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On the sonification technique

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Abstract. An introduction of the sonification theory and its applications to the medical imaging is presented in this paper. The sonification theory is known in the literature as the transformation of the image into sound by means of a linear operator based on the linear theory of sound propagation. By reversing back to image, an inverse problem has to be solved in order to find if the sound discovers or not new details in the original image. When the classical sonification operator is applied in the inverse problem, no image enhancement is achieved and no details are discovered. This is probably because the classical operator is based on the linear theory of sound propagation. In this paper a new sonification algorithm is advanced based on the Burgers equation of sound propagation. The new algorithm is able to bring improvements in the medical image by inversion. It earns gains in improvement of the medical image by capturing hardly detectable details in the unclear original images. The approach is exercised on fictive ultrasound images of human and rat *livers.*

Keywords: burgers equation, sonification, medical image.

1. Introduction

The enhance of the quality of medical images used to surgery is a problem that scientists have given a special interest [1-4]. The computed tomography, magnetic resonance imaging, nuclear imaging, and ultrasound-positioned medical imaging are implemented for medical purposes including to diagnoses and surgery. This paper introduces a new sonification operator based on the nonlinear theory of the sound propagation, the Burgers equation, in order to solve the inverse problem of sonification and to earn gains in improvement of the medical images.

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Pollack applied in 1952 the information theory to evaluate auditory displays as a visualization tool [5, 6]. Kramer (1994) and Kramet et al (1999) trace the history of the sound art by highlighting the old art such as Sonic Youth and the contemporary art that led to provocative applications, including the works of Christian Marclay, LaMonte Young, Janet Cardiff, Rodney Graham and Laurie Anderson among many others [7-9].
The nano-guitar built by Cornell University physicists from the crystalline silicon no larger than a single human blood cell and the quantum whistle is a nano-scale sound which is able to discover oscillations in superfluid gases are two examples where the sonification is applied [10, 11].
The sonification theory allows new insights into some diseases such as the Alzheimers's dementia [12] and therapies in body movements such as walking, turning, rising arms or legs [13].
The inverse problem of sonification, i.e. the reversing back the sound samples into new images is less studied so far to our knowledge. This is because the applying of known sonification operator in the inverse approach does not bring any improvement in the medical image because the theory behind the sonification operator is the linear theory of sound propagation. This paper introduces a new sonification operator based on the nonlinear Burgers equation of the sound propagation. The new operator has proved its ability to solve the inverse problem of sonification and to earn gains in improvement of the medical image.

2. Theory of sonification

We present in this paragraph the direct problem of sonification as known in the literature [14-17]. The sonification operator S^0 transforms the point data D into sound signals Y^0 $S^0 : D \to Y^0$, $S^0 : x(t) \to y^0(t^0, x(t), p^0)$, where $x(t)$ is the 1D point data to be transformed into sounds, t is the data time, t^0 is the sonification time, and $p^0 \subseteq P^0$, $P^0 = \{k^0, \Delta^0, f^0_{ref}, \alpha^0, \beta^0, \phi^0, \varepsilon^0, g^0, \gamma^0, H^0\}$ are the sonification parameters: k^0 is the time compressor factor on the time interval $T^0 = T / k^0$, $\Delta^0 \geq 0$ is the dilation factor, f^0_{ref} is the reference frequency, $\alpha^0, \beta^0 \geq 0$ are the pitch scaling parameters, $\phi^0 \geq 1$ is the power distortion factor, $\varepsilon^0 \geq 0$ is the threshold for the amplitude, g^0 is the gain function, γ^0 is the decay parameter and H^0 is the timbral control function.
The 1D data stream $x(t)$ can be divided into non-overlapping segments of different length. The variables of the data domain are t, t_i, T. The signal $x(t)$ is expressed as a sequence $x(n)$ at the sampling rate f_s of duration of T seconds. The $x(n)$ consists of $N = T \times f_s$ samples. The time points t_i are the frontiers between

segments $x_i(t)$. If $t_0 = 0$ and $t_M = T$ a possible division in M segments of $x_i(t)$ is

$$x_i(t) = \begin{cases} x(t + t_{i-1}) & 0 \le t \le (t_i - t_{i-1}) \\ 0 & \text{else} \end{cases} \quad (1)$$

