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Transformation functions and trajectories of a hypocycloid mechanism completed with a dyad

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Abstract. It starts from a planetary mechanism that generates hypocycloids and it is completed with an RPP dyad, so that transfer functions are generated, i.e. races along the x and y directions. Different hypocycloids (normal, elongated, shortened) are generated and transfer functions are established for these cases. The obtained curves and transfer functions are discussed, establishing some conclusions. All study includes various range of lengths for the cinematic links so the important and interesting trajectories are found. The related transfer functions can be used in a diversity of applications. These findings are useful for special devices for automatic lathes, horizontal forging machines, textile machinery, and kinetic art.

Keywords: hypocycloids generating mechanism, transfer functions for mechanisms.

1. Introduction

The proposed theme approaches two directions: the generation of hypocycloids and the mechanisms transfer functions. Cyclical curves have been extensively studied in the technical literature. The hypocycloidal gear that transforms the linear translation motion of the piston into rotation motion of the engine wheel, used by White and Murray in 1801 at locomotives, was studied in detail in [1]. Mathematical theory and generation of hypocycloids are described in [2]. In [3] the mathematical side and the generation of hypocycloids and epicycloids is presented,

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including substitution mechanisms each one having two driving links with correlated motions, not independent ones. The paper contains examples of planetary mechanisms generating these curves. In techniques, especially in fine mechanics, gears with cycloidal profiles, (based on the combination of epicycloids with hypocycloids) are used, which are also studied in [4]. It is mathematically studied the contact between the mating profiles, based on the differential geometry, establishing the line of contact. In [5] there were analyzed the cyclic curves by giving their equations and examples for normal, elongated and shortened variants. The spherical equivalents of some of these curves and their functional models were also presented. In [6] the cyclical curves were generated, then rotated by different axes, looking for aesthetic effects, usable in commercials, artesian fountains, mechanical toys. Regarding the transfer functions there is a monograph [7] where these functions were studied in analytical form for positions, velocities and accelerations in the case of bar mechanisms. It contains quite complicated mathematical relations, diagrams and examples. In [8] it was solved a mechanism optimizing problem to ensure the positioning function so that the driven link starting from its initial position arrived in its final position in the shortest time. The method was applied to mechanisms for electric apparatus. An analytical solution based on variational calculation was found. In [9] it was studied an aperiodic mechanism and the transfer function was established as a function dependent on several parameters; their influence on the function and the period of motion was studied, too. In [10] the variation of the torque transmitted from the driving link of a mechanism to the final driven element was treated, and the transfer function was written, starting from the motion equation. The possibility of compensating the torque for uniform rotation achievement was considered. In [11] the transfer functions for non-circular gears with parallel axis were studied by processing with the Maple program package. In [12] the transfer function was set for the four-bar mechanism, crank and rocker mechanism type, as well as to other variants according to the Grashof theorem, showing the diagram $\psi(\varphi)$. There are studied below the transfer functions for a planetary mechanism generating hypocycloids to which a RPP dyad was attached.

2. Hypocycloid mechanism. Transfer functions

The planetary mechanism shown in Fig. 1 is made up by the fixed central wheel 1 with internal teeth, on which the wheel 2 with external teeth driven by the AC satellite arm is running.

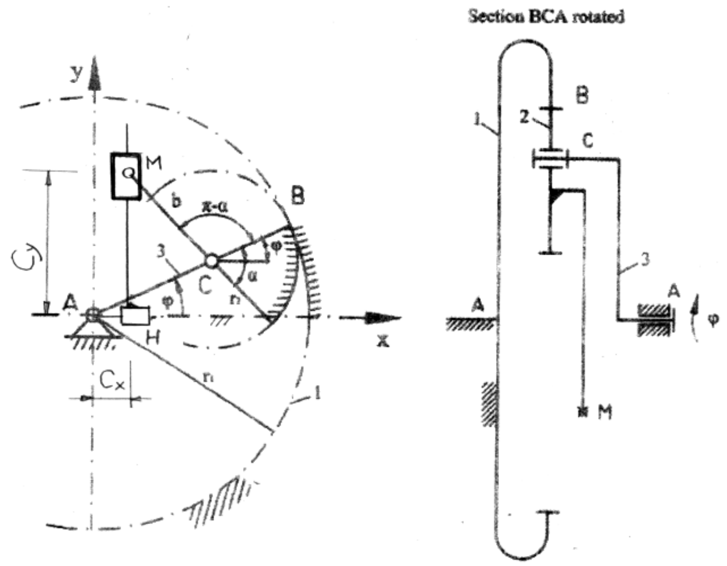

Fig. 1. The generating mechanism.

