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Stress concentration in the components of a drive coupling

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Abstract. Actuating couplings are used to transmit the torque between the ends of two shafts with or without coaxial deviations. The stress distribution and the stress concentration in a coupling loaded statically by a torque are presented in this paper. The calculation of the stress concentration coefficients in the dynamic stress regime of the machine parts is usually done with relations in whom the stress concentration coefficients in the static regime have a very important weight. For this reason, knowing the values of the stress concentration coefficients in static regime is absolutely necessary. The analysis was made using the finite element method. A calculus methodology is proposed establishing how the loads are applied and how the anti-symmetry of loading is taken into account when boundary conditions are imposed. The influence of the free space between the shaft and the sleeve on the stress field in these two parts is analysed thought the values of the stress concentration coefficient.

Key words: stress concentration, coupling, finite elements, shaft, sleeve

1. Introduction

The drive couplings [1] are used to transmit torques between the ends of two shafts with or without coaxial deviations. Fig. 1 shows a movable grooved coupling used to drive rolling mill cylinders [2]. The coupling, made of cast and hardened steel, consists of an inner grooved sleeve which is mounted on one or two grooved half-couplings fixed on the ends of two shafts. In the specialist treatises [1], [2] the strength calculations are limited to the twist check calculations of the sleeve and the shaft as well as to the contact pressure between the grooves.
Presence of stresses concentrators both in the grooved shaft and in the sleeve, in the transition zone from one groove to another, where a sudden change of

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concavity takes place, introduces strong concentration stresses. At the same time, jumps of the stresses values are registered in the contact area of the grooves. The investigation of the stresses state in the component elements of the coupling, with the highlighting of the stresses concentrations, represents an important step in establishing the overloaded areas and the measures that are required for a proper operation of the coupling.

Stress concentration [3] is the phenomenon of increasing the value of stresses that occur in certain parts of the required bodies in which the cross section shows rapid variations in shape and size. The geometric elements that produce such effects are called stress concentrators. Holes, transverse or longitudinal channels, connections, threads that are frequently found in the machine elements that make up the mechanical structures are stress concentrators.

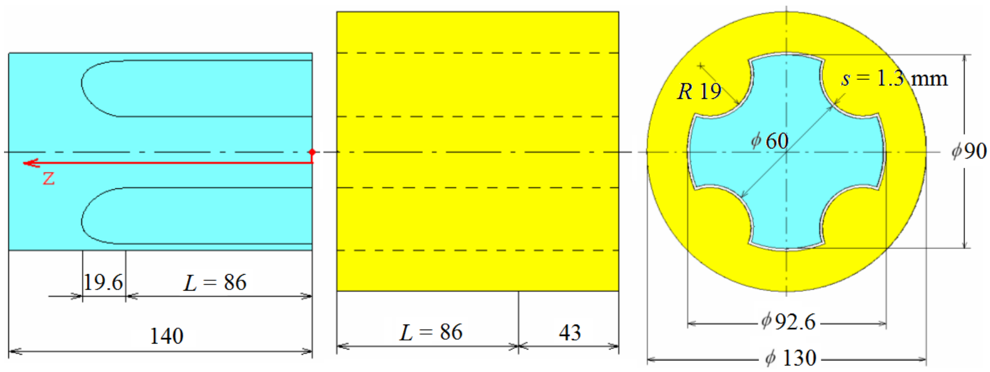


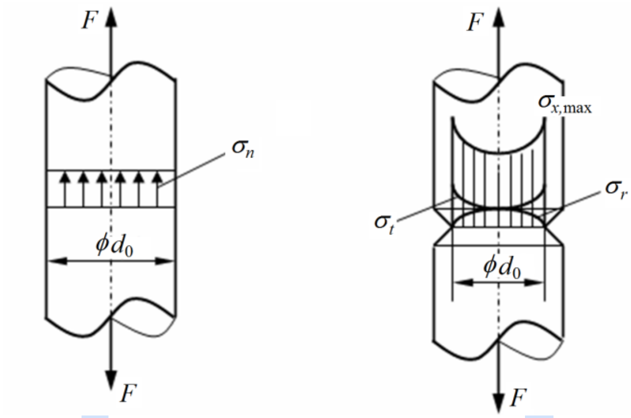
Fig.1. Coupling geometry. (a) Side view of assembly components; (b) Coupling section.