The duration of each segment is $T_i = t_i - t_{i-1}$ Each segment $x_i(t)$ is sonified as a single event $y_i^0(t^0)$ which can be longer or shorter than T_i

$$y^0(t^0) = \sum_{i=1}^{M} y_i^0(t^0 - t_{i-1}^0),\ t_{i-1}^0 = \frac{t_{i-1}}{k^0}. \quad (2)$$

The general form for the sonification signal $y^0(t^0)$ is

$$y_i^0(t^0) = |\, x_i(\Delta^0 t^0)\,| \sin\left(2\pi \int_0^{t^0} f_{ref}\, 2^{(x_{trend}(t_{i-1}) + x_i(\Delta^0 t^{0'}))} \mathrm{d}t^{0'} \right), \quad (3)$$

where the second term $x_i(\Delta^0 t^{0'})$ is the mean free segment, and $x_{trend}(t_{i-1})$ is the trend signal at the starting point for pitch modulation. Parameter Δ^0 determine the length of the sonic event T_i^0. If $\Delta^0 = k^0$ the adjacent events do not overlap but for $\Delta^0 \le k^0$ they overlap.

To introduce control of timbre, the operator H^0 acts as the sine function, so

$$y_i^0(t^0) = a_i(t^0) H^0 < \sin\left(2\pi \int_0^{t^0} f_{ref}\, 2^{b_i(t^{0'})} \mathrm{d}t^{0'} \right) >$$

$$b_i(t^{0'}) = \left(\alpha^0 x_{trend}(t_{i-1}) + \beta^0 x_i(\Delta t^{0'}) \right), \quad (4)$$

where $a_i(t^0)$ is the amplitude modulator, f_{ref} is the base frequency for the pitch range of sonification and $b_i(t^0)$ is a pitch modulator. The amplitude modulator is defined as

$$a_i(t^0) = |\, x_i(\Delta^0 t^0\, |^{\phi^0} \;,\; \phi^0 \ge 1, \quad (5)$$

where φ^0 has the role of the amplitude modulator. For exceeding a threshold ε^0 around the mean of the amplitude, a half-wave rectification is included

$$a_i(t^0) = g\left(|\, x_i(\Delta^0 t^0\, |, \varepsilon^0 \right), \qquad g(x, \varepsilon^0) = \begin{cases} x - \varepsilon^0 & x \ge \varepsilon^0 \\ 0 & \text{else} \end{cases} \quad (6)$$

3. The Burgers equation

The Burgers equation or Bateman-Burgers equation has applications in fluid mechanics, nonlinear acoustics, and other fields [18]. The equation is written as [19]

$$\frac{\partial v}{\partial x}-\frac{\beta}{c_0^2}v\frac{\partial v}{\partial \tau}-\frac{b}{2\rho_0 c_0^3}v\frac{\partial^2 v}{\partial \tau^2}=0 \quad , \qquad (7)$$

where $x=(x_1,x_2,x_3)$, $v=(v_1,v_2,v_3)$ is the acoustic velocity vector, $\tau=t-x/c_0$ is the retarded time, where t is time, c_0 is the velocity of sound propagation in the linear approximation, $b=(b_1,b_2,b_3)$ are the dissipation coefficients, ρ_0 is density of medium, $\beta=(\beta_1,\beta_2,\beta_3)$ is nonlinearity coefficient. Details on the pulse propagation in nonlinear 1D media can be found in [20-23].

The purpose of a new sonification operator is to bring gains in a medical image with potential detectable details. A replication of the image already present does not help the diagnosis and surgical of tumors. So, we are looking for a new operator to replace (3) which is based on the linear theory of sound propagation. To construct a new sonification operator, the Burgers equation of sound propagation is used.

Let us to consider a digital image B of area A, seen as a collection of N pixels $D=\{d_1,d_2,...,d_N\}, d_i\in R^N$. The B is subjected to external force $f(t)$ written as a sum of the excitation harmonic force $F_p(t)$ and the generation sound force $F_s(t)$. The last force is introduced to build the sonification operator. The response of B to $f(t)$ is a new configuration b defined of all points $P\in B$ at the time t resulting by vibration of B . The vibration of B is described by decoupled Burgers equation (7).

