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Autonomous vehicles in mixed traffic: Accounting for human reactions, modelling the Anticipated Safe Distance

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Abstract. Aiming to fill the gaps of literature that deals only with the last stage of overtaking, the present papers continues the previous work of the author, that formalised the first stage, ***Detecting–Need & Types–of-Overtaking*** and the Trio Overtaking Model (TOM) developed for the third and last stage, the ***Perform-Overtaking***. The Legal, Logic, and Engineering Analyses were employed, firstly to account for human behaviour and reactions within the overtaking model for mixed traffic. Secondly, *Anticipated Trio Overtaking Model (A-TOM)* is developed from TOM, to calculate the Anticiped-Safe-Distance, and asses prior to proceed with any maneouvre (Pre-Check-Overtaking) or anytime during it (***Perform-Overtaking***), whether the overtaking is safe or not. A-TOM capabilities are reviewed with regard to *design simulation*, ***synchronous monitoring*** and *driving control* features of the autonomous vehicle Ego, to analyse various scenarios of Accelerating, Flying, Piggy-backing, and 2+ overtaking types.

Keywords: anticipated trio overtaking model, anticipated safe distance, autonomous vehicle in mixed traffic, psychological safe distance, overtaking simulation and monitoring.

1. Introduction. Traffic codes formalisation.

The ethical and technical visions regarding the risks are irreversibly evolving toward transferring the liability from user to the system's designer [1] but the compliance assessment for both sides remains anyway of crucial importance.

The challenge when appraising the compliance for the human driver and autonomous vehicle (AV), resides in expressing the traffic code's rules from the today narrative format, specific to legal texts, to a form appropriate to be embedded in the logic of AV and automatedly checked when necessary. This process of converting an informal language as the legal one is, to the computer logic, was

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addressed for the first time in the attempt to express „The British Nationality Act" in Prolog programming language [2] and the literature refers it as *Formalization*. W.r.t. AV, where the engineering problem asks to express the traffic regulation in terms of position, speed and acceleration, the concept of *concretization* was introduced [3], and the Isabelle proving software was employed to check some aspects of overtaking sub-manoeuvres.

2. Concretization of AV's overtaking: State of the art and literature gaps

The method was extended to the entire duration of Overtaking (OT) and consolidated in three major methodological steps: i) *Legal Analysis*, which eliminates provisional redundancy of legal texts, assign responsibilities between AV and humans and identify the predicate precursors, ii) *Logic Analysis* bridges between Legal and Engineering concepts and develops predicates and logic formulas to improve the automation, iii) *Engineering Analysis* which enhances the expressivity through the *concretization* of the kinematic and dynamic aspects of the AV. The phases of entire OT process were defined, for a comprehensive approach of the manoeuvre (see Fig. 1): *Detecting–Need & Types–of-Overtaking* over [t0, t3], *Pre-Check-Overtaking* over [t3, t5], and *Perform-Overtaking* over [t5, t9] that three-step methodology being then fully employed for the first of them, the *Detecting–Need & Types–of-Overtaking.*
In [4], the focus was on the third OT's phase, the *Perform-Overtaking,* for which the same three-step methodology delivered the multi-level hierarchy Atomic Propositions Tables (APT) and the related Linear Temporal Logic (LTL) Formulas by employing Modal Logic Flow Chart and the Temporal Logic Diagram over the last time interval of OT, namely [t5, t9], according to Fig. 1. The Engineering Analyses was done, by mean of High Order Logic (HOL), to improve expressivity, by considering not only the overtaking AV (Ego) and the overtaken vehicle V_1 but also the forerunner vehicle V_2 behind of which Ego should return after overtaking V_1. The model resulted from the concretization process was denoted as Trio Overtaking Model (TOM) and dealt with 25 combined braking scenarios for the three vehicles involved, gathered within a matrix and the associated formal definitions, theorems, Collision Matrix and Safe Distance Matrix were developed.
TOM assesses the safe-distances between the involved vehicles, considering that the forerunner vehicle V_2 initiates a braking at the moment t9 when Ego completed the OT and returned behind it and in front of V_1.
This is, the model is able to assess the safety exactly at t_9, but not earlier, having the capacity to deal with freeway, divided traffic scenarios, where exist no pressure to return to initial lane but lacking so far the ability to resolve two-ways roads where Ego should anticipately assess the safe distance for the moment t9, at any earlier moment, even from the OT's phase two, the Pre-Check-Overtaking.

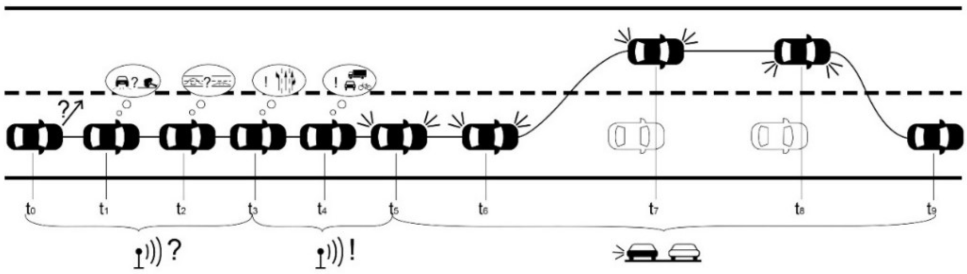

Fig. 1. Temporal Logic Diagram for Overtaking.

TOM will be therefore extended in the present work with this feature of calculating the *anticipated-safe-distance* (ASD) between the three vehicles. On this purpose, the not yet addressed phase *Pre-Check-Overtaking phase* should be also approached by the three-steps methodology and the related APT and LTL Formulas developed accordingly. The assessment of the *Safe-Distance-Incoming* predicate is not the object of the present paper but of the future work. Instead, consideration on human behaviour specific to mixed traffic (AV and non-AV) will be made and psychological- and legal-safe-distance shall be introduced.