The existence of concentrators contributes to the shortening of the life of parts or construction elements. Moving parts where the stress varies cyclically are in danger of giving way in areas with concentrators. The effect of a stress concentrator is manifested on an area in its vicinity. In the case of a bar without a concentrator (Fig. 2, a) the stress state is mono-axial characterized by the component:

$$\sigma_n = \frac{F}{A_{\min}}, \tag{1}$$

where F is the axial force and $A_{\min}$ the net cross-sectional area of the bar.

In the bar with a notch (Fig. 2, b) that has the same minimum cross section, the stress state is triaxial in the area of the concentrator. If the stress occurs in the field of linearly elastic deformations, the axial stresses σ_x circumferential σ_t and radial σ_r are distributed in the minimum cross section as seen in Fig. 2, b.



(a) (b)

Fig. 2.Stress concentration.

For reasons of symmetry the stresses σ_x, σ_r, σ_t are principal stresses and can be identified by: σ_1, σ_2, σ_3 ($\sigma_1 > \sigma_2 > \sigma_3$).
The maximum stress occurs at the top of the notch and is equal to:

$$\sigma_{\max} = \sigma_{1,\max} = \sigma_{x,\max} \,. \tag{2}$$

The theoretical stress concentration coefficient, which expresses the local effect at static stresses, is calculated as the ratio between the maximum stress in the section of a concentrator and the nominal stress σ_n (Fig. 2, a), which is determined for the same minimum cross section, but in which there is no concentration effect, i.e.

$$K_t = K_{t,\sigma} = \frac{\sigma_{\max}}{\sigma_n} \tag{3}$$

For parts loaded in torsion or shear, the following theoretical stress concentration coefficient is defined:

$$K_t = K_{t,\tau} = \frac{\tau_{\max}}{\tau_n} \,. \tag{4}$$

In the case of bars, the nominal stresses are calculated with the relations in the Strength of materials that do not take into account the concentration effect, nor the fact that the area of the section changes during the stress, especially after exceeding the yield limit of the material. The characteristic curve of the material shows that the real stress is higher than the engineering stress. Relationships (3) and (4) are valid for fragile materials or for tenacious materials that have a characteristic linear curve until the yield limit is reached. It is noted that at the top of the notch the stresses σ_x and σ_t take maximum values, but applying the theory of strength of the maximum normal stress, in relations (3) and (4) only the stress σ_x is taken into account. In general, if the stress state in the concentrator area is spatial, with the main stresses σ_1, σ_2, σ_3, the most stressed point in relation (2) shall be considered, i.e.

$$\sigma_{\max} = \max\left(\left|\sigma_1\right|, \left|\sigma_3\right|\right) . \tag{5}$$

If there are two intersecting concentrators in the bar section, the total stress concentration coefficient is calculated with the relation

$$k_t = k_{t,1} \cdot k_{t,2}\,, \tag{6}$$

where $k_{t,1}$ and $k_{t,2}$ represent the stress concentration coefficients for each concentrator. For bars, the nominal stresses σ_n and τ_n produced in the case of simple stresses, are calculated with the relations in Strength of materials which are based on the hypothesis of plane sections and do not take into account the effect of stress concentrators. The analytical calculation of the effective stress $\sigma_{\max}$ ($\tau_{\max}$) is estimated accurately, using the methods applied for an elastic linear model. The local effects that occur in the shell-type structures characterized by geometric-structural discontinuities at the joints between the component parts, a coefficient similar to that of stress concentration is defined, called the stress amplification coefficient k_t which represents the ratio between the maximum stress in the area with discontinuity and a reference stress in an area far enough away from it. This coefficient generally has values greater than two. Such shell like structures is frequently found on machine elements for the process industry. It has been found [4] that approximately 80% of damage to pressure vessels is due to local stress concentration effects in areas with various geometric discontinuities, holes, connections, support brackets, sudden changes in thickness, defective welds. For this reason, in the design phase of such machines, constructive aspects must be taken into consideration to reduce stress spikes, thus reducing the risk of cracks. An example of this is the stiffening of the edges of the holes.