One way to convert (7) into a linear equation is applying the Cole-Hopf transformation [24]

$$v=-2u\frac{1}{\phi}\frac{\partial\phi}{\partial x}.$$

Another method to solve (7) is the cnoidal method [25]. The cnoidal solutions are localized waves that conserve their properties even after interaction among them, and then act somewhat like particles. We introduce the function transformation

$$\theta=2\frac{\mathrm{d}^2}{\mathrm{d}t^2}\log\Theta_n(t), \qquad (8)$$

where the theta function $\Theta_n(t)$ are defined as

$\Theta_1=1+\exp(\mathrm{i}\omega_1 t+B_{11})$,

$\Theta_2=1+\exp(\mathrm{i}\omega_1 t+B_{11})+\exp(\mathrm{i}\omega_2 t+B_{22})+\exp(\omega_1+\omega_2+B_{12})$,

$\Theta_3=1+\exp(\mathrm{i}\omega_1 t+B_{11})+\exp(\mathrm{i}\omega_2 t+B_{22})+\exp(\mathrm{i}\omega_3 t+B_{33})+\exp(\omega_1+\omega_2$

$\quad +\exp(\omega_1+\omega_3+B_{13})+\exp(\omega_2+\omega_3+B_{23})+\exp(\omega_1+\omega_2+\omega_3+B_{12}$

and

$$\Theta_n=\sum_{M\in(-\infty,\infty)}\exp(\mathrm{i}\sum_{i=1}^{n}M_i\omega_i t+\frac{1}{2}\sum_{i<j}^{n}B_{ij}M_iM_j), \qquad (9)$$

$$\exp B_{ij} = \left(\frac{\omega_i - \omega_j}{\omega_i + \omega_j} \right)^2, \ \exp B_{ii} = \omega_i^2 . \tag{10}$$

The solution of (7) is taken under the form

$$\theta(t) = 2\frac{\partial^2}{\partial t^2}\log\Theta_n(\eta) = \theta_{lin}(\eta) + \theta_{int}(\eta), \tag{11}$$

for $\eta = -\omega t + \phi$.

The first term in (11) represents a linear superpositions of cnoidal waves, while the second term represents a nonlinear superposition or interaction among cnoidal waves. So, the solution of (7) can be written as

$$v_i = \sum_{j=1}^{l} a_j \mathrm{cn}^j(m_i, \eta_i) + \frac{\sum_{j=1}^{l} b_j \mathrm{cn}^j(m_i, \eta_i)}{1 + \sum_{j=1}^{l} c_j \mathrm{cn}^j(m_i, \eta_i)}, \tag{12}$$

where $\eta_i = k_{1i}x_1 + k_{2i}x_2 + k_{3i}x_3 - \omega_i t + \tilde{\phi}_i$, l is a finite number of degree of freedom of the cnoidal functions, $0 \le m \le 1$ is the moduli of the Jacobean elliptic function, ω is frequency and $\tilde{\phi}$ the phase, k_1, k_2, k_3 are components of the wave vector.

As a result, the cnoidal method yields to solutions consisted of a linear superposition and a nonlinear superposition of cnoidal waves.

The equation (7) and other equations of the same kind (Schrödinger, Korteweg–de Vries equations etc.) have an infinite number of local conserved quantities, an infinite number of exact solutions expressed in terms of the Jacobi elliptic functions (cnoidal solutions) or the hyperbolic functions (solitons), and the simple formulae for nonlinear superposition of explicit solutions.

4. New sonification algorithm

The main idea derives from the fact that the linearized equation can be solved by an ordinary Fourier series as a linear superposition of sine waves. The generalization of Fourier series as a linear superpositions of cnoidal waves seems to be a natural way to solve the equation (7).

Now, given a known force F_p, we determine the unknown force F_s such that the acoustic power radiated from B to be minimum. The acoustic power radiated from B is written as

$$W = \frac{A}{2} v^T p, \tag{13}$$

where v is the velocity verifying (7) and p the acoustic pressure vector, A is the area of the rectangular picture, and the subscript T represents the Hermitian transpose [26].