The element CM is linked to the wheel 2. The point M of the element CM generates (in the plane of the base) a normal, elongated or shortened hypocycloid, according to the length of the element CM during the movement of the wheel 2. The rolling circle of the wheel 2 runs without sliding on the rolling circle of the wheel 1, so that the circular arcs hatched in Fig. 1 are equal, which allows the writing of the following expressions:

$$b=CM \tag{1}$$

$$r_2 \cdot \alpha = r_1 \cdot \varphi \tag{2}$$

$$x_M=(r_2-r_1)\cos\varphi + b\cos(\varphi+\pi-\alpha) \tag{3}$$

$$y_M=(r_2-r_1)\sin\varphi + b\sin(\varphi+\pi-\alpha) \tag{4}$$

A RPP-type dyad MH was added to the hypocycloidal mechanism shown in Fig. 1, so that there are obtained the values of Cx and Cy path, i.e. the transfer functions Cx (φ) and Cy(φ).

Because hypocycloids have many forms, with many arcs, depending on the initial data, it is clear that a wide variety of transfer functions will result, as this paper aims to show. Using the notations from Fig. 1, it was considered C x = x_M; Cy = y_M. It was initially adopted r_1=100 and r_2 was cycled. The analyzed cases were for b = r_2 (normal curve), b > r_2 (elongated curve) and b < r_2 (shortened curve).
In Fig. 2 it is shown the hypocycloid traced for the values shown below the figure. Considering $r_1/r_2 = 10$, the resulting hypocycloid has 10 arcs.

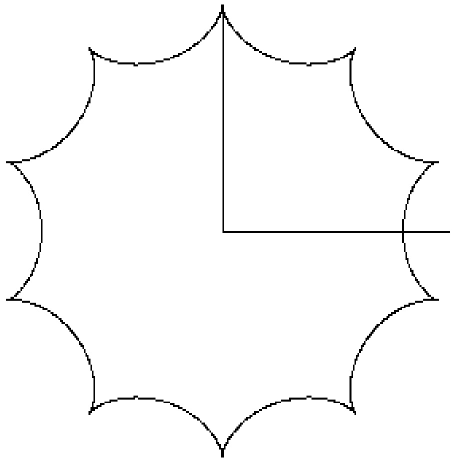

Fig. 2. r_2=10; b=10 – normal curve

The resulting transfer functions are shown in Fig. 3, each arc on the hypocycloid generating a loop in the diagram of Cx and Cy, respectively. Just these loops are the specific features of this mechanism.

In the case of the elongated hypocycloid shown in Fig. 4, the arcs are no longer connected by nodes but by small loops; the diagrams of the transfer functions are similar (Fig. 3 and Fig. 5) but at different scales, the loops shown in Fig. 5 being higher.

