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Total size minimization of cam mechanisms with translating follower

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Abstract. Usually, the size optimization by the cam mechanisms is reduced to the cam size only, which is not completely correct because the cam is only one of the two important entities working together. The cam follower optimization is taken into account, but only as distinct part of the cam mechanism. Total size regarded as a minimization criterion is an important problem in cam mechanism synthesis. The paper shows a total size minimization of the cam mechanism, where both parts are equally considered. The mathematical generalization is developed for cam mechanisms with translating roller-follower and flat-face follower.

Keywords: cam mechanism, general optimization, translating roller/flat-follower, minimum size of the cam mechanism.

1. Introduction

Fundamentals of cam mechanism design, regarding cam types, cam dimensioning computation, various follower types and their opportunity of choice, aspects regarding materials selection, technology, tribology and reliability – [1-7] – are deepened through thorough analysis of various features on purpose of optimization the parts of the mechanism, following different criteria. The interest in cam mechanisms is very visible in the specialty literature because of intensive use of such mechanisms in important technical areas, such as automotive or robotics [8,9]. Optimization of cam mechanisms refers to computer-aided analysis regarding the pressure angle, contact stress and/or wear, cam-follower separation.
Chablat and Caro [8] presented an interesting workflow of calculus based on minimization of Hertz pressure. They analyze the influence of the design

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parameters on pressure angle and Hertz pressure, and then describe a successful implementation of their results in the design of a Slide-o-Cam transmission for high-speed machines. Yong and Yanping [10] developed a so-called Auxiliary Angle Method, which overcomes the weak points of the traditional Velocity Instantaneous-Pole Method in deriving the pressure angle expression. A particular type of cam mechanism, namely the cam with negative radius roller-follower is discussed by Carra et al [11]. The proposed synthesis achieves both pressure angle minimization and rise angle lower limitation in order to avoid undercutting occurrence. The contact stress and loading conditions are taken into account in developing the cam design algorithm described by Golovin et al [12].

An interesting approach of cam wear was performed by Golovin et al. [13] through a comprising experimental program and a theoretical development based on retrieving the original cam profile by transfer function of the follower's velocity.

An original approach assuming variable cam speed, expressed as Bezier functions, is proposed by Yan and Tsai [14] in order to prevent cam-follower separation.

Modeling of cam mechanisms tends to preserve a central position within the modern research. Different original models ware presented by a large series of authors, which focus on various aspects and purposes. Lanni et al [15] developed a set of models involving stiffness and damping parameters. Bouzakis at al [16] imagined a model using a transfer function, which is compliant with a numerical code creation, further sent on to a milling machine. Zhao et al [17] presented a compact generalized mathematical model for the design of cam mechanisms by using some new, original geometric and kinematic formulas. Petropoulou et al [18] introduced an iterative method to compute several follower motion properties such as oscillation, velocity and acceleration, using the vector differences for three successive points on the cam profile and aiming to transfer the results to a NC milling machine. Moustafa [19] proposed a model using the vector analysis in order to obtain equations describing the cam profile.

However, it is obvious that most researches aim to optimize the cam size.

Flores [20] proposed an optimization algorithm of the cam size based on three central parameters (base circle radius, roller radius and offset of the follower). Ji and Mana [21] focused on minimization of the roller-follower under the constraint of maximum pressure angle.

Chan and Sim [22] developed a computer-aided design tool, which can be used to design and optimize disk cams with different follower configurations. The main parameter involved is the radius of the cam base circle, which is optimized using the search method known as the Monte Carlo method.

Terauchi and Shakery [23] optimized the cam size on a principle which states equal maximum contact stress of both rise stroke and fall stroke, and proposed an elegant iterative mathematical solving process.

Sim and Chan [24] tested a genetic algorithm in optimizing cam mechanisms and got positive results in relationship with a range of parameters such as base circle, thickness and volume of the cam.

Naskar and Acharyya [25] before the attempt to develop an extensive experimental program of measuring various kinematic and dynamic sizes, started with a cam size and mass optimization taking into account limited pressure angles and geometrical shape conditions on the cam.

Moise et al [26] conceived an algorithm of cam size minimization on the basis of elimination the angular points of the directory curve, developing a sophisticated mathematical approach. The demarche is dedicated to translating flat-face follower case. Also to flat-face follower but oscillating case, refers Yu [27], who proposed an eccentric cam.

