Journal of Engineering Sciences and Innovation
Volume. 2, Issue 1/ 2017, pp. 113-120

Technical Sciences Academy of Romanian
www.jesi.astr.ro

E. Civil Engineering and Transporting Engineering

Received ***6 December 2016*** *Accepted* ***9 March 2017***
Received in revised from ***31 January 2017***

EXPRESSING THE INFLUENCE AREA OF SLABS THROUGH DEFORMED SHAPES OF THE STRUCTURE

TOPA NICOLAI[1*], DOVANCESCU MIHAI[2*]

[1]*Member of the Technical Sciences Academy of Romania, 26, Dacia Blvd., Bucharest 010413, Romania*
[2]*Consitrans SRL, 56, Polona Street, Bucharest 010504, Romania*

Abstract: The design of bridge decks and in particular the ones with slabs, implies the determination of the stresses introduced by mobile loads, which often leads to the existence of influence areas (or, as case may be, influence lines). This article is intended to show that for the slab elements the influence area of a cross-sectional effort can be treated like a deformed shape of the structure. For this demonstration, the bending moment will be used, usually being the most important cross-sectional effort.

Keywords: influence area, slabs, deformed shapes.

1. The influence area of the bending moment (theoretical aspects)

Given the needed cross-sectional effort M_x in point *i* in Fig. 1.

$$(M_x)_{ij} = -D\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right)_{ij} = -D\left(\frac{\psi^x_{ij}}{\partial x} + \mu\frac{\psi^y_{ij}}{\partial y}\right) \qquad (1)$$

The derivative of second order of the *w* function shows in one point the opposite of the radius of gyration, and for a limited area ∂x or ∂y the relative

*Correspondence address: nicvar1929@gmail.com; mihai.dovancescu@consitrans.ro

rotation between the tangents to the deformed surface of the outer points of ∂x or ∂y segment – Fig. 1b. ψ angle also represents the relative rotation of the orthogonals from the extremities of ∂x or ∂y.

Fig. 1c shows the conventional vector representation of the bending moments and rotations that will be used in this article.

Taking a system of forces Q_i depending on a single parameter as the one in Fig. 1a.

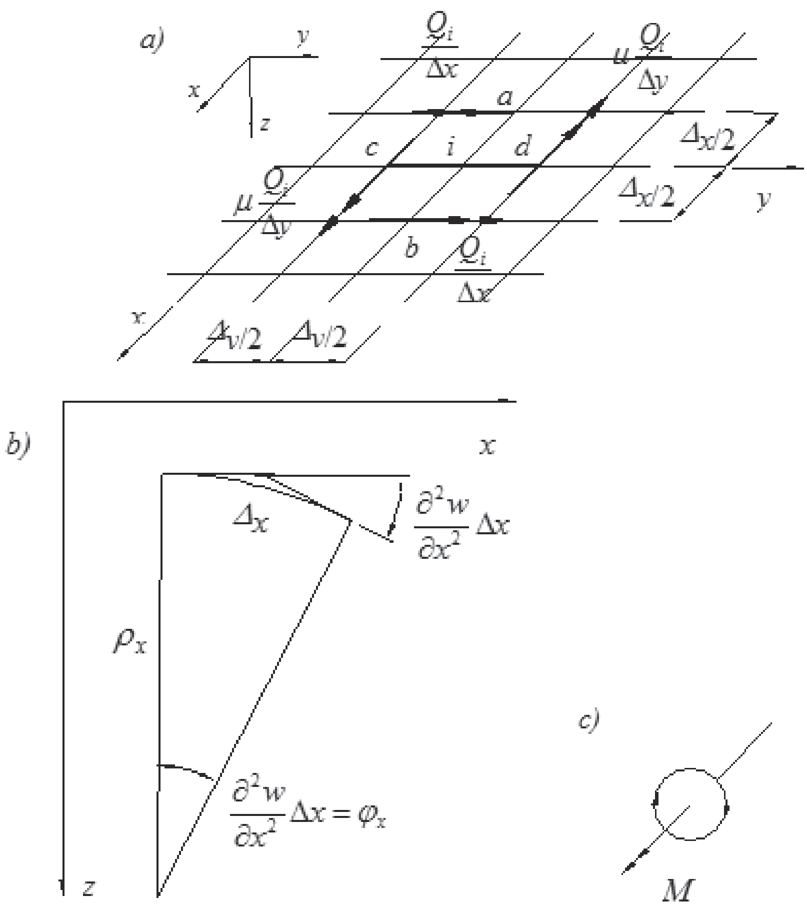


Fig. 1

The other system of forces is reduced to a single force P_j in the current point, of a variable position *j*.