3. Logic Analysis of Pre-Check-Overtaking phase

The Legal Analysis of the relevant provisions contained in Traffic Code [4], employed for the level 1. Atomic Proposition (AP) *Pre-Check-Overtaking* (second OT's phase), delivers two level 2. APs, namely *Pre-Check-Infrastructure* over time interval $[t_3, t_4]$ and *Pre-Check-Road-Users* over $[t_4, t_5]$. The level 3. APs are either defined, for the former, to logic assessment of the individual legal provisions (R. Art.120 (1) from a) to i) or, for the latter, to logic assessment of the predicates resulted from interactions with other road users. Their logic interpretation is presented in Table 1.

All atomic propositions of Pre-check phase are assigned with Boolean values, indicating whether the envisaged overtaking is safe, value 1, or not, value 0. The *Pre-Check-Infrastructure* makes sure that none of the infrastructure situations where the overtaking is forbidden by Traffic Code R.Art.120 (1) {a,b,c,d,e,f,g,i}, takes place within the calculated distance needed for OT manoeuvre. The present work does not detail the differences between an uncontrolled environment and an operational design domain (ODD) although the inputs for logic evaluation of the later are sensibly easier to obtain. For this purpose, semantic map of the environment during the operation could be developed, representing the road as lanelets [5] or using tools for translating specialized (OpenDrive, RoadXML) or commercial (LandXML, OpenStreetMap) packages into lanelets or enriched maps dedicated to automated vehicle navigation [6].

The weather-related road conditions are also not included in present analysis, as the author considers more appropriate to deal with them in the *Adapt-Speed* procedure.

It is obvious that the allowed speed on an icy road or under heavy rain will be significantly lower and an overtaking assessed as “safe” for normal weather and certain speed could eventually change to “unsafe”, asking for an adapted (reduced speed) in order to stay safe.
As for the *Pre-Check-Road-Users* assessment, this is a more complex issue, due to the infinite number of scenarios to be considered comparing with the *Pre-Check-Infrastructure* where in the ODD case could be only about accessing a database.
Regarding the predicates *Safe-Distance-Rear* and *Safe-Distance-Front,* each are conceived for assessment of certain individual situations, displaying specific traits but not so different to justify defining a new predicate.

Table 1. Atomic propositions, level and interpretations for ***Pre-Check of Overtaking***

Level	Time	Atomic Propositions (AP)	Logic Interpretation
1.	t_3-t_5	*Pre-Check-Overtaking*	Check if safe to proceed with overtaking
2.	t_3-t_4	*Pre-Check-Infrastructure*	Check if infrastructure allows a safe overtaking
3.		*Check R. Art. 120, (1) a)*	Check if safety condition mentioned by article are met (no unsignalized crossing detected in the distance required for overtaking)
3.		…	Check if safety condition mentioned by every article from b to h are met. (not listed for brevity reasons)
3.		*Check R. Art. 120, (1) i)*	Check if safety condition mentioned are met (no simple/double central line detected in the distance required for overtaking)
2.	t_4-t_5	*Pre-Check-Road-Users*	Check if other road users allow safe overtaking
3.		*Check-Blinker-Front*	Check if the forerunning vehicle is engaged in an overtaking (check its blinkers and position)
3.		*Check-Blinker-Rear**	Check if the following vehicle is engaged in an overtaking (check its blinkers and position)
3.		*Safe-Distance-Rear*	Check if safety distance is maintained from all the following vehicles.
3.		*Safe-Distance-Incoming***	Check if the traffic from opposite direction allows a safe overtaking (no other road user detected in the distance required for overtaking) (including that AV has visibility over the whole distance required for OT.
3.		*Safe-Distance-Front****	Check if safety distance is maintained from all the forerunners vehicles.
3.		*Check-Official Cars*	Check if the forerunner is not an official car

* The main difference between check-blinker-front and –rear, and the reason for defining as separate atoms is that the former is checked only when ego vehicle intends to take over whilst the latter is a run-time procedure, being checked continuously to detect when the following vehicle intends to take over and Ego should accordingly turn itself to defensive driving.
** There are major differences between freeway (divided traffic) and two-way road. On the former it is possible not to check/plan these conditions prior to proceed with overtaking, as the AV has in most cases no pressure to return to initial lane. For two-way road, the traffic from opposite direction

could change in any moment the situation and urge shortening the overtaking and then a continuous monitoring is necessary in addition to initial check/plan, prior to proceed with overtaking. However, the assessment of *Safe-Distance-Incoming* is not the object of present paper.
*** Special attention is then required when the situation planned before OT changes during OT (e.g. the gap in the front of forerunner intended for return is occupied in the meantime by another vehicle like the fore-forerunner that brakes)

Therefore, the author prefers to implement each of them rather as families of similar predicates than as separate predicates for each situation and they will be evaluated in different stages, as *Pre-Check-Overtaking* and *Perform-Overtaking*. For example, Safe-Distance-Rear is evaluated during Pre-Check-Overtaking phase with the aim to check if distance to the other vehicles approaching Ego from behind is safe (*sd*) and to allow Ego to leave its lane for overtaking. The same Safe-Distance-Rear is assessed for the case when the Ego shall, *at the end* of the *Perform-Overtaking phase*, makes sure that the distance to the overtaken vehicle V1 is safe to allow the return in front of him (*sd*). A third case is the newly proposed one, when Ego shall assess in advance or during *Pre-Check-Overtaking,* the Anticipated *Safe Distance (asd)* for the same return-sub-manoeuvre in front of V1, which is going to happen at a later moment, during the next phase, *Perform-Overtaking*.
The *LTL formulas* of traffic rules, for *Pre-Check Overtaking*, specific to above defined time intervals, are:
1. Overtaking is safe from the point of view of infrastructure conditions - R. Art.120, (1)a)..i):