Korunoski et al [28] achieved the minimization of the cam profile area by choosing a certain eccentricity of the follower face and considering the minimum value of the follower rotation angle. Loeff and Soni [29] proposed an optimization algorithm of cam size based on the condition of keeping the pressure stress always less than the maximum admitted value for the compressive stress of fatigue and wear. Angeles and Cajun in [7] shows a minimization method of the cam mechanisms with translating follower based on the minimization of the pressure angle for an optimal eccentricity of the follower. Navarro et al [30] presented a valuable analysis of a set of cam mechanisms, taking into account specific linear and angular parameters regarding both cam and follower. They provided a software application to design cams with minimum base circle radius and safe pressure angle using the minimization method proposed in [7].

Simionescu et al [31] described a robust mathematical model, applicable to multiple cam mechanisms types, conceived to optimize the cam size under a large set of constraints, among which the pressure angle is placed on a central position. Hidalgo et al proposed in [32] an optimization procedure based on adapted Bézier curves to minimize the sliding velocities in planar cam mechanisms with flat-faced translating followers. A modified adaptive differential evolution algorithm is proposed by Ferhat et al in [33] using a multi-objective optimization of a cam mechanism with offset translating roller follower for three objectives: minimum congestion, maximum efficiency and maximum strength resistance of the cam. Djeddou et al in [35] shows preliminary deterministic optimization to find the optimum size of a cam system and to ensure its high operating performance by using an objective function with constraints on performance and resistance indicators. Redjechta et al in [35] continue the deterministic optimization algorithm by considering as well as the geometric conditions. Nguyen et al in [36]. A complex cam profile described by Lagrangian finite elements for imposed displacements of the follower is proposed by Nguyen et al in [36]

Taking into account the state-of-art presented above, one may conclude that all analysis and optimization solutions in the literature focus mainly on minimization of cam size and, in fewer cases, on minimization of the area covered by the follower. The present paper considers as global minimization criterion the total size of the cam mechanism, including cam and follower (translating roller follower, translating flat-face follower). Some of the authors have previously published preliminary studies regarding minimizing the size of both types of cam mechanisms with translating roller and flat-face follower [37], [38], [39].

2. Cam mechanisms with translating roller-follower

The total size minimization of the cam mechanism with translating follower implies:
- the determination of the optimum value of the eccentricity of the cam in order to reduce the maximum admissible pressure angle of the cam mechanism, thus finding the minimum base radius of the cam
- the determination of the guiding length.

The following paragraphs develop the influence of different parameters on the total size of the cam mechanism with translating follower.

2.1. The pressure angle

The pressure angle α is defined as the angle between the line of the acting force F and the translation direction. In Fig. 1, for a current position of cam mechanism with translating follower, the geometrical parameters and the forces are represented [34].

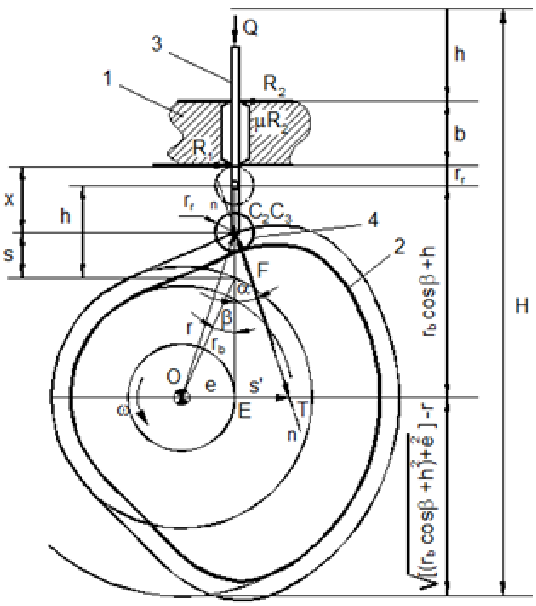

Fig. 1. The geometry, kinematics and force equilibrium on a current position of a cam mechanism with roller translating follower [34].

The follower's velocity ($\overline{v_F}$) can be written as:

$$\overline{v_F} = \overline{v_{C3}} = \overline{v_{C2}} + \overline{v_{C3C2}}\,, \qquad (1)$$

where:

$\overline{v_{C2}} = \overline{\omega} \times \overline{r}$ is the velocity of the follower's roller, center point belonging to cam,

$\overline{v_{C3C2}}$ - the relative velocity between cam and follower at center point level.