Knowing the coefficients $k_{t,\sigma}$ and $k_{t,\tau}$ of stress concentrators that often occur in engineering practice, the maximum stresses are calculated with the relations

$$\sigma_{\max} = k_{t,\sigma} \cdot \sigma_n\,; \tag{7}$$

$$\tau_{\max} = k_{t,\tau} \cdot \tau_n\,. \tag{8}$$

Values of the stress concentration coefficients $k_{t,\sigma}$ and $k_{t,\tau}$ depend only on the type of stress (Fig. 2) and the geometry of the part in the weakened area and are independent of the material. In specialized works [3], [5], for each type of concentrator and stress are drawn, in parametric coordinates, the variation of the stress concentration coefficients. The study of the stress state in the vicinity of concentrators can be performed for simple cases with the methods of Theory of Elasticity and for complex cases, with very good accuracy, by the Finite Element Method (FEM) [6] and experimentally with photoelasticity [7], electrical strain gauge tensometry [8], holographic interferometry [9] or moiré fringes [10]. Analytically, using the stress function method, Kirsch [11] determined the stress distribution around a hole in a plate for traction load. Mushelishvili [12] used the complex variable function method to solve two-dimensional elasticity problems. Neuber [13] and Savin [5] applied the method of potential displacement functions to the study of stress concentrations in flat bands with lateral or central cuts and in axially-symmetrical bodies loaded in stretching, twisting and bending establishing expressions of stress concentration coefficients. Savin [5] deepens the study of the concentration of stresses around holes of different shapes by using conformal transformations and the functions of complex variable. Experimentally, Frocht [14] by photoelasticity, establishes a relationship for the calculation of stress concentration coefficients for steel shafts with annular recesses or with shoulders and connections. Leikin [15], by electrical strain gauge analysis, studied the effect of stress concentration in tubular drive shafts with step varying diameters in twisting or bending and established relationships for the calculation of stress concentration coefficients. Based on the established relationships, curves of variations of these coefficients were drawn depending on different dimensionless geometric parameters of the concentrators.

The analysis of the stress state and of the stress concentrations in the two elements of the coupling by analytical way, using the Airy function [16], [17], is laborious in the engineering practice. However, finite element analysis allows the rapid determination of stresses distribution in different operating situations. Research by FEM regarding the concentration of stresses in bars with different types of concentrators, required for torsion, are presented in [18], [19], [20], [21].

2. Distribution of stresses in the coupling

The adopted calculation model is shown in Fig. 3. The conceptual model took into account the existence of geometric symmetry and loading antisymmetry, singularities, connections, location of tasks and reactions. Adaptive discretization was done with BRICK type elements with 20 nodes resulting in a number of 99153 nodes and 21432 finite elements as in Fig. 4. Because the problem of contact between the shaft and the sleeve is strongly influenced by the "quality" of the discretization of the probable contact area, this area was very finely discretized. At the same time, the nodal coordinates must approximate as closely as possible the geometry of the surfaces in contact.

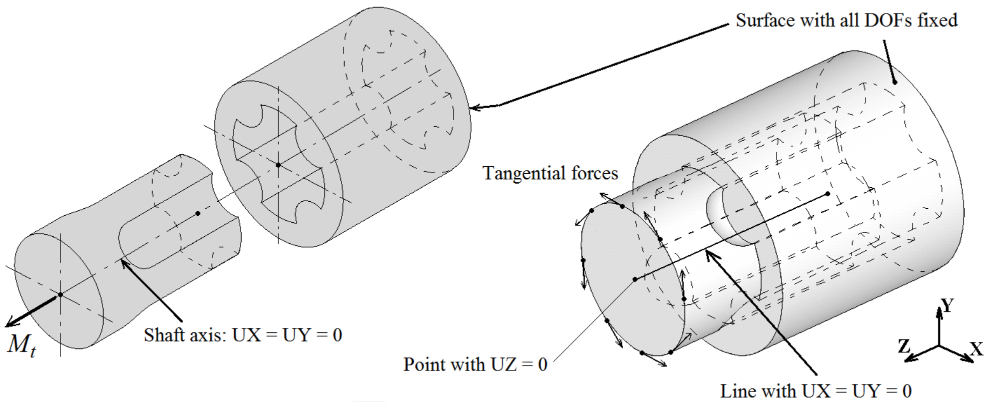


Fig 3. The conceptual model.