$$\mathcal{F}_{(3-4)} = f(\textit{Pre-Check-Infrastructure} \rightarrow (\textit{Check R.Art. 120,(1) a)} \wedge \textit{Check R.Art. 120(1), b)} \wedge \ldots \wedge \textit{Check R.Art. 120(1), k)} \wedge \textit{Check R.Art. 120,(1) i)})) \quad (1)$$

where $\mathcal{F}_{(3-4)}$ is the formula evaluated between the time points $[t_3, t_4]$;
$f(AP)$ - the function assessing the atomic proposition AP always as true.
2. Overtaking is safe from the point of view of the other road users - R. Art.120, (1)k)-l), (2):

$$\mathcal{F}_{(4-5)} = f(\textit{Pre-Check-Road-Users} \rightarrow \textit{Check-Blinker-Front} \wedge \textit{Check-Blinker-Rear} \wedge \textit{Safe- Distance-Rear} \wedge \textit{Safe-Distance-Incoming} \wedge \textit{Safe-Distance-Front}) \wedge \textit{Check R. Art. 120 (2)}) \quad (2)$$

3. Overtaking is safe as both conditions related to infrastructure and other users are fulfilled R.Art. 120 (1), (2):

$$\mathcal{F}_{(3-5)} = f(\textit{Pre-Check-Overtaking} \rightarrow (\textit{Pre-Check-Infrastructure} \wedge \textit{Pre-Check-Road-Users}), \quad (3)$$

which is the logic formula for the Level 1. *Pre-Check Overtaking.*
With formula 3 the Logic Analysis is completed and the required preparedness for automation is fully achieved. Next step is the Engineering Analysis (Concretization), to increase expressivity of the model. The Trio Overtaking Model will pe extended in order to calculate Anticipated-Safe-Distance, prior to start manoeuvres for OT, as a part of *Pre-Check-Road-Users* assessment. For a realistic model, the *Kinematic Safe Distance* initially delivered by TOM should be first

integrated with the *Psychological Safe Distance*, accounting for human behaviour, and Legal Safe Distance, accounting for already existing legal provisions.

4. Concretization: Considering the human behaviour for mixed traffic

When all the vehicles involved in overtaking are autonomous, the reaction times for braking have smaller values and they tends to zero when V2V communication is enabled. Thus, the *spatial separation* derived from *safe distances,* and *temporal headway* between AVs decrease accordingly. E.g. $d_{1,2} = 3\ m$ [7] or even smaller $d_{1,2} = 2.5m$ and the related time gap of t = 0.3 s [8] for AVs comparing with t = 1.8 s, accepted as a longest human reaction time.
For mixed traffic, an AV returning to initial lane after overtaking, at only 3m in front of a vehicle operated by a human driver, could result, in the best case, in a discomfort for driver or could precipitate an undesired reaction of him, such as a sudden brake or swerve, leading to accidents. Therefore, the *kinetic safe distance (KSD)* calculated in literature [9] should be amended by a *psychological safe distance (PSD)* specific to the driving culture of the geographic area where the travel takes place. Such implementation of the PSD should be done for the *rear-view mirror rule* practiced in Germany and even systematically taught in their driving schools: "*An overtaking vehicle will be allowed to return to the initial lane only when the overtaken vehicle can be entirely seen in the rear-view mirror*". As can be observed in Figure 2, the rule can be concretized by calculating the necessary distance d_{long} resulted from the view angle 2α, having the vertex on the position of rear-view mirror and the sides crossing the left and right extremities of the rear windshield. The 2α angle defines the area of visibility where the vehicle should be observed whilst the 360-2α angle is known as dead-angle. Then:

$$ctg\ \alpha = \frac{d_{long} + l_m}{d_{lat} + \frac{w_v}{2} + w_v} \tag{4}$$

and

$$d_{long} = \left(d_{lat} + \frac{3}{2} w_v\right) ctg\ \alpha - l_m\ , \tag{5}$$

where	$ctg\ \alpha$	is	cotangent value of the half visibility angle ($ctg\ \alpha = 2l_r/w_r$);
	d_{long}	-	the psychological safe distance;
	l_m	-	the distance from the rear-view mirror to the rear-most point of the vehicle;
	d_{lat}	-	lateral (safe) distance;
	w_v	-	width of the vehicles;
	l_r, w_r	-	length and width of the Ego's cabin,