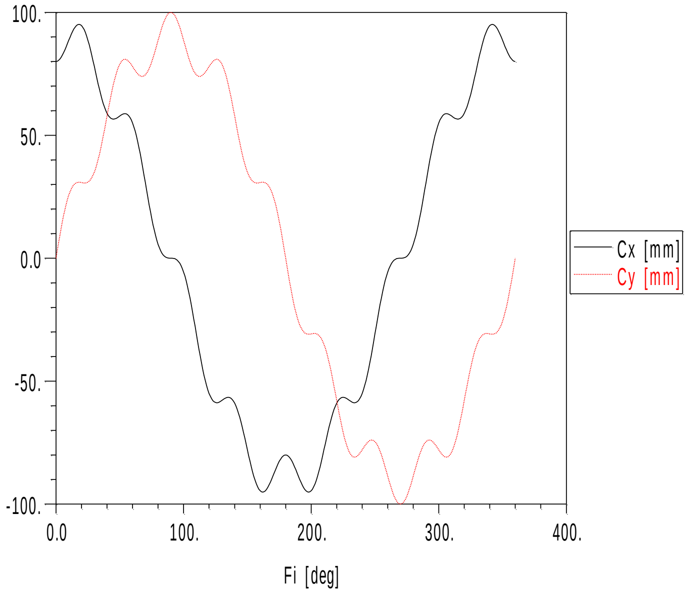

Fig. 3. r_2=10; b=10 - normal curve

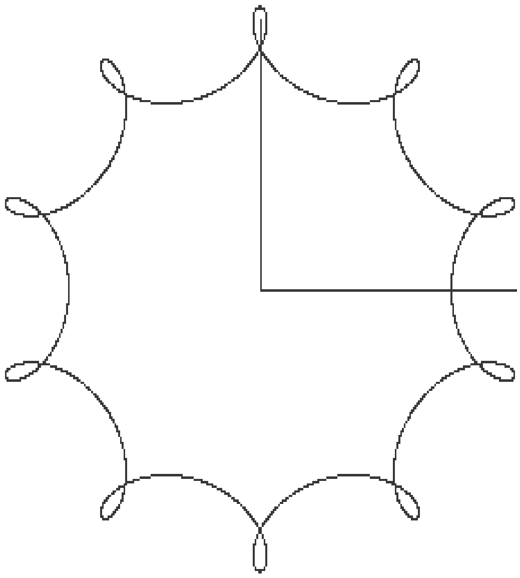

Fig. 4. r_2=10; b=15 – elongated curve

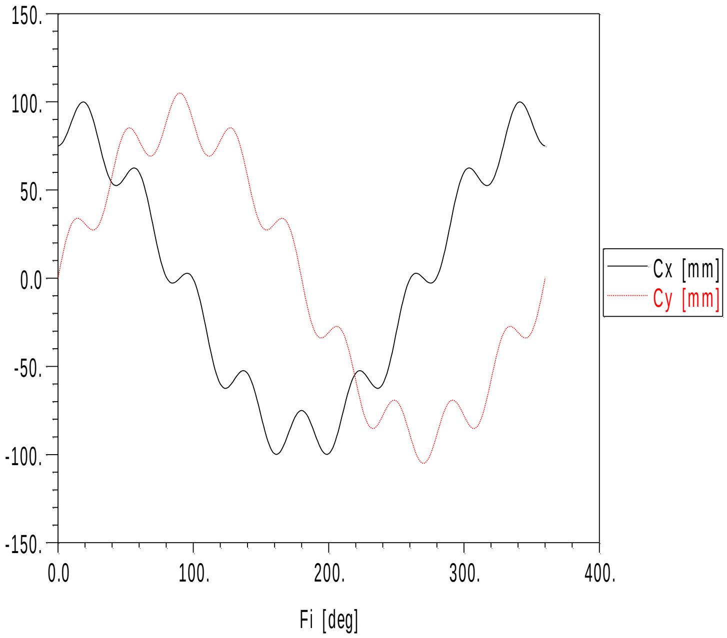

Fig. 5. r_2=10; b=15 – elongated curve

In the case of shortened hypocycloids (Fig. 6), the filleted corners between the arcs have no loops and the curvature radii of the arcs are larger than in the previous cases.

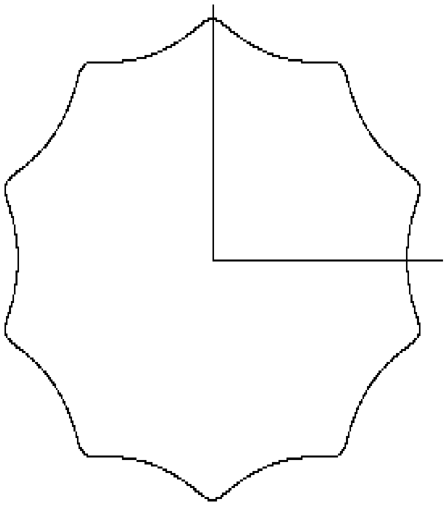

Fig. 6. r_2=10; *b*=5 – shortened curve

The transfer function diagrams (Fig. 7) are curves with only small loops.