We suppose that the solutions v_i, $i = 1,2,3$ of (7) are expressed by (12).

In the following we stop to $l = 2$, and we will see that there are no sensible improvements in solutions for $l > 2$ By setting

$$\frac{\partial W}{\partial F_s} = 0\,, \tag{14}$$

the function $F_s(t)$ is determined.

The unknown parameters $P_j = \{m_j, \omega_j, k_{1j}, k_{2j}, k_{3j}, \tilde{\phi}_j, a_1, b_1, c_1, a_2, b_2, c_2\}$, $j = 1,2,3$, are find by a genetic algorithm which minimizes the objective function $\Upsilon(P_j)$ written with respect to residuals of (7) and (14)

$$\Upsilon(P_j) = 3^{-1}\sum_{j=1}^{3}\delta_{1j}^2 + \delta_2^2\,, \tag{15}$$

with

$$\delta_{1j} = \frac{\partial v_j}{\partial x_j} - \frac{\beta_j}{c_0^2} v_j \frac{\partial v_j}{\partial \tau} - \frac{b_j}{2\rho_0 c_0^3} v_j \frac{\partial^2 v_j}{\partial \tau^2}\,,\quad \delta_2 = \frac{\partial W}{\partial F_s}\,. \tag{16}$$

The genetic algorithm is running until it is reached a non-trivial minimizer, which will be a point at which (15) admits a global minimum.

The quality of results depends on the values of Υ . The required precision is taken to be six places after the decimal point. The genetic parameters are assumed to be as follow: number of populations 200, ratio of reproduction 1.0, number of multi-point crossover 1, probability of mutation 0.5, and maximum number of generations 500.

Once determined the function F_s, the sonification operator S is written as

$$S(D,t) = F_s(\tilde{D},t) + \frac{F_s(\tilde{D},t)}{1 + F_s(\tilde{D},t)}\,, \tag{17}$$

where $D = \{d_1, d_2, ..., d_N\}, d_i \in R^N$ is the point data domain of the original image, $\tilde{D} = \{\tilde{d}_1, \tilde{d}_2, ..., \tilde{d}_N\}, \tilde{d}_i \in R^N$ is the point data domain of the sonified image, and t is the sonification time.

Data D is arranged under the form of a matrix with arbitrarily number of elements. The grid is assumed to be sufficiently fine and special care is devoted to eliminate (or at least to keep under control) the sources of numerical errors. Each element of the matrix may contain interfaces or borders separating the colors and nuances, line and curved lines, that is everything that appears in the image.

The equation (7) is solved for continuity requirements expressed as sharp interface conditions for which a matching of both displacements and stresses at the image interfaces is imposed. In order to assure that the sonified image contain new relevant detail and not noise, the reflections by the edges of the discretization are removed by the Dirichlet and Neumann boundary conditions and a strongly attenuative buffer. The outward-moving wavefield is separated from the inward-moving one, and reflections along the boundaries are reduced by the outward-moving components. The reflection coefficient is [27]

$$r_j = \left(\frac{1-\cos\theta}{1+\cos\theta} \right)^j , \qquad (18)$$

where j is the degree of approximation, and θ is the incidence angle. The reflections from boundaries are removed by coupling the Dirichlet fixed boundary conditions solution to the Neumann free boundary conditions solution. For more than one component of displacement, the Dirichlet and Neumann conditions alternate components at the boundaries. When more than one boundary is nonreflecting, more solutions are added to eliminate multiple reflections.

The continuity requirements are used to fill in blank spaces in the sonified image by continuity of solutions in the vicinity of interfaces and edges. After sonification, the mapped data typically contain small blurred areas with cavities and white dots due to the inaccuracies of the original medical images. These areas are filled with color and geometric lines, through continuity requirements with the help of solutions in adjacent areas, so that the final image may contain new elements, new details that do not appear in the original image.

Utility of the new inverse sonification technique was highlighted on some medical images used to surgical operations. In the next Section we present the applications.

5. Applications

The first exercise consists of some ***2D fictive ultrasound images of human*** liver with some interest regions inspired from [28]. Ultrasound imaging is a noninvasive tecnique and real-time diagnostic modality for diagnosing the diffuse liver diseases, in the frequency range of [2, 18] MHz [28, 29]. Early diagnosis of the liver diseases is important to improve the treatment and to save the human lives.