and

$$PSD = d_{long} = (2d_{lat} + 3\ w_v)\frac{l_r}{w_r} - l_m\ . \tag{6}$$

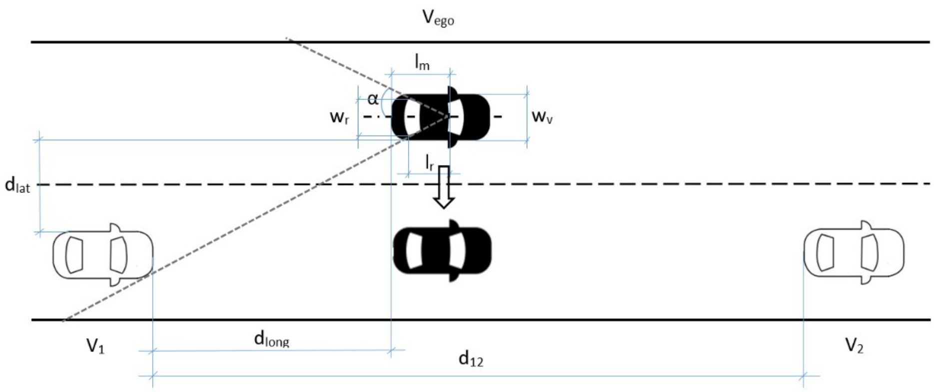

Fig. 2. “Rear-view mirror” rule concretization

Some national traffic codes have specific provision for safe distance that must be considered. E.g. in Germany the rule of “half speedometer distance”, implemented in the German traffic code, states that *the driver of a passenger car should maintain outside of urban area, a safe distance in meters to the vehicle ahead, at least equal with the half of its current speed in kilometers per hour*. The rule lies on a reaction time of 1.8 seconds requested to travel exactly the specified distance to the vehicle ahead. Within urban area the reaction time of 1 second is employed. For trucks over 7.5 tones travelling at speeds over 50 kilometers per hour, a special safe distance of 50 meters, applies. The *Legal Safe Distance (LSD)*, according to the German Traffic Code should be then defined as follows:

$$LSD = \begin{cases} \approx \frac{v}{2}[m]\ or\ the\ distance\ in\ [m]\ traveled\ in\ 1.8\ s - outside\ urban\ area \\ \approx \frac{v}{3}[m]\ or\ the\ distance\ in\ [m]\ traveled\ in\ 1\ s - inside\ urban\ area \\ 50\ [m]\ for\ truck\ over\ 7.5\ tones\ running\ with\ over\ 50\frac{km}{h} \end{cases} \quad (7)$$

and the Duo (involving only two vehicles) Safe Distance (SD) accounting for kinetic, psychologic and legal considerations is

$$SD = max\ (KSD, PSD, LSD) \quad (8)$$

which is the German safe distance between two vehicles used in any further calculation for mixed traffic. When the overtaken vehicle is an AV then the PSD = KSD.

Formula [8] can be adapted to any other Traffic Code (and related LSD). When specific regulations on safe distance are missing one may consider that LSD ≤ KSD.

5. Concretization: Computing the Anticipated Safe Distance prior to, or during overtaking

For a better understanding of the gaps the proposed model aims to fill, the taxonomy proposed by [10] and [11] was adapted at this point for describing the Ego’s *overtaking driving strategies:*

Normal overtaking - Ego approaches the forerunner vehicle and adapts the speed behind of him, waiting for an overtaking opportunity. When the Pre-check-Overtaking is assessed as true, Ego can overtake the forerunner vehicle and will need to accelerate during the overtaking manoeuvre. The normal overtakers are sometime called *accelerative overtakers* (overtaking after a wait).
Flying overtaking - Ego is traveling at it cruise speed and does not brake when detecting the forerunner vehicle, being able to directly overtake the forerunner vehicle without adjusting its speed. (the on-fly assessment of Pre-check-Overtaking predicate as true, allows Ego the overtaking without a pause).
Piggy backing overtaking - A vehicle overtakes the forerunner vehicle and the Ego follows this vehicle, while they both overtake the forerunner vehicle. Alternatively, Ego is the lead vehicle and the follower, autonomous or not, does the overtaking on its tail.
2 + overtaking - Ego overtakes one or more vehicles driving behind a forerunner vehicle and in the same move, it also overtakes the forerunner vehicle (minimal number of vehicles that are overtaken is 2).
On the motorway, with divided traffic, where at least two lanes are available for the same running direction, there is no critical pressure from the opposite direction (incoming) traffic, on Ego to conclude the overtaking and return to initial lane. A pressure of this nature could still arise from the vehicles behind Ego, traveling to the same direction at a larger speed, but considering the current regulation and common practice that cannot be deemed as critical. However, the assessment of the safe distance between the vehicles V1 and V2, where Ego must return after overtaking (Fig. 3), should be done prior to start the merging sub-manoeuvre and the (time, distance) to initial lane should be considered for predicting the evolution of the safe distance (SD) value.

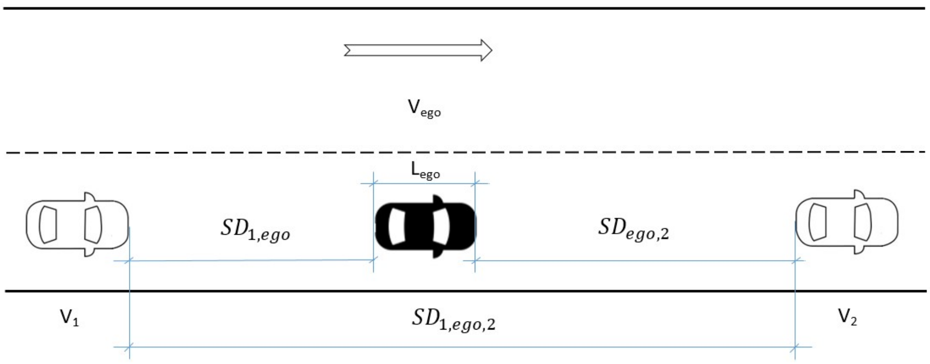

Fig. 3. Vehicle's position for Trio Overtaking Model at the end of manoeuvre (t_9).

On the two-ways roads, the traffic from opposite direction makes a big difference and therefore the assessment by Ego of the SD between the vehicles V1 and V2, before initiating the overtaking, is required as a prerequisite of a safe planning. Back to the motorway case, the (time, distance) Ego has to travel from the beginning of overtaking to its end, when returns to the initial lane, should be calculated in advance and introduced in the equations of the SD model, obtaining the *Anticipated Safe Distance*. Thus, becomes possible its *initial evaluation* (since *Pre-Check-Overtaking Phase*) and also its *constant monitoring* at any moment during the manoeuvre (also during *Perform-Overtaking Phase*).