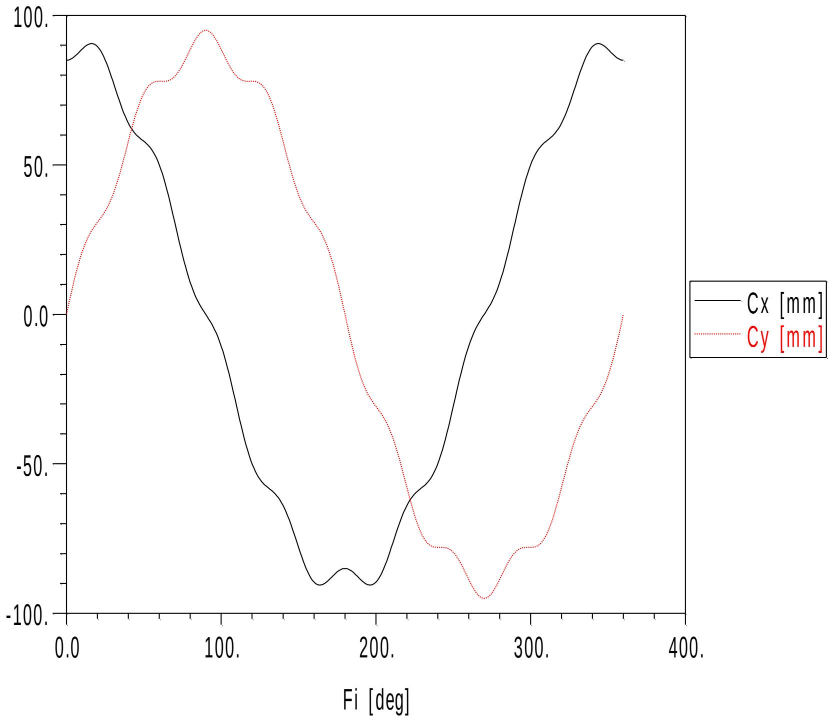

Fig. 7. r_2=10; *b*=5 – shortened curve

If the ratio of the gears radii is 5, the normal hypocycloid shown in Fig. 8 is obtained having 5 arcs with the transfer functions shown in Fig. 9, where there are noticed 5 loops on each curve. Outside the loops, on the diagrams the curves are quite uniform, without curled zones.