The ultrasound image is generated by ultrasound waves into the human body, which are reflected as echoes and converted into signals by a transducer. In recent years, 3D ultrasound images has also been developed. 29, 39 Accurate information about the space, shape, size, and texture of tumor is provided by 3D/4D ultrasound. This procedure may also be useful to evaluate the real tumor volume as well as the healthy surrounding organs.

The sonification operator S given by (17) is exercised on fictive ultrasound images of normal and diffuse liver diseases. Fig. 1 shows an ultrasound image of normal human liver. Fig. 2 presents the ultrasound image of a chronic liver before sonification (left) and after sonification (right). Fig. 3 presents the ultrasound image of a fatty liver before sonification (left) and after sonification (right). Fig. 4 presents the ultrasound image of a cirrhotic liver before sonification (left) and after sonification (right), and Fig. 5 presents the ultrasound image of a cirrhosis on which hepatocellular carcinoma liver before sonification (left) and after sonification (right) . New details in sonified images can be useful for examining the liver abnormalities.

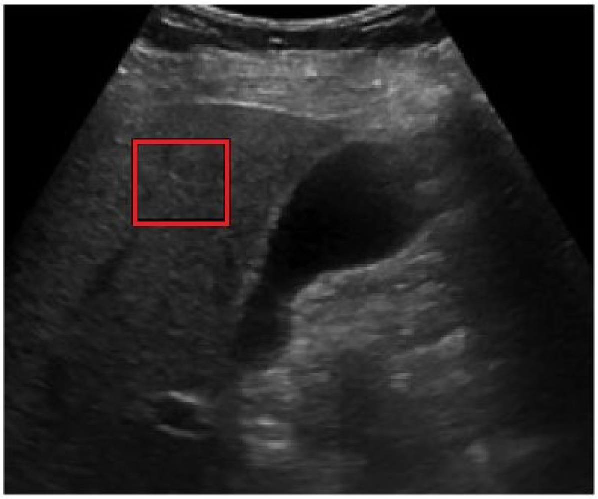

Fig. 1. Ultrasound image of a normal human liver before and after sonification.

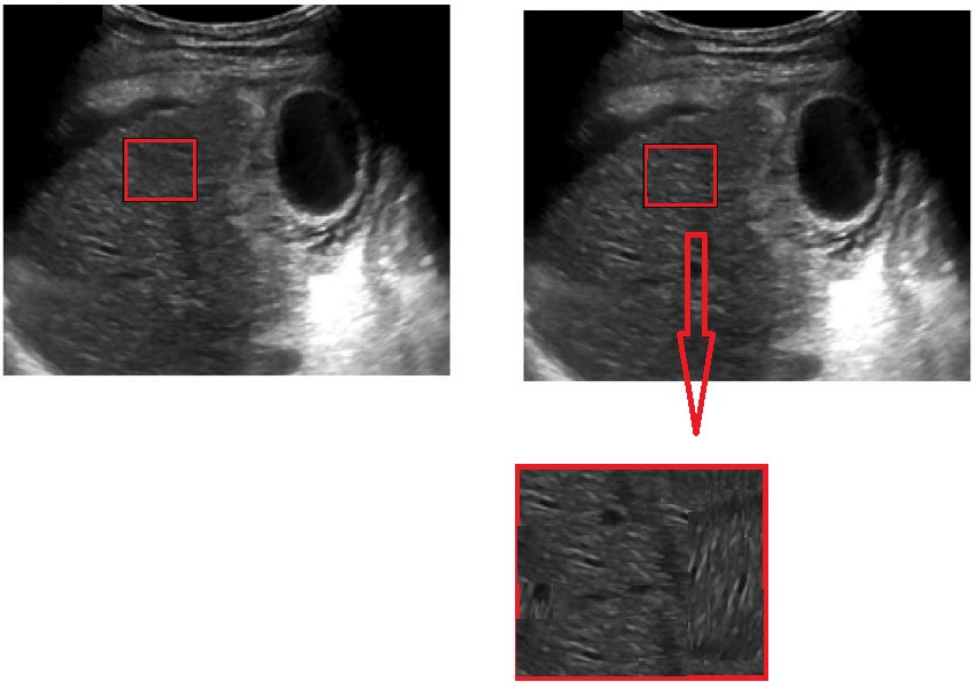

Fig. 2. Ultrasound image of a chronic liver before and after sonification.