On this purpose, the braking equations for Ego, V1 and V2 vehicles, having so far in literature, [9], their initial moment at t_9 (see Fig.1.) when overtaking ends, will be backwardly extended on the time axis of Temporal Logic diagram toward the moment t_4. On the time interval $[t_4, t_5]$, the predicate *Pre-Check-Road-Users* is assessed, as per Table 1., such that, at the moment t_5 the Ego should be allowed either to start the overtaking or obliged to adapt the speed to that of the forerunner V1, waiting for an OT opportunity. Permission for overtaking will not be for now accounting for the traffic from the opposite direction (predicate Safe-Distance-Incoming will be introduced later) but it will be focusing on the estimation at the moment t_5 of the *anticipated-safe-distance* $asd_{1,ego,2}$, between the V1 and V2 at the moment t_9 when the Ego should arrive between them.

To keep the model's degree of generality high, the formulation will include, for every time interval, all terms of kinematics equations, including acceleration, which is anyway deemed as *constant over the intervals* $[t_i, t_{i+1}]$ of LTD.

The motion equations of vehicle V_2, over the time interval $[t_4, t_9]$ and after it, to still stand, given that the vehicle V_2 starts to brake at t_9, are referred as following Eq. (9):

$$d_2(t) = \begin{cases} D_{2,4}(t) = d_{2,4} + v_{2,4}(t-t_4) + 0.5a_{2,4}(t-t_4)^2 & for\ t_4 \le t \le t_5 \\ D_{2,5}(t) = D_{2,4}(t_5) + v_{2,5}(t-t_5) + 0.5a_{2,5}(t-t_5)^2 & for\ t_5 \le t \le t_6 \\ D_{2,6}(t) = D_{2,5}(t_6) + v_{2,6}(t-t_6) + 0.5a_{2,6}(t-t_6)^2 & for\ t_6 \le t \le t_7 \\ D_{2,7}(t) = D_{2,6}(t_7) + v_{2,7}(t-t_7) + 0.5a_{2,7}(t-t_7)^2 & for\ t_7 \le t \le t_8 \\ D_{2,8}(t) = D_{2,7}(t_8) + v_{2,8}(t-t_8) + 0.5a_{2,8}(t-t_8)^2 & for\ t_8 \le t \le t_9 \\ D_{2,8}(t_9) + v_{2,9}(t-t_9) + 0.5a_{2,9}t(t-t_9)^2 & for\ t_9 \le t \le t_9 + t_{2,stop} \\ D_{2,8}(t_9) + v_{2,9}\left(t_9 + t_{2,stop}\right) + 0.5a_{2,9}\left(t_9 + t_{2,stop}\right)^2 & for\ t \ge t_9 + t_{2,stop} \end{cases}$$

where $d_{2,4}$ is the initial position of V_2 at the moment t_4;
$D_{2,i}(t)$ - the position of V_2 at the moment t, where $t_i \le t \le t_{i+1}$;
$v_{2,i}$ - speed of V_2 at the moment t_i, for i = 4..9;
$a_{2,i}$ - acceleration of V_2 at the moment t_i, for i = 4..9;
$t_{2,stop}$ - time to stop of V_2 ($t_{2,stop} = -v_{2,9}/a_{2,9}$).

Including in expression of $D_{2,8}(t)$ the parameters of previous equations, a more compact form of (9) results as (10):

$$d_2(t)=\begin{cases} D_{2,i}(t)=D_{2,i-1}(t_i)+v_{2,i}(t-t_i)+0.5a_{2,i}(t-t_i)^2 \\ D_{2,8}(t_9)=d_{2,4}+\sum_{i=4}^{8}v_{2,i}(t_{i+1}-t_i)+0.5\sum_{i=4}^{8}a_{2,i}(t_{i+1}-t_i)^2 \\ D_{2,8}(t_9)+v_{2,9}(t-t_9)+0.5a_{2,9}(t-t_9)^2 \\ D_{2,8}(t_9)\ +v_{2,9}\left(t_9+t_{2,stop}\right)+0.5a_{2,9}\left(t_9+t_{2,stop}\right)^2 \end{cases} \quad (10)$$

Valid each for the following corresponding time intervals:

$$d_2(t)\ for\begin{cases} t_i\le t\le t_{i+1};\ 4\le i\le 8 \\ t_4\le t\le t_9 \\ t_9\le t\le t_9+t_{2,stop} \\ t\ge t_9+t_{2,stop} \end{cases} \quad (11)$$

Once the V_2 initiated the braking, Ego that just returned on the same lane behind of him is obliged also to brake. The motion equations of Ego vehicle over the time interval $[t_4, t_9]$ and after it are similarly expressed as Eq. (12):

$$d_{ego}(t)=$$
$$\begin{cases} D_{Ego,8}(t_9)=d_{Ego,4}+\sum_{i=4}^{8}v_{Ego,i}(t_{i+1}-t_i)+0.5\sum_{i=4}^{8}a_{Ego,i}(t_{i+1}-t_i)^2\ -\delta_{d} \\ D_{Ego,8}(t_9)+v_{ego,i}(t-t_9) \\ D_{Ego,8}(t_9)+v_{ego,i}t+0.5a_{ego,i}(t-t_9-\tau_{ego})^2 \\ D_{Ego,8}(t_9)+v_{ego,i}\left(t_9+\tau_{ego}+t_{ego,stop}\right)+0.5a_{ego,i}(t_9+t_{ego,stop})^2 \end{cases}$$

Valid each for the following corresponding time intervals:

$$d_{ego}(t)for\begin{cases} t_4\le t\le t_9 \\ t_9\le t\le t_9+\tau_{ego} \\ t_9+\ \tau_{ego}\le t\le t_9+\tau_{ego}+t_{ego,stop} \\ t\ge t_9+\tau_{ego}+t_{ego,stop} \end{cases} \quad (13)$$

where	$d_{ego,4}$	is	the initial position of V_{ego} at the moment t_i, for i = 4..9;
	$D_{Ego,i}(t)$	-	the position of V_2 at the moment t, where $t_i \le t \le t_{i+1}$;
	$v_{ego,i}$	-	speed of V_{ego} at the moment t_i;
	$a_{ego,i}$	-	acceleration of V_{ego} at the moment t_i;
	τ_{ego}	-	reaction time of V_{ego};
	$t_{ego,stop}$	-	time to stop of V_{ego} ($t_{ego,stop}=-v_{ego,9}/a_{ego,9}$);
	δ_{dev}	-	change lane correction, accounting for deviation of V_{ego} from moving axis of V_1 and V_2.