A general deformed shape of the structure and also v_{ji} corresponds to Q_i system of forces. A general deformed shape of the structure and also ψ_{ij} (the two rotations ψ_{ij}^{x} and ψ_{ij}^{y} together, between points *a* and *b* on *x* direction and, respectively, points *c* and *d* on *y* direction) corresponds to P_j system of forces. All the points *a*, *b*, *c*, *d* are shown in Fig. 1a.

The influence area of $M_{x,i}$ is, therefore, the influence area of the member in brackets in relation (1), with -*D* scale.

According to BETTI theorem:

$$-\frac{Q_i}{\Delta x}\psi_{ij}^{x} - \mu\frac{Q_i}{\Delta y}\psi_{ij}^{y} = P_j v_{ji} \qquad (2)$$

The negative sign comes from the fact that for a positive M (bottom fiber in tension) and *ρ* positive (in the positive direction of *z* axis), the rotation has an opposite sign from M vector.

Considering $Q_i = P_j = 1$:

$$\frac{\psi_{ij}^{x}}{\Delta x} + \mu\frac{\psi_{ij}^{y}}{\Delta y} = -\frac{P_j}{Q_i} v_{ji} = -v_{ji} \qquad (3)$$

Relation (1) becomes:

$$\left(M_x\right)_{ij} = Dv_{ji} \qquad (4)$$

meaning that the influence area of M_x in *i* point is the deformed v_{ji} with *D* scale (v_{ji} being the deformed surface from the load $Q_i = 1$, Q_i system being the one shown in Fig. 1a.

For $Q_i = D$, instead of $Q_i = 1$:

$$\left(M_x\right)_{ij} = v_{ji} \qquad (\text{for } Q_i = D) \qquad (5)$$

a remarkable equation for its simplicity.

Due to type of the special load, v_{ji} is dimensionless.

The results will be more accurate if ∂x and ∂y are smaller in order to obtain the value of M_x as close to point *i*.

2. Numerical example

For a practical exemplification of the above described determination of the influence area of the bending moment M_x in one point of a slab, a 5.00 x 5.00 m slab was chosen, of 0,10m thickness. The modulus of elasticity of the concrete is 2400000 tf/m^2 and Poisson ratio is 0.2. In order to obtain the influence area in the chosen point, the slab was divided into a mesh of 25 x 25 elements of 0.20 x 0.20 m.

An orthogonal axes system *x*, *y* was assigned to the slab. The chosen point for the determination of the influence area is located in node 536 and has the coordinates (1.60, 3.80) m.

The calculation was conducted in two ways:

a) The classic way of determining the influence area, according to the definition, by applying a unitary force in the nodes of the mesh and finding the value of the bending moment M_x in the required point, using the finite element method;

b) The determination of the influence area according to the theory mentioned in the above paragraphs, also using the finite element method for the deformed shape.

The determination of the influence area using the classic method is not spectacular.

For the determination in the second way, it was used the same finite element method analysis software. In the above mentioned second method of determination, the displacements are the most important result, which represent the required influence area.

In order to determine the influence area of the bending moment M_x in point 536, two couples of forces were applied, with the vectors oriented towards both *x* and *y* axes. The distance between the couples from the same pair will be equal to the mesh element size (the distance between two nodes), in this case 0.20m.

The bending moments acting in node 536 is shown in Fig. 2.

For a more precise calculation the bending moments are applied between two nodes (so that the distance in between is reduced) and therefore they will be replaced by couples of forces, applied into the nodes of the mesh.

According to Fig. 3, for the determination of the influence area of M_x bending moment in node 536, the bending moments acting as a load in *xoz* plan and having the vector oriented along *y* axis, are decomposed into:

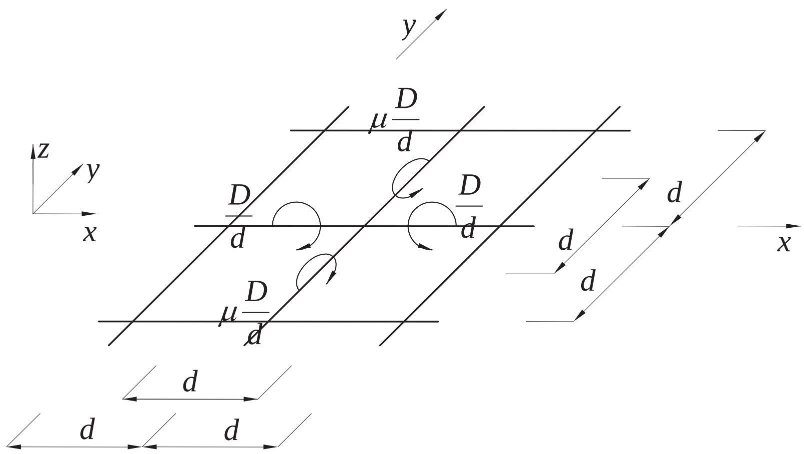


Fig. 2

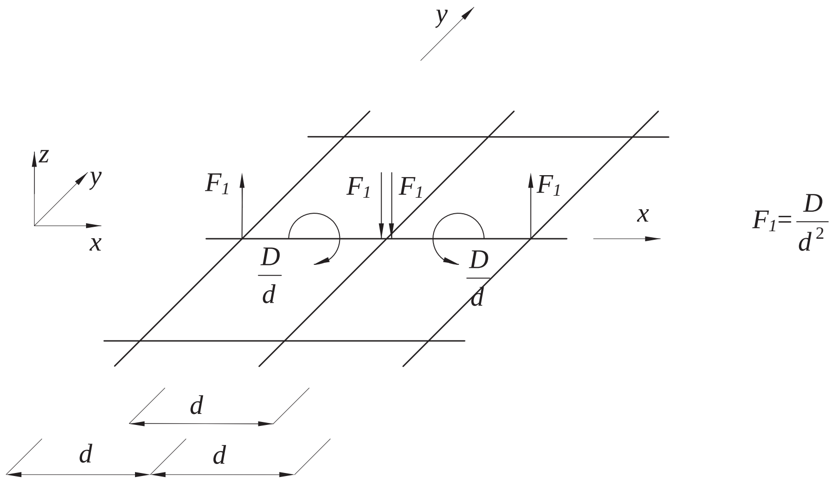


Fig. 3

The bending moments acting as a load in *yoz* plan and having the vector oriented along *x* axis, are decomposed as shown below, in Fig. 4:

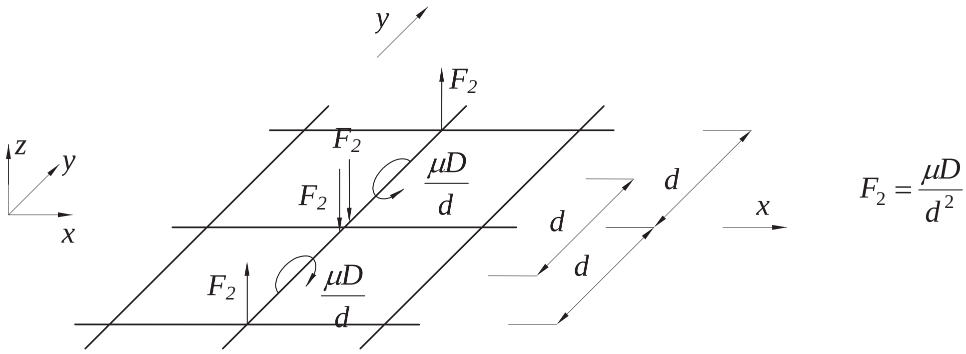


Fig. 4

As shown in Fig. 3, the value of the bending moment that needs to be applied is $\frac{D}{d}$, meaning that the value of the bending cylindrical stiffness of the slab is divided by the distance between the couples.

Taking into account that the thickness of the slab in the considered point is 0.10m, the modulus of elasticity of the concrete is 2400000 tf/m^2 and Poisson ratio is 0.2, it results:

$$D = \frac{Eh^3}{12\left(1-\mu^2\right)} = \frac{2400000 \times 0.10^3}{12\left(1-0.2^2\right)} = 208.33\ \text{tfm}$$

F_1 equals to: $F_1 = \dfrac{\frac{D}{d}}{d} = \dfrac{D}{d^2} = \dfrac{71516.927}{0.40^2} = 5208.33$

F_2 equals to: $F_2 = \mu F_1 = \dfrac{\mu D}{d^2} = 1041.66$

A plan view of the slab around the considered point and the values of the applied forces is shown in Fig. 5:

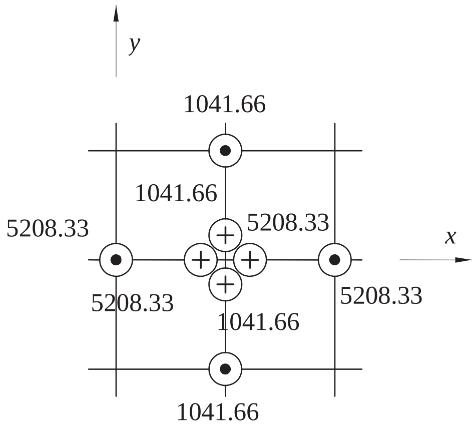


Fig. 5

The sign convention is:

$\odot$ Force pointing upwards

$\oplus$ Force pointing downwards

After the summation, the load matrix in Fig. 6 is obtained:

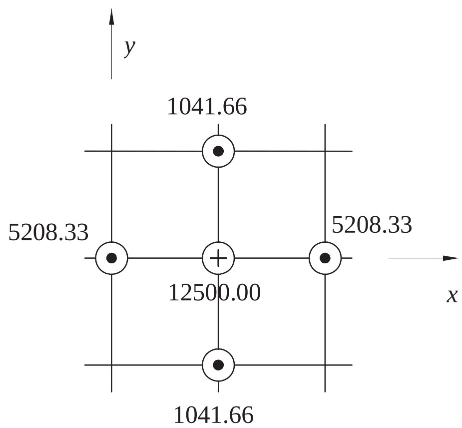


Fig. 6

Fig. 7 shows a perspective view of the influence area.