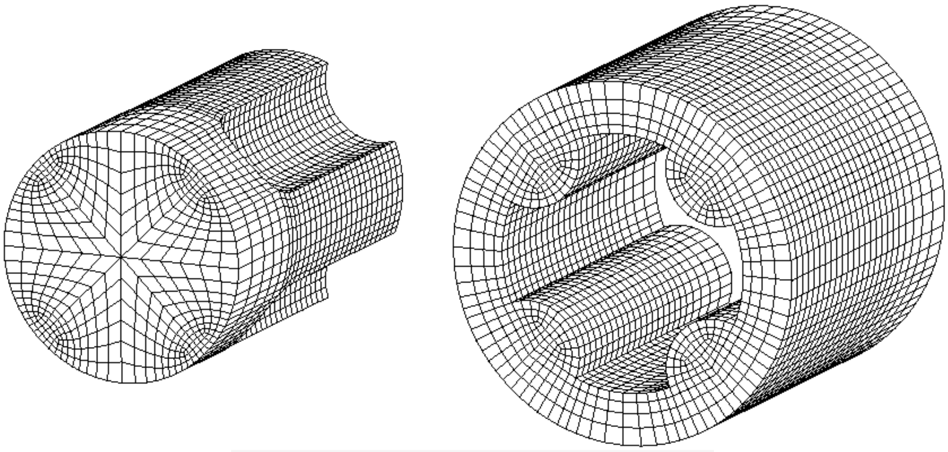


Fig. 4. Discretization of the coupling components.

For the material from which the coupling is made, the following values for the Young's modulus and Poisson's ratio were considered: E = 210 GPa and ν = 0.3.
The stress distribution in the shaft is analyzed to define a shaft stress concentrator. The maximum stress produced by the torsion in the shaft of full circular section (without groove clearances) is considered as the reference stress.
The sleeve was considered to be embedded at one end and at the free end of the grooved shaft, a torque was applied which produces in the full shaft a maximum tangential stress of 100 MPa. The torque was achieved by applying on the outer radius of the shaft end, in the 176 nodes resulting from the discretization, tangential forces. The value of a force results from the application of the relationship in Strength of materials [3]:

$$\tau_{\max} = \frac{M_t}{W_p}, \tag{9}$$

or

$$\tau_{\max} = \frac{16M_t}{\pi d^3}. \tag{10}$$

For 100 MPa and d = 90 mm it results $M_t = 14.31 \cdot 10^6$ Nmm. But $M_t = 176\frac{d}{2}$, resulting for a nodal force F = 1807.3 N.
The research carried out took into account the following variants of analyzing the distribution of stresses in the coupling:

- Variant 1: uniform constant gap space s = 1.3 mm between shaft and sleeve;
- Variant 2: no gap, i.e. s = 0 between shaft and sleeve;
- Variant 3: a simplified model that considers the grooved shaft embedded at one end and loaded at the other end with the torque $M_t = 14.31 \cdot 10^6$ Nmm.

The contact between the grooves was considered without friction, applicable to well-lubricated surfaces. This type of contact only introduces a normal pressure to the contact surfaces. The contact was modeled by surface-to-surface elements [6]. This type of element is designed to work if one of the surfaces, called the contact, tends to penetrate the other surface, called the target. The surface-to-surface contact elements have the advantage that they are superimposed over the faces of the finite elements that shape the structure. The approach to the problem of contact must take into account that finite element modeling transforms the continuum into a discrete one. Bar and plate models are introduced, for which lines and surfaces are used, so it is not possible to discuss only the contact of two surfaces, as it is in reality.

Before running the calculation program, the shaft was rotated so that it came in contact with the sleeve.