According to Fig. 4, the correction δ_{dev} is calculated to adjust the $D_{Ego,8}(t)$ by the difference between the real distance travelled along the trapezoidal path of overtaking and its projection on the reference moving axis of vehicles V_1 and V_2. The literature mentions various complex methods for trajectory description or planning, such as the lanelets [5] by splitting the real rounded path of AV into a minimal number of monotone polygonal chains each having its own coordinate system [12], where the decision procedure for linear arithmetic segment-intersection problem is used [13]. However, for the sake of model's brevity the approximation to the trapezoidal form expressed by following eq. (14) would be considered precise enough:

$$\delta_{dev} = \varepsilon_p\{[D_{Ego,6}(t_7) - D_{Ego,5}(t_6)](1-\cos\delta_1) + [D_{Ego,8}(t_9) - D_{Ego,7}(t_8)](1-\cos\delta_2)\}$$

where	δ_1	is	the steering angles when V_{ego} leaves initial lane, over interval $[t_6, t_7]$;
	δ_2	-	the steering angles when V_{ego} returns to initial lane, over interval $[t_8, t_9]$;
	ε_p	-	approximation error to trapezoidal forms.

The trapezoidal scheme was implemented to mimic the human overtaking behavior on the highway [14] and the generality of the model was maintained by allowing different values for the steering angles presuming they will be used as manoeuvre control parameter in different phases (e.g. when suddenly a cut-in manoeuvre is requested after the departure from the initial lane took place at a reasonable acceleration). However, studies to mitigate safety, transversal and longitudinal comfort should be performed [15]

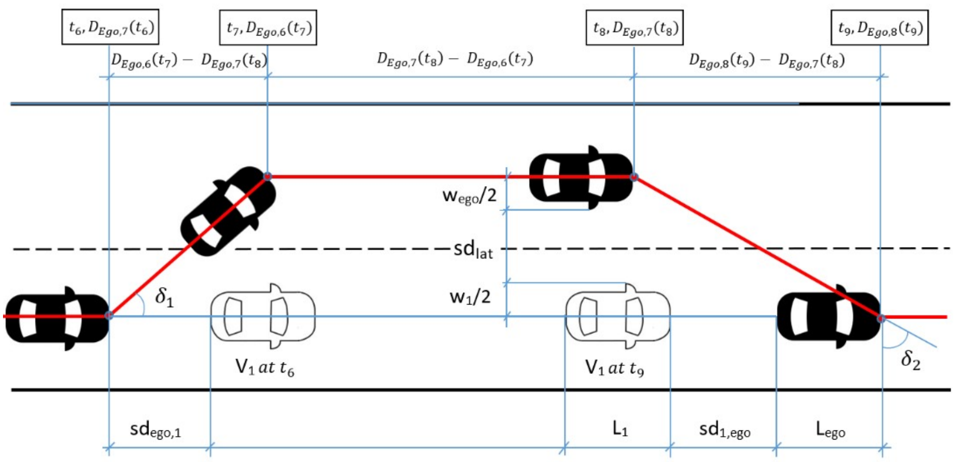

Fig. 4. Perform Overtaking phase.

The moving equations of vehicle V_1 over the time interval $[t_4, t_9]$, given that t_9 coincides with the moment when V_{ego} returns to initial lane and V_2 starts to brake, is:

$$d_1(t) = \begin{cases} D_{1,8}(t_9) = d_{1,4} + \sum_{i=4}^{8} v_{1,i}(t_{i+1} - t_i) + 0.5 \sum_{i=4}^{8} a_{1,i}(t_{i+1} - t_i)^2 \\ D_{1,8}(t_9) + v_{1,i}(t - t_9) \\ D_{1,8}(t_9) + v_{1,i}t + 0.5a_{1,i}(t - t_9 - \tau_{ego} - \tau_1)^2 \\ D_{1,8}(t_9) + v_{1,i}(t_9 + \tau_{ego} + \tau_{1,i} + t_{1,stop}) + 0.5a_{1,i}(t_9 + t_{1,stop})^2 \end{cases} \quad (15)$$

Valid each for the following corresponding time intervals:

$$d_1(t)\ for \begin{cases} t_4 \le t \le t_9 \\ t_9 \le t \le t_9 + \tau_1 + \tau_{ego} \\ t_9 + \tau_1 + \tau_{ego} \le t \le t_9 + \tau_{ego} + \tau_1 + t_{1,stop} \\ t \ge t_9 + \tau_{ego} + \tau_1 + t_{1,stop} \end{cases} \quad (16)$$

where $d_{1,4}$ is the initial position of V_1 at the moment t_i, for i = 4..9;
$D_{1,i}(t)$ - the position of V_2 at the moment t, where $t_i \le t \le t_{i+1}$;
$v_{1,i}$ - speed of V_1 at the moment t_i;
$a_{1,i}$ - acceleration of V_1 at the moment t_i;
τ_1 - reaction time of V_1;
$t_{1,stop}$ - time to stop of V_1 ($t_{1,stop} = -v_{1,9}/a_{1,9}$).