If relationship (1) is divided to cam's angular velocity (ω) and it is represented 90° clockwise rotated, it becomes:

$$\overline{OT} = \overline{OC_2} + \overline{C_3T} \ , \tag{2}$$

where:

$$\left|\overline{OT}\right| = \frac{v_F}{\omega} = \frac{ds/dt}{d\varphi/dt} = \frac{ds}{d\varphi} = s' \ . \tag{3}$$

The pressure angle α can be calculated from the triangle *OCT*:

$$\tan\alpha = \frac{ET}{EC} = \frac{s' - e}{s + r_b \cdot \cos\beta} = \frac{s' - r_b \cdot \sin\beta}{s + r_b \cdot \cos\beta} \ , \tag{4}$$

where: r_b is theoretical minimum/basic radius,

s - current position of roller follower's center.

e - eccentricity, which is considered positive for follower's translation line on right side of the cam's rotation centre O,

$$\beta = \arcsin(e / r_b) \ . \tag{5}$$

In the initial/starting position of the follower ($s' = s = 0$) $\tan\alpha_i = -\tan\beta$ or $\alpha_i = -\beta$ and for $s' = e \rightarrow \alpha = 0$.

2.2. The minimum/base radius

The base radius of a cam is defined as the minimum circle radius with the centre in the cam rotation point inscribed in the theoretical cam profile. As usual, the base radius is found imposing the non-blocking condition, i.e.:

$$\alpha \le \alpha_a \quad \text{or} \quad \tan\alpha \le \tan\alpha_a \ , \tag{6}$$

where α_a is maximum admissible pressure angle for active or passive stroke, respectively.

From relationship (4) with condition (6) the base radius results in the form [34]:

$$r_b \ge \frac{s' - s \cdot \tan\alpha_a}{\tan\alpha_a \cdot \cos\beta + \sin\beta} \ . \tag{7}$$

The minimum value for r_b is obtained for the cam position $\varphi = \varphi_M$ for which $\partial r_b / \partial\varphi = 0$, resulting the condition:

$$s''(\varphi_M) = s'(\varphi_M) \cdot \tan\alpha_a \ . \tag{8}$$

The maximum value of the pressure angle ($\alpha_{\max}$) can be found for the cam's position, where:

$$\partial\alpha/\partial\varphi = 0\,. \tag{9}$$

From relationship (4) results:

$$\frac{d}{d\varphi}(\tan\alpha) = \frac{1}{\cos^2\alpha}\cdot\frac{d\alpha}{d\varphi} = \frac{s''(r_b\cdot\cos\beta + s) - s'(s' - r_b\sin\beta)}{(r_b\cdot\cos\beta + s)^2}\,, \tag{10}$$

from which concludes:

$$\frac{d\alpha}{d\varphi} = \frac{s''(r_b\cdot\cos\beta + s) - s'(s' - r_b\cdot\sin\beta)}{(r_b\cdot\cos\beta + s)^2}\cdot\frac{1}{1+\tan^2\alpha}\,. \tag{11}$$

The condition (9) taking into account the relationships (4), (7) and (11), becomes:

$$(s'' - s'\cdot\tan\alpha_a)\left(s' + s\cdot\tan\beta\right) = 0\,, \tag{12}$$

or:

$$s''(\varphi_M) = s'(\varphi_M)\tan\alpha_a\,, \tag{13}$$

which is identical with condition (8) i.e. the base radius has a minimum value if the maximum value of pressure angle satisfies condition (6), in cam's position where condition (8) or (13) is fulfilled.

Observation: The second bracket in relationship (12) cannot equal zero because both terms have the same sign.

2.3. The guiding size

The translating follower is guided and the guiding length is an important parameter in order to determinate the total size of the cam mechanism with translating follower. The guiding length results from forces equilibrium of the follower [34]:

$$F\cdot\cos\alpha \ge Q + \mu\cdot(R_1 + R_2)\,, \tag{14}$$

where: Q is the total force acting on the follower,

F – the acting force from cam to follower,

$\mathrm{R_1}$ and $\mathrm{R_2}$ – the reaction forces in the guide way,

$$R_1 = F\cdot\sin\alpha\cdot\left(1 + x/b\right),\qquad R_2 = F\cdot\sin\alpha\cdot(x/b)\,. \tag{15}$$

calculated by neglecting transversal follower's dimension.

With these values, the condition (14) becomes:

$$F\cdot\cos\alpha \ge Q + \mu\cdot F\cdot\sin\alpha\cdot[1 + 2(x/b)]\,. \tag{16}$$

x is the current distance from roller's center C to the first edge of the guiding.