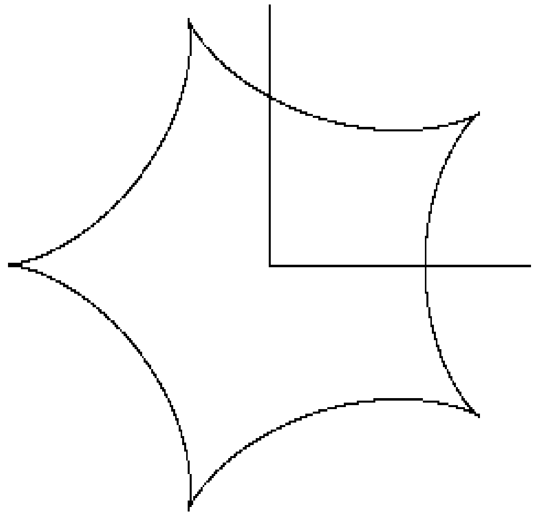

Fig. 8. r_2=20; b=20 – normal curve

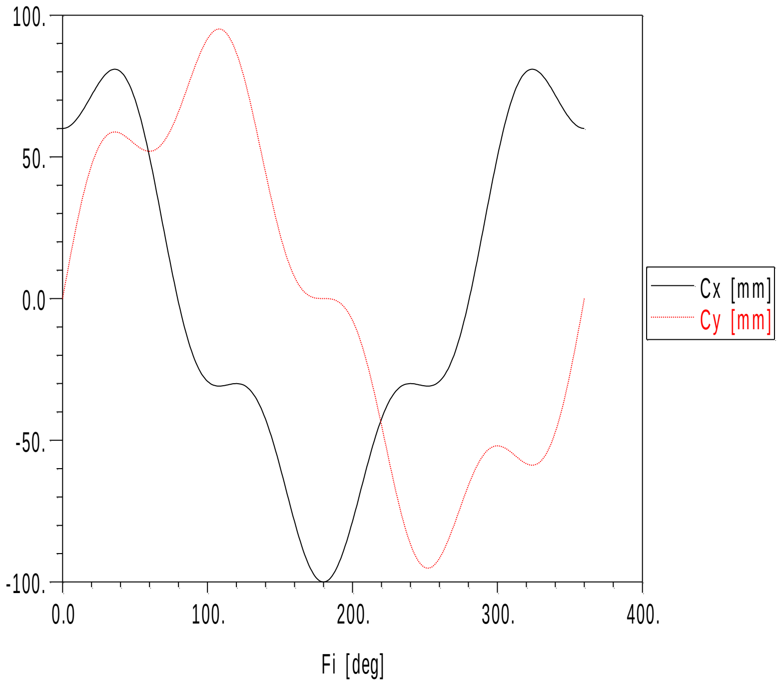

Fig. 9. r_2=20; b=20 – normal curve

For elongated and shortened curves (Fig. 10 and Fig. 12), the transfer function diagrams (Fig. 11 and Fig. 13) are similar to the previous ones at different scales.