The braking ways of the three vehicles, V_2, V_{ego}, V_1 can be expressed now as:

$$\begin{cases} BW_2 = d_{2,stop} = D_{2,8}(t_9) - 0.5\frac{v_{2,9}{}^2}{a_{2,9}} \\ BW_{ego} = d_{ego,stop} = D_{Ego,8}(t_9) + v_{ego,9}\,\tau_{ego} - 0.5\frac{v_{ego,9}{}^2}{a_{ego,9}} \\ BW_1 = d_{1,stop} = D_{1,8}(t_9) + v_{1,9}(\tau_{ego} + \tau_1) - 0.5\frac{v_{1,9}{}^2}{a_{1,9}} \end{cases} \quad (17)$$

As the goal is to anticipate the safe distances (ASDs) at the end of overtaking, even from the moment t_4 and to account since then for kinematics of vehicles, the Equations [10], [11], [12], [13] and [16] are used to extend the above Eq. [17], to the following Eq. [18] for the braking ways of V_2, V_{ego} and V_1, respectively:

$$\begin{cases} d_{2,4} + \sum_{i=4}^{8} v_{2,i}(t_{i+1} - t_i) + 0.5\sum_{i=4}^{8} a_{2,i}(t_{i+1} - t_i)^2 - 0.5\frac{v_{2,9}{}^2}{a_{2,9}} \\ d_{Ego,4} + \sum_{i=4}^{8} v_{Ego,i}(t_{i+1} - t_i) + 0.5\sum_{i=4}^{8} a_{Ego,i}(t_{i+1} - t_i)^2 + v_{ego,9}\,\tau_{ego} - 0.5\frac{v_{ego,9}{}^2}{a_{ego,9}} - \delta_{dev} \\ d_{1,4} + \sum_{i=4}^{8} v_{1,i}(t_{i+1} - t_i) + 0.5\sum_{i=4}^{8} a_{1,i}(t_{i+1} - t_i)^2 + v_{1,9}(\tau_{ego} + \tau_1) - 0.5\frac{v_{1,9}{}^2}{a_{1,9}} \end{cases}$$

Starting from the five elemental scenarios identified for two vehicles (Duo) [16], where the vehicle following, stops before or after the positions the forerunner vehicle initiates braking or stops itself, and extending it to the three vehicles case (Trio) [17], a *combined scenarios matrix CBS* results. CBS includes n=25 scenario, 16 of them leading to collision and 9 to no-collision. For each case, a theorem should be expressed to assess the predicate *Avoid-Rear-End-Collision (AREC)* and then derive Safe Distances between all involved vehicles.

For brevity, only the first scenario is detailed in the present paper. It considers the case when, after the vehicle V_2 initiates the chain braking, V_{ego} stops before the point V_2 initiated braking and V_1 stops before the point V_{ego} initiated braking. The theorem formalizing this combined scenario is:

***Theorem* 1.** : $(d_{ego,stop} < D_{2,8}(t_9) \wedge (d_{1,stop} < D_{Ego,8}(t_9)) => (AREC(TD))$
It follows that:

$$\begin{cases} d_{ego,stop} - D_{2,8}(t_9) < 0 \\ d_{1,stop} - D_{Ego,8}(t_9) < 0 \end{cases} \qquad [19]$$

Using Eq. [10], [11], [12], [13] and [15] and regrouping the terms, results in [20]:

$$\begin{cases} d_{2,4} - d_{Ego,4} > \sum_{i=4}^{8} v_{Ego,i}(t_{i+1} - t_i) + 0.5 \sum_{i=4}^{8} a_{Ego,i}(t_{i+1} - t_i)^2 - \delta_{dev} + v_{ego,9}\, \tau_{ego} \\ \qquad -0.5 \dfrac{{v_{ego,9}}^2}{a_{ego,9}} + \sum_{i=4}^{8} v_{2,i}(t_{i+1} - t_i) + 0.5 \sum_{i=4}^{8} a_{2,i}(t_{i+1} - t_i)^2 \\ d_{1,4} - d_{Ego,4} > -\sum_{i=4}^{8} v_{1,i}(t_{i+1} - t_i) - 0.5 \sum_{i=4}^{8} a_{1,i}(t_{i+1} - t_i)^2 - v_{1,9}(\tau_{ego} + \tau_1) \\ \qquad +0.5 \dfrac{{v_{1,9}}^2}{a_{1,9}} - \sum_{i=4}^{8} v_{Ego,i}(t_{i+1} - t_i) - 0.5 \sum_{i=4}^{8} a_{Ego,i}(t_{i+1} - t_i)^2 + \delta_{dev} \end{cases}$$

Which are the expressions of the required distances between Ego and other vehicles at the assessment moment (t_4 in this case), to secure that, the distances at t_9, the end of overtaking, calculated by the original TOM, will be also safe.

$$\begin{cases} sd_{Ego,2}(t_4) = d_{2,4} - d_{Ego,4} \\ sd_{1,Ego}(t_4) = d_{1,4} - d_{Ego,4} \end{cases} \qquad [21]$$

Ego and V_1 changed the places during overtaking. and the convention for distance calculation „between the foremost point of the follower and rear most point of the forerunner vehicle" should be amended by substracting the lenghth $\boldsymbol{L_1}$ of V_1. For practical reasons when applying different overtaking strategies, a safety margin SM, should be accounted as well. Then, the (anticipated-safe-) distance between vehicles V_1 and V_2 at moment t_4 ,that will secure a safe return of Ego on the initial lane, between them, at moment t_9, should be calculated as follows:

$$\boldsymbol{asd}_{1,ego,2}(t_4)(\mathbf{1},\mathbf{1}) = \boldsymbol{sd}_{ego,2}(t_4) - \boldsymbol{sd}_{1,ego}(t_4) - \boldsymbol{L_1} + \boldsymbol{SM} \qquad [22]$$