Using the so called loading coefficient $\Phi = Q/F \in (0,1)$ the relationship (16) can be written in the form:

$$\Phi \le \cos\alpha - \mu\cdot\sin\alpha\cdot[1 + 2(x/b)]\,. \tag{17}$$

From this condition, in the critical position of the follower, the guiding length can be found:

$$b \geq 2 \cdot x_{cr} \frac{\mu \cdot \sin \alpha_a}{\cos \alpha_a - \mu \cdot \sin \alpha_a - \Phi} . \tag{18}$$

The critical position is reached at:

$$\alpha_{\max} = \alpha_{\mathrm{a}} = \alpha(\varphi_{\mathrm{M}}), \quad \varphi = \varphi_{\mathrm{M}} , \tag{19}$$

see condition (8) or (13), and Fig.1, so that:

$$x_{cr} = h + r_r - s(\varphi_M) . \tag{20}$$

The relationship (18) with condition (20), becomes:

$$b \geq \frac{2 \cdot \mu \cdot \sin \alpha_a}{\cos \alpha_a - \mu \cdot \sin \alpha_a - \Phi} \left(h + r_r - s(\varphi_M)\right), \tag{21}$$

or in non-dimensional form:

$$\frac{b}{h} \geq \frac{2 \cdot \mu \cdot \sin \alpha_a}{\cos \alpha_a - \mu \cdot \sin \alpha_a - \Phi} \left(1 + \frac{r_r}{h} - \frac{s(\varphi_M)}{h}\right). \tag{22}$$

2.4. The optimum value for the eccentricity and the base radius

From the total size minimization point of view the eccentricity is better to be as large as possible. In this case (Fig.1) the starting/end point of the active/passive stroke, on the base circle, goes down. But, in this case, the pressure angle on basic circle β increases, according to Fig.1 and relationship (5). In order to avoid the mechanism´s blockage in this position, the pressure angle β has to be less or equal to the admissible value α_a. Accepting $\beta = \alpha_a$ the critical position for mechanism´s blockage becomes the above written one because x_{cr} is maximum ($x_{cr} = h + r_r$ according to relationship (20)). With the new value of x_{cr}, the follower´s guiding length b will increase according to relationship (18).

Now it is obvious that eccentricity e (or corresponding angle β) has to be limited such as the guiding length b remains at the same value given by relationship (18), for the critical position on active/passive stroke, i.e.:

$$\mathrm{b}(\alpha_{\mathrm{a}}, \mathrm{x}_{\mathrm{cr}}) = \mathrm{b}_0(\beta, \mathrm{x}_{\mathrm{cr}0}) , \tag{23}$$

where: $b(\alpha_a, x_{cr})$ - see relationship (21)

$$b_0(\beta, x_{cr0}) = \frac{2 \cdot \mu \cdot \sin \beta}{\cos \beta - \mu \cdot \sin \beta - \Phi} \left(h + r_r\right). \tag{24}$$

With these values, from relationships (23), (21) and (24) results:

$$A \cdot \cos \beta - B \cdot \sin \beta - A \cdot \Phi = 0 . \tag{25}$$

where:

$$A = C \cdot (h + r_r - s(\varphi_M))\ , \quad B = (h + r_r) \cdot (1 + \mu C) - \mu C \cdot s(\varphi_M) \tag{26}$$

and

$$C = \frac{\sin \alpha_a}{\cos \alpha_a - \mu \cdot \sin \alpha_a - \Phi}\ . \tag{27}$$

From equation (25) results:

$$\beta = 2 \cdot \arctan \frac{-B \pm \sqrt{B^2 + A^2 \cdot (1 - \Phi^2)}}{A \cdot (1 + \Phi)}\ . \tag{28}$$

From the two obtained values will be taken into consideration one which it obeys the condition:

$$|\beta| < \alpha_a\ . \tag{29}$$

Now with this value of the angle of eccentricity β the minimum/base radius of the cam can be calculated with relationship (7) as an optimum one, as well as the eccentricity e with relationship:

$$e = r_b \cdot \sin \beta\ . \tag{30}$$

The base radius is calculated for both strokes and, of course, will be accepted the biggest one together with corresponding eccentricity.