Following the running of the calculation program for the grooved shaft and sleeve, with *s* = 1.3 mm, are given:

- variation of the maximum main stress, in Fig. 5;
- distribution of tangential stress in cylindrical coordinates, in Fig. 6;
- variation of the contact pressure on the grooves, in Fig. 7;
- distribution of the tangential stress on the middle line of a groove, in Fig.8.

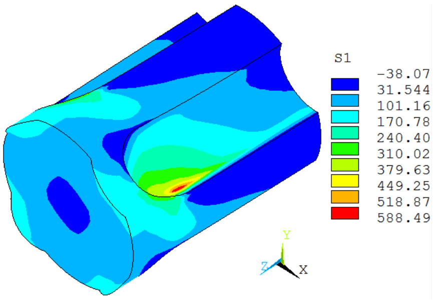


Fig 5. Variation of the maximum principal stress.

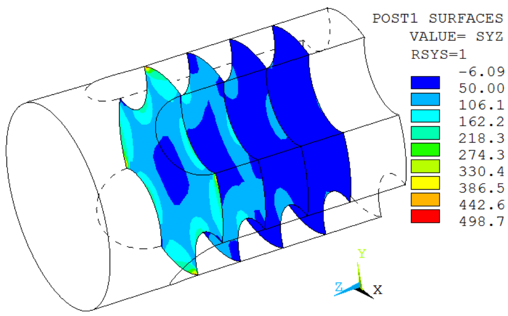


Fig. 6. Tangential stress variation in cylindrical coordinates $\tau_{\theta z}$.

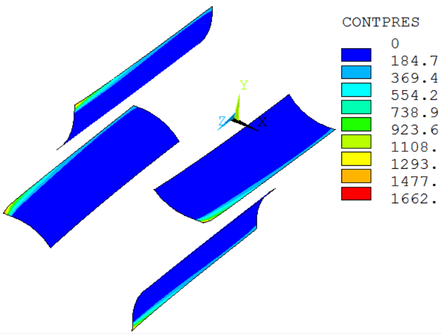


Fig. 7. Distribution of contact pressure between the grooves.

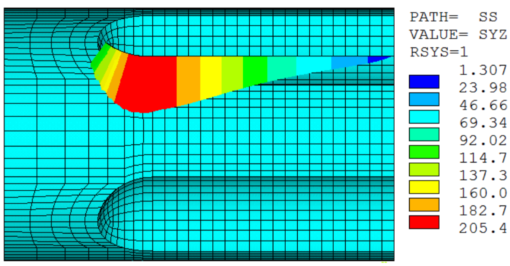


Fig. 8. Distribution of the tangential stress $\tau_{\theta z}$ on the middle line of a groove.

It is found that the torque is transmitted unevenly from the shaft to the sleeve. The distribution of the torque for different values of the *z* coordinate points (Fig. 8), from table 1 is obtained if the equivalence relation [17] is applied

$$M_r = \int_A r\tau_{\theta z} dA. \tag{11}$$

Table 1. Torque distribution along the shaft

z [mm]	21.5	43	64.5	75.25	82	86	96
M_t [kNm]	1.94	4.05	7.48	10.27	12.64	14.65	14.48

It is observed that the calculated values of the torques are slightly higher than the applied moment (from the calculations M_t = 14.48 kNm compared to the applied moment M_t = 14.31 kNm). The differences recorded are due to errors in the numerical calculation.

Distribution of the sleeve stresses are shown in Fig. 9 and Fig. 10.

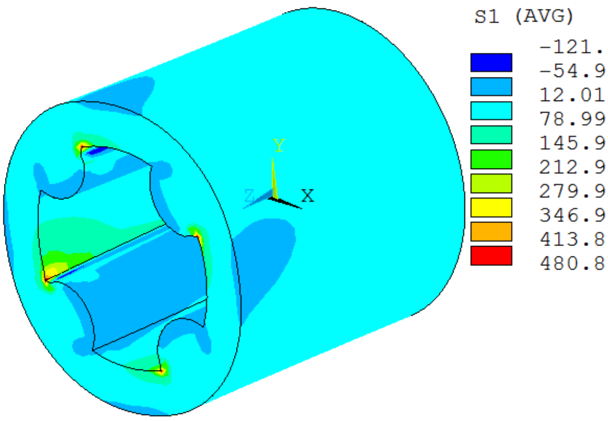


Fig. 9. Variation of the maximum principal stress.