In the same vein, the theorems and related anticipated-safe-distances will be expressed for the rest of 8 no-collision scenarios and assembled in *Anticipated-Safe-Distance matrix*, as non-trivial elements. The safe distances for the 16 collision cases were conventionally considered ∞. Extending the generality for any moment $\boldsymbol{t} \in [t_4, t_9]$ the matrix is formulated as:

$$[ASD_{1,ego,2}] = \begin{vmatrix} asd_{1,ego,2}(1,1) & asd_{1,ego,2}(1,2) & asd_{1,ego,2}(1,3) & \infty & \infty \\ asd_{1,ego,2}(2,1) & asd_{1,ego,2}(2,2) & asd_{1,ego,2}(2,3) & \infty & \infty \\ asd_{1,ego,2}(3,1) & asd_{1,ego,2}(3,2) & asd_{1,ego,2}(3,3) & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{vmatrix} \quad [23]$$

The Anticipated-Safe-Distance matrix extends the Trio Overtaking Model (TOM), [17] to Anticipated Trio Overtaking Model (A-TOM).

6. A-TOM's capabilities for Control, Monitoring and Scenario Simulation

The kinematics (position, speed, acceleration) of vehicles V_1 and V_2 will be regarded as input data, while the similar set of Ego will be regarded as control parameters, used to keep the manoeuvre safe when the variation of the former requires it (e.g. the case when the overtaken V_1 accelerates slightly, obliging the Ego to adapt itself in order to safely conclude the overtaking).

A-TOM is meant to be used as a *runtime procedure*, where the parameters of V_1 and V_2 are continuously measured by Ego, and periodically updated in the model when passing to the next interval. Consequently, the way the motion equations were formulated for V_1 and V_2, separately over every time interval, does not mean the related parameters are known in advance, which is obviously not possible, but means that, they could experience variation as interval changes. One could maintain the subintervals as functionally defined hereabove or could conveniently refine the sub-division, usually related to sample rate of AV's kinematic sensors.

For *mixed traffic*, where unpredictable human behaviour shall be accounted, the kinematic parameters of non-AV should be continuously monitored and their instantaneous measured values, should be assumed to be the same for all remaining subintervals, until the next measurement updates them. For *all-AV-traffic* the speed can be negotiated before OT through V2V interface and maintained by all vehicles over entire manoeuvres' subintervals.

A-TOM was also conceived to be used for *simulation procedures* during the development phase of Ego. Various scenarios are modelled by assigning values to kinematic parameters of V_1 and V_2 and then, the response kinematic parameters of Ego required to maintain the manoeuvre safe, by complying with ASDs, are elicited. The simulation can be performed on *Checking Mode* to assess the capability of an already designed Ego and its subsystems, or on *Design Mode* to specify the ranges of kinematics the subsistems should be conceived to cover. E.g., an A-TOM based simulation for some scenarios of V_2 (reducing acceleration, slowing down by service/engine braking or braking on emergency), during different OT's subintervals, will help setting the required tresholds for service- and emergency-deceleration of Ego, and appropriately design the braking system. Similarly, when V_1 increases speed or acceleration during OT, an Ego's power capability analysis is deployed, to determine whether the required acceleration for Ego can be delivered or not. Constraints as longitudinal and transversal acceleration (comfort or safety) tresholds can be accounted for.

7. ATOM's overtakings portfolio

The degree of generality the model was provided with, by including position, velocity and acceleration in the kinematic parameters' set, combined with the division of manouvers in time subintervals, allow the approaching of all types of OT such as Accelerating, Flying, Piggy-backing and 2+ Overtakings.
Emergency or defensive driving strategies (most probable the AV will be overtaken than will do overtake during its operation) can be simulated, analysed and proposed, based on already existing or AV specific scenarios such as AV swap overtaking or dealing with short overtaking gaps, lane sharing and cut-in manoeuvres of human drivers.

8. Conclusions and future work

A-TOM adds to the time being existing models in literature, the capability of calculating with anticipation the safe distance between Ego and the overtaken vehicle V_2 and between Ego and the forerunner vehicle V_1. That means the distance between all involved vehicles, at the critical moment when Ego returns to initial lane, can be known in advance, prior to start the OT or at any moment during OT. The ASDs computing makes possible the initial *OT planning*, for different OTs types such as Accelerating, Flying, Piggy-backing and 2+, by accounting for their kinematics (position, velocity, acceleration) known, as measurements, at the assesment moment.
Embedding A-TOM in a run-time procedure, allows Ego to operate in the *synchronous monitoring* and *driving control modes* and deal with variation of other vehicles' kinematics, by adjusting Ego's kinematics, in order to safely complete the OT. The *design simulation mode* is also up-graded, enlarging the sets of capability analyses for Ego and its subsystems even from the early stages of development.
The human behaviour and reaction specific to mixed, AV and non-AV traffic, are accounted by introducing in the generalized formula of safe distance not only safe-kinematic-distance but the psychological-safe-distance as well. In the same vein, the eventual national regulation constraints are accounted through the legal-safe-distance.
As for the future work*, the modeling* of *incoming traffic* is envisaged, by assessing the *safe-distance-incoming predicate*. This is, not only making sure in advance, prior to begin the overtaking, the Ego will be safe when return to initial lane,

between V1 and V2 (where a set of safe inter-vehicles distances is secured) but also making sure in advance the distance to the first vehicle V3 approaching from the opposite direction is large enough to allow Ego to safely finish the overtaking.

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