2.5. The guiding size

Taking into account the dimensions indicated in Fig.1, the total size of the mechanism in the follower's translation direction is [34]:

$$H \geq 2 \cdot h + r_b \cdot \cos \beta + b + \sqrt{(r_b \cdot \cos \beta + h)^2 + e^2}\ , \tag{31}$$

or in non-dimensional expression:

$$\frac{H}{h} \geq 2 + \frac{r_b \cdot \cos \beta}{h} + \frac{b}{h} + \sqrt{\left(1 + \frac{r_b \cdot \cos \beta}{h}\right)^2 + \left(\frac{e}{h}\right)^2}\ . \tag{32}$$

For a centric cam mechanism ($e = 0$) these relationships become:

$$H \geq 3 \cdot h + 2 \cdot r_b + b\ , \tag{33}$$

or in non-dimensional expression:

$$\frac{H}{h} \geq 3 + 2 \cdot \frac{r_b}{h} + \frac{b}{h}\ . \tag{34}$$

Obviously, on the transverse direction the size is:

$$T = 2 \cdot (\sqrt{(r_b \cdot \cos \beta + h)^2 + e^2} - r_r)\ , \tag{35}$$

according to Fig.1, or in non-dimensional expression:

$$\frac{T}{h} = 2 \cdot \left(\sqrt{\left(\frac{r_b}{h} \cdot \cos\beta + 1\right)^2 + \left(\frac{e}{h}\right)^2} - \frac{r_r}{h} \right). \tag{36}$$

It is interesting to notice that the longitudinal dimension of the cam mechanism is only a little bit influenced by the roller radius (by means of dimension b (21), (22)), so that it can be chosen as large as is possible, being limited by cam´s profile undercutting ($r_r = \rho_{\min}$). A large roller radius is also favorable both for reducing the transversally size of cam mechanism and reducing the wear and contact stress between cam and roller as well.

The size of the total aria in the mechanism´s plane of movement, in non dimensional expression, can be put in the form:

$$\frac{A}{h^2} = \frac{H}{h} \cdot \frac{T}{h}, \tag{37}$$

which may represents an indicator and criteria for size minimization.

3. Cam mechanism with translating flat/tangential follower

3.1. The basic radius

In this case the pressure angle is zero and the base radius is computed from geometrical condition, namely the avoiding of singularities of the cam's profile. In this condition, the basic radius is computed for both strokes and then is chosen the largest one [33]:

$$r_{b\,a} \geq \left| s''_{\max a} \right| - s\big|_{s''_{\max\ a}}, \quad r_{b\,p} \geq \left| s''_{\max p} \right| - s\big|_{s''_{\max\ p}}, \tag{38}$$

$$r_b = \max(r_{b\,a}, r_{b\,p}), \tag{39}$$

or in non dimensional form:

$$\frac{r_b}{h} \geq \left| \frac{s''_{\max i}}{h} \right| - \frac{s_i}{h}\bigg|_{s''_{\max}}, \qquad i = a, p\,. \tag{40}$$

The indices *a* and *p* represents the active or passive/returning stroke respectively.

3.2. The guiding size

In Fig. 2 the cam mechanism's main parameters are presented in a current position of it. Using the same procedure as in §2.1 for contact points velocities one can write:

$$\overline{v_{Fl}} = \overline{v_{C3}} = \overline{v_{C2}} + \overline{v_{C3C2}}, \tag{41}$$

By dividing to cam´s angular velocity and representing the triangle rotated 90° counter clockwise [8], [9], [13], one obtains:

$$\overline{C_3F_l} = \overline{C_2O} + \overline{OF_l} \ , \qquad (42)$$

with

$$\left|\overline{CF_l}\right| = v_{Fl} / \omega = s' \ , \quad \left|\overline{OF_l}\right| = v_{C3C2} / \omega = r_b + s \ . \qquad (43)$$

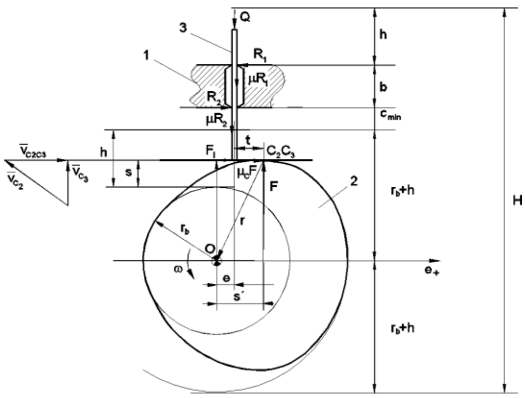

Fig. 2. Kinematics and forces equilibrium of the cam mechanism with flat follower [33]

By equilibrium conditions of the follower one gets:

$$R_1 = \frac{F}{b} \cdot \left[t - \mu_C (h + c_{\min} - s)\right], \quad R_2 = \frac{F}{b} \cdot \left[t - \mu_C (b + h + c_{\min} - s)\right] \ , \qquad (44)$$

where: $t = s' \pm e$, as the follower's guiding way is on the right or on the left side, in respect with cam's rotation center O, respectively,

μ_C - the friction coefficient between cam and follower,

$c_{\min}$ – the minimum clearance between follower's disc and the guide way.