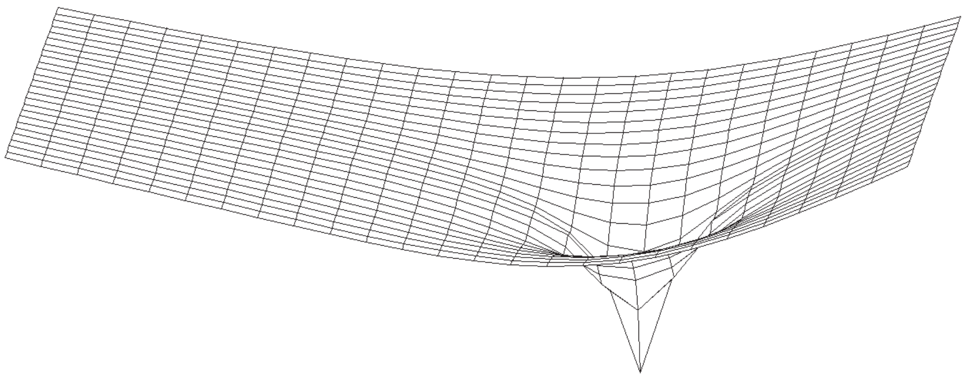
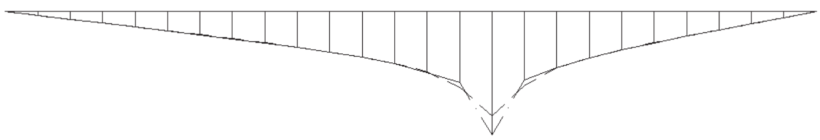


Fig. 7

Fig. 8 shows a section through the influence area, which intersects the considered point.



NODES	Mx FROM UNITARY FORCES	Mx FROM COUPLES OF FORCES	DISTANCE BETWEEN NODES	TOTAL DISTANCE
521	0.0000	0.0000		0.00
522	0.0172	0.0173	0.20	0.20
523	0.0348	0.0347	0.20	0.40
524	0.0523	0.0521	0.20	0.60
525	0.0701	0.0698	0.20	0.80
526	0.0873	0.0877	0.20	1.00
527	0.1064	0.1059	0.20	1.20
528	0.1252	0.1246	0.20	1.40
529	0.1446	0.1440	0.20	1.60
530	0.1653	0.1644	0.20	1.80
531	0.1866	0.1863	0.20	2.00
532	0.2115	0.2109	0.20	2.20
533	0.2406	0.2401	0.20	2.40
534	0.2804	0.2771	0.20	2.60
535	0.3259	0.3498	0.20	2.80
536	0.5698	0.4825	0.20	3.00
537	0.3165	0.3414	0.20	3.20
538	0.2604	0.2603	0.20	3.40
539	0.2147	0.2148	0.20	3.60
540	0.1773	0.1770	0.20	3.80
541	0.1447	0.1439	0.20	4.00
542	0.1136	0.1133	0.20	4.20
543	0.0842	0.0840	0.20	4.40
544	0.0559	0.0560	0.20	4.60
545	0.0278	0.0270	0.20	4.80
546	0.0000	0.0000	0.20	5.00

Fig. 8

Comparing the results that represent the ordinates of the influence area determined by two different methods, differences of maximum 1% are obtained, which can be considered as satisfactory.

It is important to notice that the two influence areas show differences only in the considered point, due to the already known fact in literature that the value of the bending moment is infinite in a point where a concentrated force is applied.

Considering that in all the other points of the mesh the differences between the ordinates are below 1%, we can draw the conclusion that the determination of the influence areas by the proposed method is correct, the duration is short and it offers accurate results both theoretical and practical.

References

[1] Timoshenko, S.P., Woinowsky – Krieger, S. *Teoria plăcilor plane şi curbe.* Editura Tehnică, Bucureşti 1968.

[2] Mazilu, P., Țopa, N., Ieremia, M. *Aplicarea teoriei elasticităţii şi a plăcilor în calculul construcţiilor.* Editura Tehnică, Bucureşti, 1986.