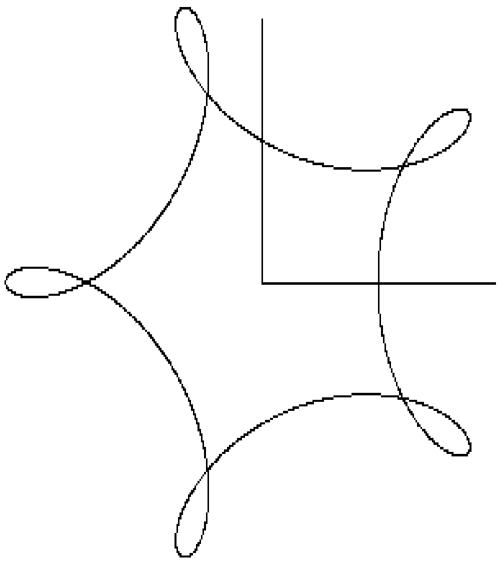

Fig. 10. r_2=20; *b*=30 – elongated curve

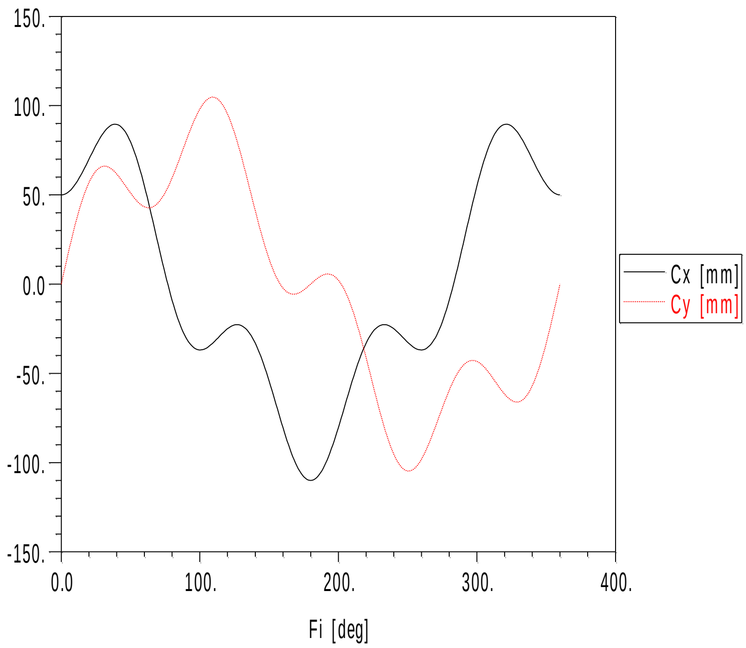

Fig. 11. r_2=20; *b*=30 – elongated curve

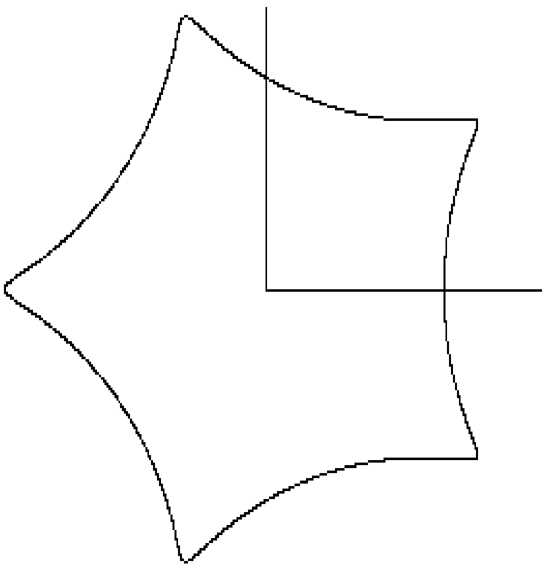

Fig. 12. r_2=20; b=15 – shortened curve

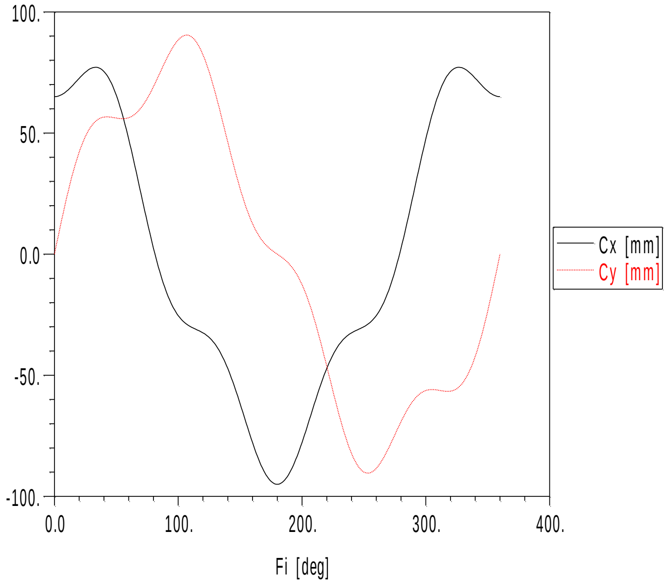

Fig. 13. r_2=20; b=15 – shortened curve

The radii ratio was considered equal to 2 and it was obtained the *diametral line* shown in Fig. 14, knowing this property of this planetary mechanism. The resulting curve is a straight line overlapping the y-axis, in fact a degenerate ellipse with width equal to 0.

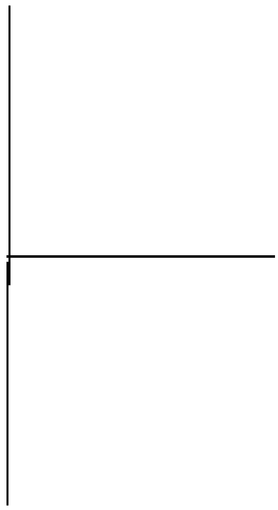

Fig. 14. r_2=50; b=50 – normal curve (diametral line)

In Fig. 15 there are shown the normal variances of Cx and Cy; so, the variance of Cx is null and that of Cy is a uniform sinusoid.

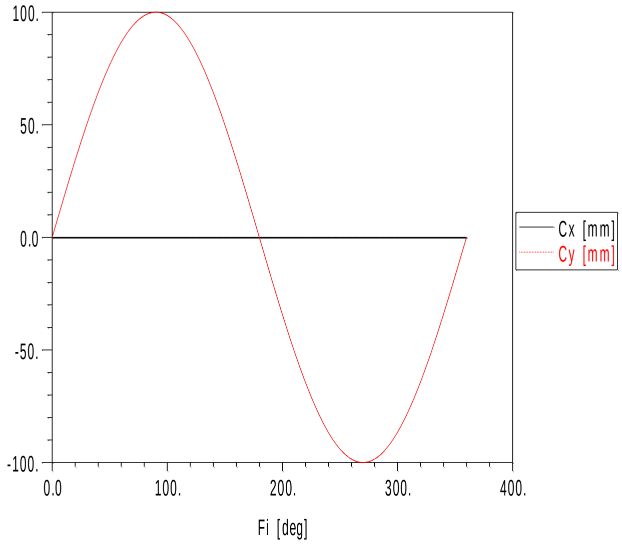

Fig. 15. r_2=50; b=50 – normal curve

In Table 1 it may be noticed (for a subinterval of the rotation cycle) that Cx is not rigorously equal to 0, it has changes in the 5th or 6th decimal, due to truncation errors of the trigonometric function.