The working condition is obvious:

$$\mathrm{F} \geq \mathrm{Q} + \mu \cdot (\mathrm{R}_1 + \mathrm{R}_2) \ . \qquad (45)$$

With the values given in relationships (44) this condition can be written in the form:

$$2 \cdot \mu \left[s' \mp e - \mu (h + c_{\min} - s) / b - \mu_C / 2\right] + \Phi \leq 1 \ . \qquad (46)$$

$\Phi = Q/F$ as was defined in §2.3.

From relationship (46) the minimum guiding length results for active (b_a) and passive (returning) stroke (b_p) respectively:

$$b_a \geq 2 \cdot \mu \frac{s'_{\max a} - e - \mu_C (h + c_{\min} - s|_{s'_{\max a}})}{1 - \mu \cdot \mu_C - \Phi}, \tag{47}$$

$$b_p \geq 2 \cdot \mu \frac{\left|s'_{\max p}\right| + e - \mu_C (h + c_{\min} - s|_{s'_{\max p}})}{1 - \mu \cdot \mu_C - \Phi}. \tag{48}$$

The optimum value for the eccentricity is obtained for:

$$\mathrm{b_a} = \mathrm{b_p}, \tag{49}$$

which represents the critical blocking condition for the two strokes.
Considering the maximum values of the follower's velocity at $s = h/2$, as at the most of the usual motion curves (symmetrical motion event), from condition (49), taking into account the relationships (47) and (48), it results:

$$e_{opt} = \frac{1}{2}(s'_{\max a} - \left|s'_{\max p}\right|). \tag{50}$$

In this case, the minimum guiding length will be:

$$b \geq \frac{2 \cdot \mu}{1 - \mu \cdot \mu_C - \Phi}\left[s'_{\max a} - e_{opt} - \mu_C (h + c_{\min} - s|_{s'_{\max a}})\right], \tag{51}$$

or in non dimensional expression:

$$\frac{b}{h} \geq \frac{2 \cdot \mu}{1 - \mu \cdot \mu_C - \Phi}\left[\frac{s'_{\max a}}{h} - \frac{e_{opt}}{h} - \mu_C (1 + \frac{c_{\min}}{h} - \frac{s|_{s'_{\max a}}}{h})\right]. \tag{52}$$

3.3. The guiding size

The total size of a cam mechanism on follower's translating direction can be deduced from Fig. 2, i.e

$$H = 3 \cdot h + 2 \cdot r_b + b + c_{\min}, \tag{53}$$

or in non dimensional expression:

$$\frac{H}{h} = 3 \cdot + 2 \cdot \frac{r_b}{h} + \frac{b}{h} + \frac{c_{\min}}{h}. \tag{54}$$

$c_{\min}$ is the minimum clearance necessary for spring emplacement [33].
On transverse direction, the mechanism's size is:

$$T = 2 \cdot (h + r_b), \tag{55}$$

or in non dimensional expression:

$$\frac{T}{h} = 2 \cdot (1 + \frac{r_b}{h}). \tag{56}$$

Also in this case, the total aria in the mechanism's plane of movement, in non dimensional representation, can be expressed by the relationship (37).

4. Case studies

In order to compare different usual motion curves from the mechanism's size point of view, the two analyzed types of cam mechanisms with translating follower took into consideration the following main design parameters:

$$\begin{array}{l} \varphi_a = 90°, \quad \varphi_p = 180°, \quad \Phi = 0.5, \quad r_r = 0.1 \cdot h, \quad \alpha_a = 40° \; for\, roller\ follow.) \\ c_{\min} = 0.5 \cdot h, \quad \mu = \mu_C = 0.1 \quad for\ flat\ follower \end{array} . \quad (58)$$

The normalized motion curves taken into account [10], [11], [13] for the two strokes are presented in Table 1.