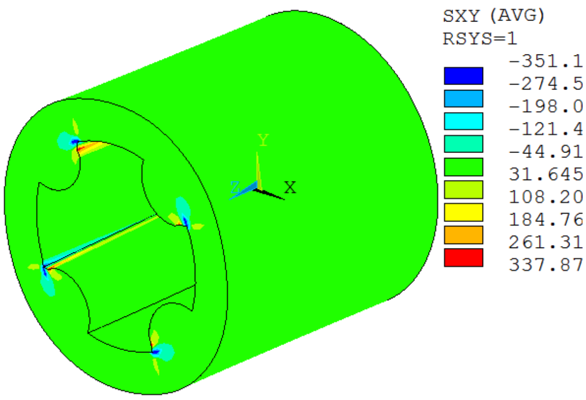


Fig. 10. Tangential stresses variation in cylindrical coordinates $\tau_{r\theta}$.

By analyzing the stresses distribution, the maximum values of the main, tangential and contact stresses in the shaft of the coupling were obtained: 588.9 MPa; 498.8 MPa; 1663 MPa. The maximum values of the stresses take place in the end area of the groove, at the sudden change of the concavity By applying the relation (4) it resulted for the stress concentration coefficient k = 4.988. At the same time it is found that the maximum stresses in the sleeve are lower than those in the shaft and will not be commented in the paper. The net reference stress was considered the maximum torsion stress in the shaft of full circular section, without grooves.

If the gap between the grooves becomes zero, then the stress distribution changes, the stress values decrease much as can be seen in Fig. 11, Fig. 12 and Fig.13. Thus, the maximum principal stress, the maximum tangential stress and the contact stress decrease from the values of 588.9 MPa; 498.8 MPa; 1663 MPa in the case of a free space between the shaft and the sleeve s = 1.3 mm to 362.3 MPa; 303.0 MPa; 854.7 MPa. At the same time the value of the stress concentration coefficient decreases from 4.98 to 3.03.

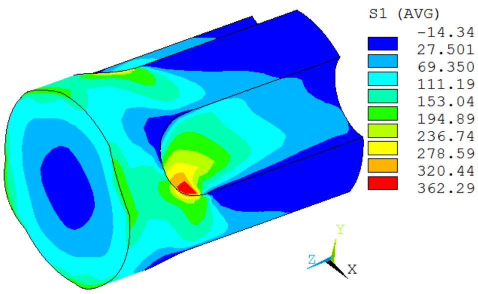


Fig.11. Variation of the maximum principal stress.

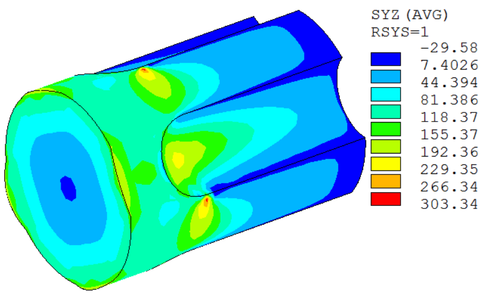


Fig.12. Tangential stress variation in cylindrical coordinates $\tau_{\theta z}$.

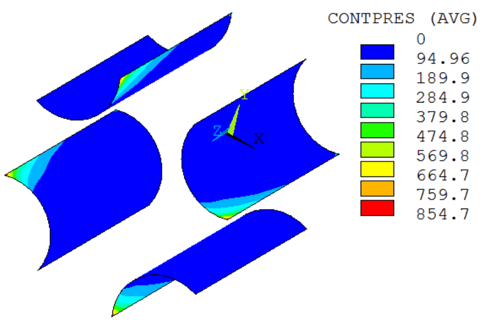


Fig. 13. Distribution of contact pressure between the groove and shaft (s = 0).