Table 1

φ [deg]	Cx [mm]	Cy [mm]
0	0	1.267591E-04
1	3.814697E-06	1.745374
2	3.814697E-06	3.490078
3	7.629395E-06	5.233718
4	7.629395E-06	6.975776
5	1.144409E-05	8.715696
6	1.525879E-05	10.45296
7	1.525879E-05	12.18704
8	1.144409E-05	13.91743
9	1.907349E-05	15.64356
10	1.907349E-05	17.36492
11	1.907349E-05	19.08101
12	3.051758E-05	20.79129
13	2.288818E-05	22.49521
14	3.433228E-05	24.1923
15	3.433228E-05	25.88202
16	3.433228E-05	27.56384
17	3.814697E-05	29.23727
18	3.814697E-05	30.9018
19	4.577637E-05	32.55692
20	4.196167E-05	34.20211
21	4.577637E-05	35.83689
22	4.577637E-05	37.46075
23	5.340576E-05	39.0732
24	4.577637E-05	40.67375
25	5.340576E-05	42.26192

The elongated hypocycloid shown in Fig. 16 is an ellipse with the minor axis given by Cx and the major axis given by Cy.

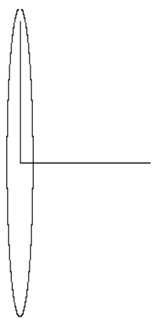

Fig. 16. r_2=50; b=60 – elongated curve (ellipse)

In Fig. 17 it may be noticed that the variation of Cx is small relative to that of Cy, the ellipse having a small width.

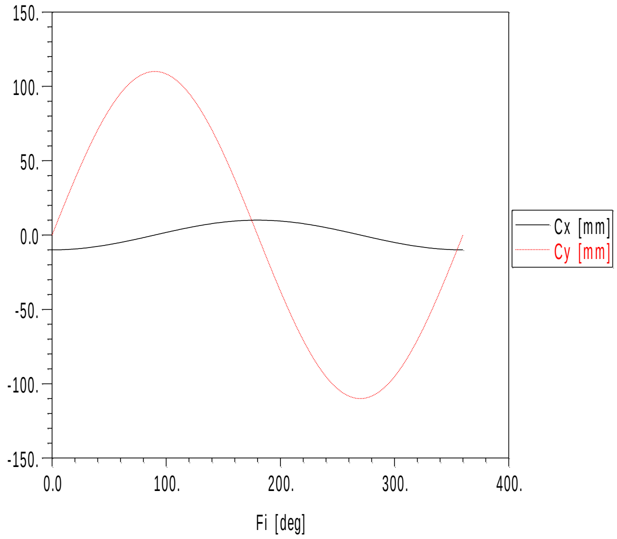

Fig. 17. r_2=50; b=60 – elongated curve

Further, there were studied cases with r_1 = 120, the findings being like that in the above cases; here the range of hypocycloids is higher because 120 divide both with 3 and multiples of 3.

After other tests there were noted other forms of hypocycloids and transfer functions (for r_1 = 100 mm), too.

Thus, Fig. 18 shows a shortened hypocycloid having a square form that can be used, for example, by the devices for squares generation on automatic lathes or horizontal forging machines.

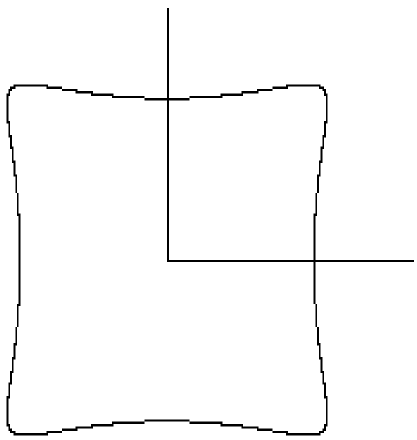

Fig. 18. r_2=25; b=15 – shortened curve

Transfer functions shown in Fig. 19 are different from the previous ones; each of them has two loops and close to straight zones.