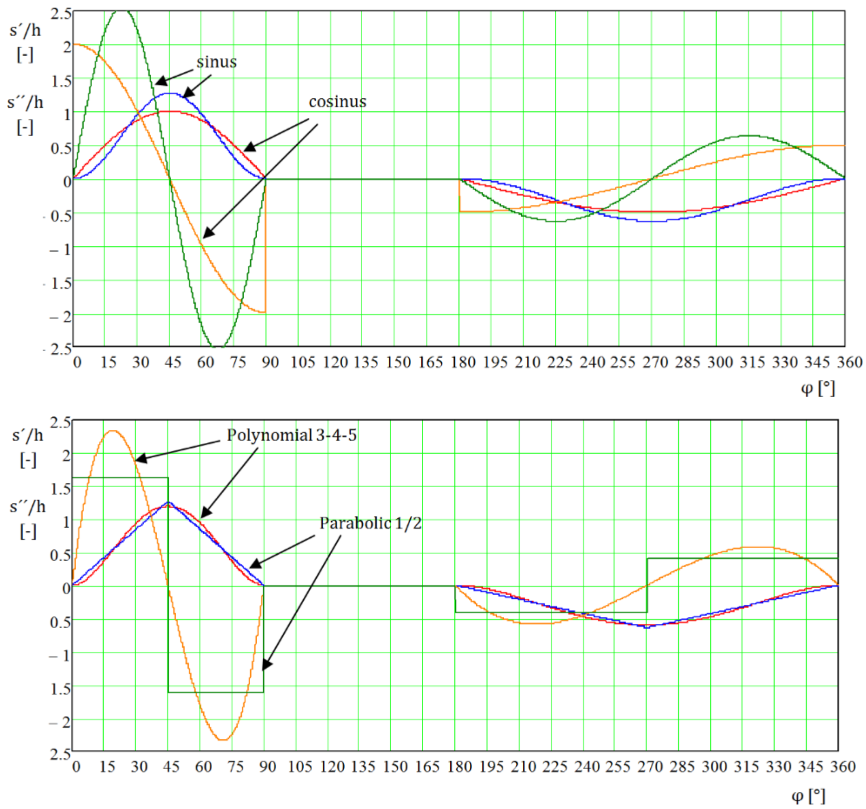

Fig.3 The normalized motion curves of first and second order for the considered example.

Table 1. The analyzed normalized motion curves.

Nr	Motion curves	Active/rise stroke (a)	Passive/returned stroke (p)
1.	Cosine	$y = (1 - \cos \pi x)/2$	$y = (1 + \cos \pi x)/2$
2.	Sine	$y = x - \sin 2\pi x / 2\pi$	$y = 1 - x + \sin 2\pi x / 2\pi$
3.	Parabolic 1/2	$y = 2x^2$ for $x \in (0{,}0.5)$	$y = 1 - 2x^2$ for $x \in (0{,}0.5)$

		$y = 2(-x^2+2x-0.5)$ for $x \in (0.5,1)$	$y = 2(x-1)^2$ for $x \in (0.5,1)$
4.	Polynomial 3-4-5	$y = 10x^3 - 15x^4 + 6x^5$	$y = 1 - 10x^3 + 15x^4 - 6x^5$

* with y= s/h and x=φ/$\varphi_{a/p}$

The main kinematical parameters of these motion curves for the two normalized strokes (active with $\varphi_a = 90°$ and passive/returned with $\varphi_p = 180°$, respectively) are represented in Fig. 3.

4.1. Cam mechanism with roll follower

In order to establish the cam mechanism size and to evaluate the eccentricity influence, the procedure presented in § 2 is applied in the following succession:

- first it is established the cam position φ_M where the pressure angle has the maximum value (relationship (8)),
- the guiding length (in absolute or relative value) is found by means of the relationships (21) and (22), respectively,
- the angle of eccentricity β is calculated with relationship (28),
- now the basic circle's radius can be calculated using relationship (7), as well as the eccentricity e with relationship (30),
- finally, the total size of the cam mechanism on the follower's translation direction H, and on the transversal one on it T, is established.

The above relationships are applied for the stroke, giving the maximum value for the base radius (relationship (7)), i.e. the most "abrupt" one, as it is the active stroke in these example problems.

The main relative dimensions of the cam mechanism with translating roller follower with no eccentricity and with the optimum one are presented in Table 3.

In Figure 4 the variation of pressure angle on active stroke in respect with the cam position (φ) for the centric mechanism (a) and for the optimum eccentric one (b) are presented.

As one can see in Table 3, the total size (H·T) of a cam mechanism with an optimum eccentricity is reduced within the range of (24.6... 32.2)% in respect with the centric cam mechanism, and (10.5... 13.5)% only in the translating direction (H).

