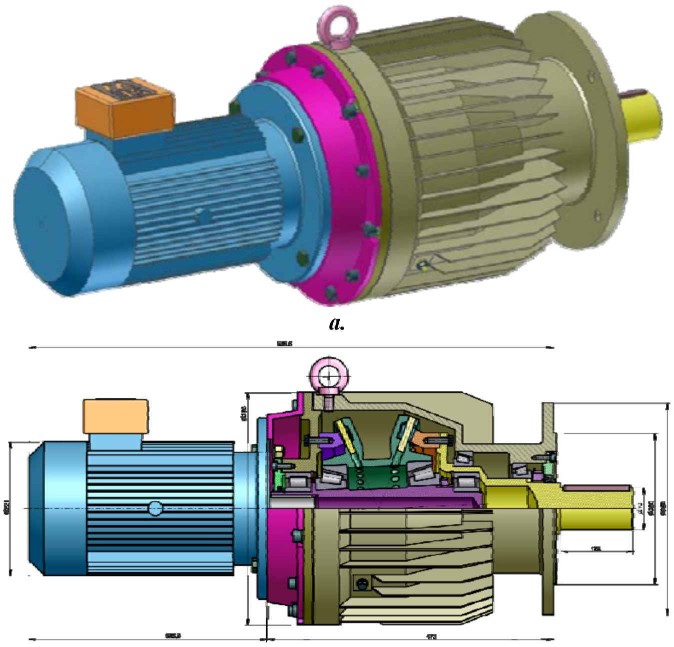
where: Z_{g_1}, Z_{g_2} are number of rollers of the satellite crowns g_1 and g_2; Z_a and Z_b are number of teeth of the gears a and b.



a. *b.*

Fig. 5. Conceptual diagrams (a) and dynamic computerised model of precesional transmissions.

As a result of analysis of a wide range of tooth profiles with different geometrical parameters of gear by using the mathematical modelling package MathCAD 2001 Professional, the optimum tooth profiles were selected with account of their functioning in conditions of reducer. Also, in MathCAD 2001 Professional software the calculation of geometrical parameters of precessional gear was done. The structures of precessional reducer were designed in SolidWorks software. The precessional reducer (figure 6 and 7) is connected by flange with an electric motor, which allows obtaining a compact coaxial module.

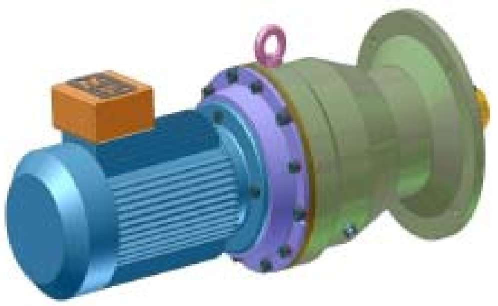


a.

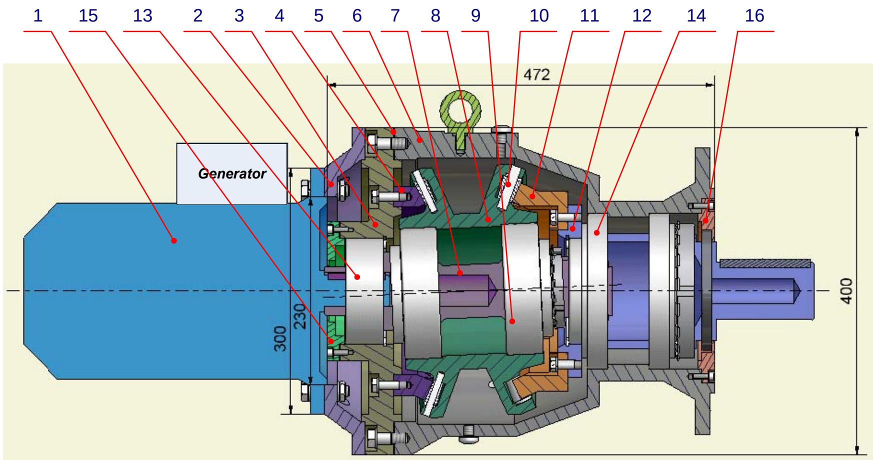
b.

Fig. 6. Planetary precessional multiplier:
a – general view; b – section view**.**

To simulate the reducer assembly and functioning, the dynamic computerised model of the precessional reducer was developed in AutoDesk MotionInventor (figure 5, *b*). The precessional reducer has small dimensions and mass, high loading capacity and ratio up to i = 3600.

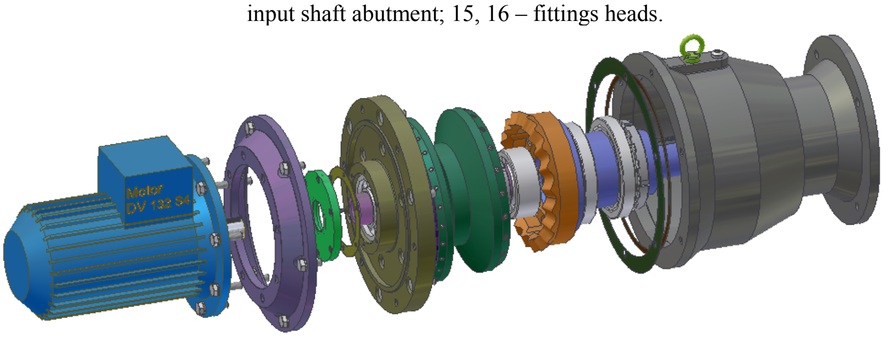


a.



b.

1 – electrical motor; 2 – flange: motor - reducer; 3 – cover; 4 – fixed gear wheel; 5 – adjustment pitch (washer); 6 – reducer housing; 7 – crank shaft; 8 – satellite gear; 9 – bearings for the satellite abutment on the crank shaft; 10 – unit (subsystem): conical roller – axes – disk; 11 – mobile wheel gear; 12 – input shaft; 13 – bearings for the central abutment of the crank shaft; 14 – bearings for the input shaft abutment; 15, 16 – fittings heads.



c.

Fig. 7. Planetary precessional reducer: a – general view; b – section view; c – components in progress.

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