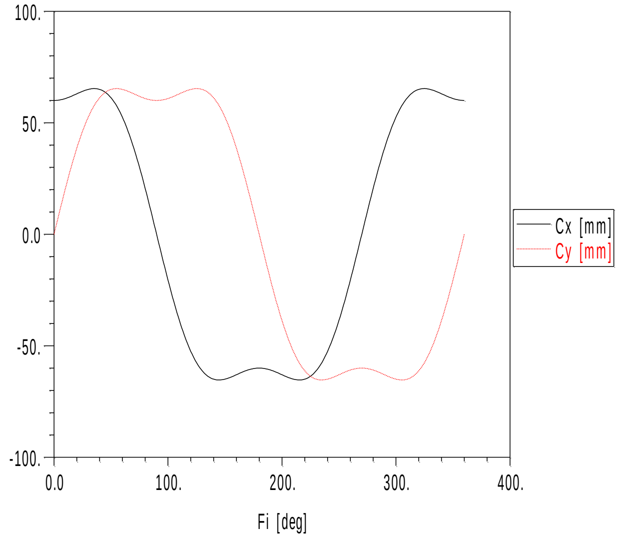

Fig. 19. r_2=25; b=15 – shortened curve

In the case shown in Fig. 20 hypocycloid is similar to a triangle, and Cx (Fig. 21) has two approximately straight zones.

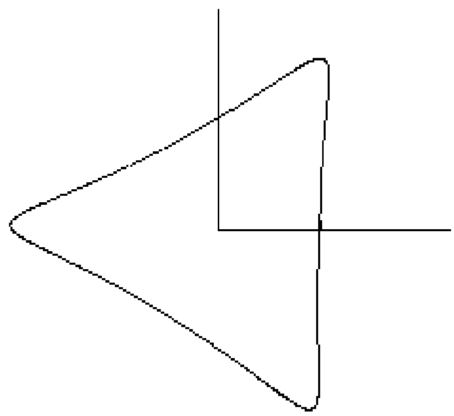

Fig. 20. r_2=33; b=23 – shortened curve

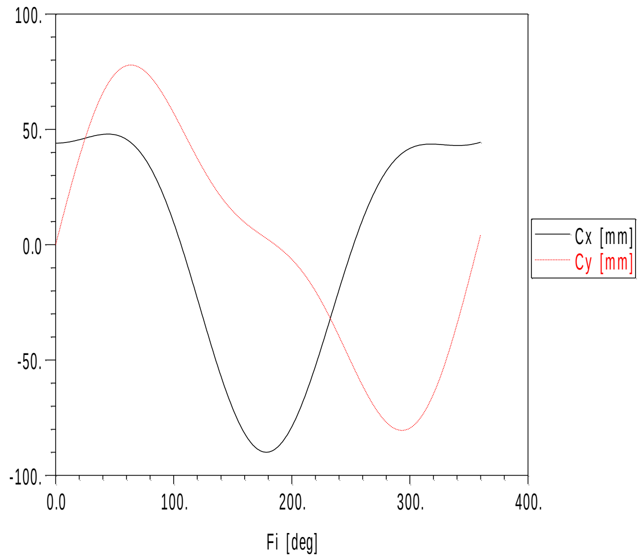

Fig. 21. r_2=33; b=23 – shortened curve

3. Conclusion

A mechanism for hypocycloids generation has been completed with a RPP dyad in order to obtain different forms of transfer functions. The following issues were found:

– the number of arcs of generated hypocycloids is equal to the ratio of the gears radii r_1 / r_2;

– transfer functions are curves that have a number of small loops on them equal to the number of arcs of the respective hypocycloid;

– elongated curves have arcs and loops on the drawn curves;

– shortened curves have no loops, but finer filleted corners between the arcs;

– for the radii ratio equal to 2, the diametral line is obtained, the transfer function Cx being null;

– if the radii ratio is 2, the elongated curve and the shortened curve are ellipses;

– if the radii ratio is a decimal number, the curve of the transfer function Cy no longer closes to the starting value;

– many types of curves with different transfer functions can be obtained by changing the values of r_2 and b.

Such transfer laws may be used for:

a) devices to generate squares on automatic lathes;

b) devices to generate triangles or hexagons on automatic lathes;

c) devices to generate squares, triangles or hexagons on horizontal forging machines;

d) devices to generate squares, triangles or hexagons on automatic textile machines;
e) in kinetic art.

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