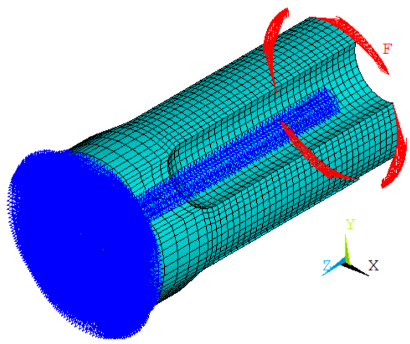


Fig. 14. Shaftless sleeve model loaded at one end.

For the calculation of the simplified model that considered the grooved shaft embedded at one end and loaded at the other end with the torque M_t = 14.31·10^6 Nmm, totally different results were obtained from those previously determined. If the shaft is extended by 50 mm, in order to remove the concentration effects of the stresses produced by the load application mode and to load the free end (Fig. 14), a distribution of the maximum main stress and the tangential stress results, as those in Fig. 15 and Fig. 16.

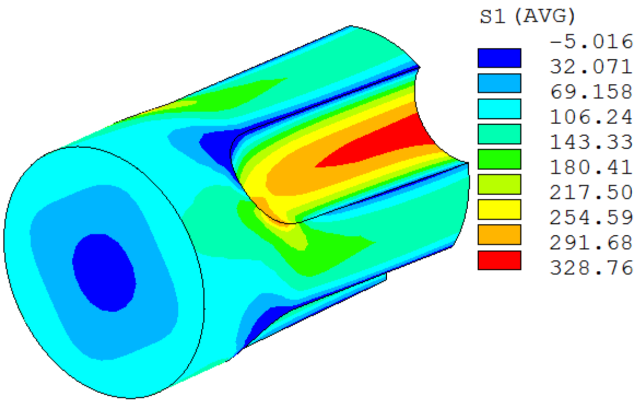


Fig. 15. Variation of the maximum principal stress (simplified model).

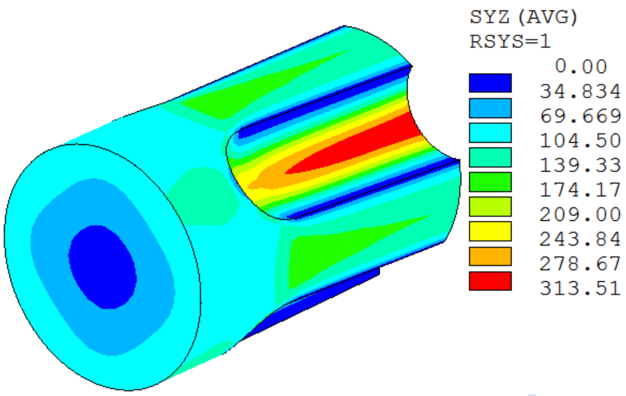


Fig. 16. Tangential stresses variation, in cylindrical coordinates (simplified model).

3. Conclusions

The investigation of the stresses concentrations based on the numerical analysis of the stresses state for a drive coupling was performed on three calculation models: with free space s = 1.3 mm between shaft and sleeve, without free space and a simplified model that considers only the coupling shaft. The analysis of the results shows the following:

- the values of the stresses in the shaft are higher than the values of the stresses in the wall of the sleeve;
- the torque is transmitted unevenly from the shaft to the sleeve;

- the transitions from one concavity to another of the groove surfaces represent strong areas of concentration of stresses;
- reducing the play between the shaft and the sleeve contributes to the decrease of the stresses values and of the stress concentration coefficient;
- application on the outer radius of the shaft end in a large number of points (176 in the studied model) of the forces requiring torsional coupling, reduces the importance of local problems created by concentrated forces;
- the simplified model, in which the shaft fully takes over the torque, deviates from the actual case of the coupling formed by the shaft-sleeve assembly;
- the realization of an adaptive discretization facilitates the study of the influence of the concentrators on the stresses distribution in the coupling.

Numerical analysis of stresses distributions allowed the study of several geometric and mechanical models for coupling.

The research highlighted at the same time the influence that different geometric and technological parameters have on the stress state and the values of the stresses concentration coefficients in the two component elements of the coupling.

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