Table 3 The main relative dimensions and the total relative size of the analyzed cam mechanisms

Motion curves	φ_M °	b/h	$e=0$				$e = e_{opt}$					
			$\frac{r_b}{h}$	$\frac{H}{h}$	$\frac{T}{h}$	$\frac{H \cdot T}{h^2}$	$\beta°$	$\frac{e_{opt}}{h}$	$\frac{r_b}{h}$	$\frac{H}{h}$	$\frac{T}{h}$	$\frac{H \cdot T}{h^2}$
1. Cosine	33°35′	0.500	0.792	5.09	3.37	17.08	37°00′	0.307	0.526	4.404	2.71	12.0

2. Sine	39°05´	0.650	1.083	5.63	3.88	21.57	35°40´	0.490	0.723	4.905	3.08	15.1
3. Parabolic	45°00´	0.382	1.017	5.41	3.83	20.72	21°20´	0.360	0.690	4.839	3.23	15.6
4.Polynomial 3 -4 - 5	37°46´	0.477	0.998	5.47	3.80	20.78	36.10´	0.387	0.665	4.758	2.96	14.1

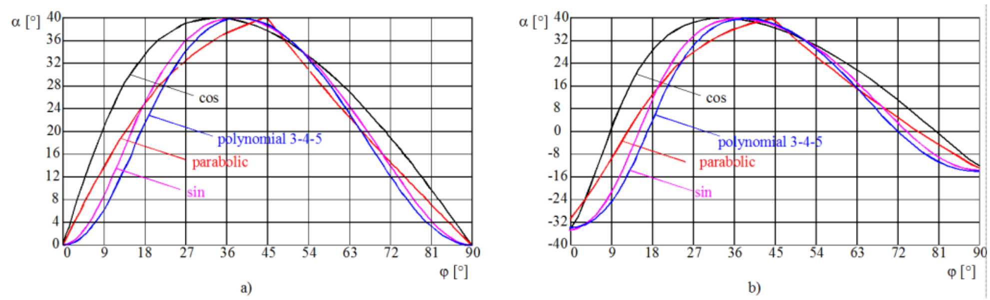

Fig. 4. The variation of pressure angle on active stroke with e=0 (a) and with e=e$_{opt}$ (b)

4.2. Cam mechanism with flat follower

With the four transmission functions taken into consideration (Table 2) 16 combinations may result, some of them being presented in Table 4.

Table 4. Several possible combinations of motion curves for a cam mechanism with flat-face follower

Motion curves		r_b / h	$e = 0$				$e = e_{opt}$				
active stroke $\varphi_a = 90°$	returned stroke $\varphi_p = 180°$		$\frac{b}{h}$	$\frac{H}{h}$	$\frac{T}{h}$	$\frac{H \cdot T}{h^2}$	$\frac{e_{opt}}{h}$	$\frac{b}{h}$	$\frac{H}{h}$	$\frac{T}{h}$	$\frac{H \cdot T}{h^2}$
Sine	parabolic 1/2	1.960	0.456	7.87	5.92	46.60	0.315	0.33	7.75	5.92	45.9
Cosine	polynomial 3-4-5	1.000	0.350	5.85	4.00	23.40	0.185	0.279	5.78	4.00	23.12
Parabolic 1/2	cosine	1.120	0.456	6.20	4.24	26.30	0.135	0.403	6.14	4.24	26.03
Polynomial 3-4-5	sine	1.385	0.325	6.59	4.77	31.45	0.275	0.318	6.58	4.77	31.42

The basic radius was calculated for both strokes and it was accepted the largest one, namely for the active stroke which is more “abruptly” than the passive/returning one.
The length of the guiding way was calculated for two cases: without eccentricity and with the optimum eccentricity (Table 4). In the second case, the guiding length is reduced with (2.15...27.6)%, for the analyzed combination, but only with (0.15...1.52)% for the total size on the translating direction. That means the eccentricity has a very small influence in the total size of a cam mechanism with translating flat follower.

5. Conclusions

The proposed methods allow to minimize a certain cam mechanism size with translating roller/flat-face follower taking into account both cam and follower’s guide way dimension as well. The most important parameter in the mechanism's size establishment it is the follower's stroke, followed by the base circle's radius, guiding length and, indirectly, the eccentricity. The cam mechanism's size, from motion curves point of view, indicates that the "smooth" motion (like sine or polynomial) have a larger size than the "dour" motion (like cosine or parabolic), as one can notice in the presented example problems. That is to confirm the universal valuable rule according to which, in any technical problem there are not only advantages. The results and conclusions regarding the cam mechanism's size presented in this paper are universally valid, being expressed in non dimensional form with the follower's stroke as referential.

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