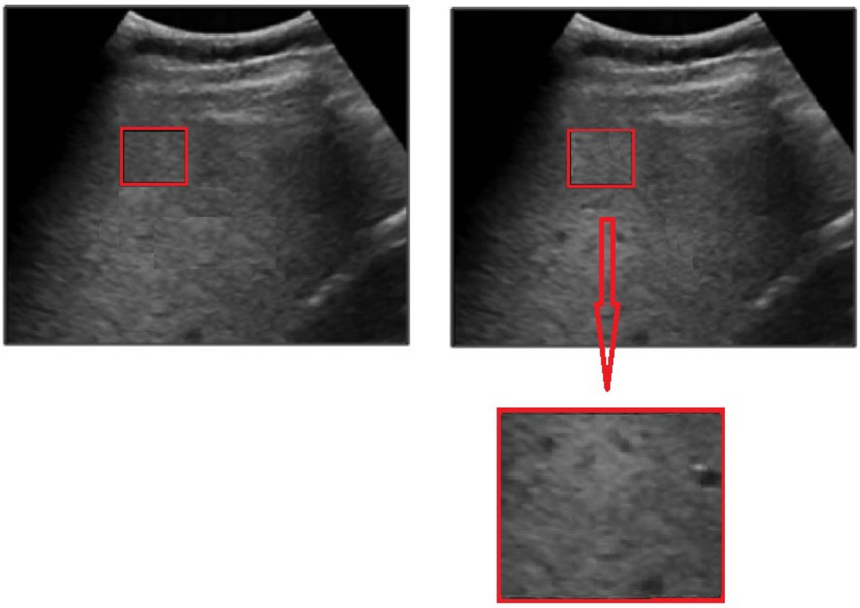

Fig. 3. Ultrasound image of a fatty liver before and after sonification.

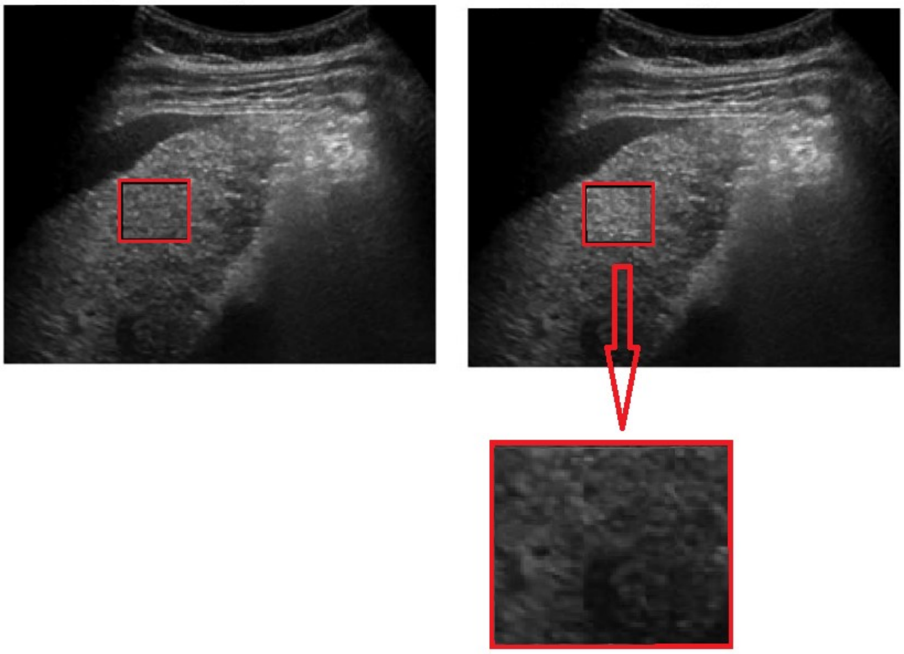

Fig .4. Ultrasound image of a cirrhotic liver before and after sonification.

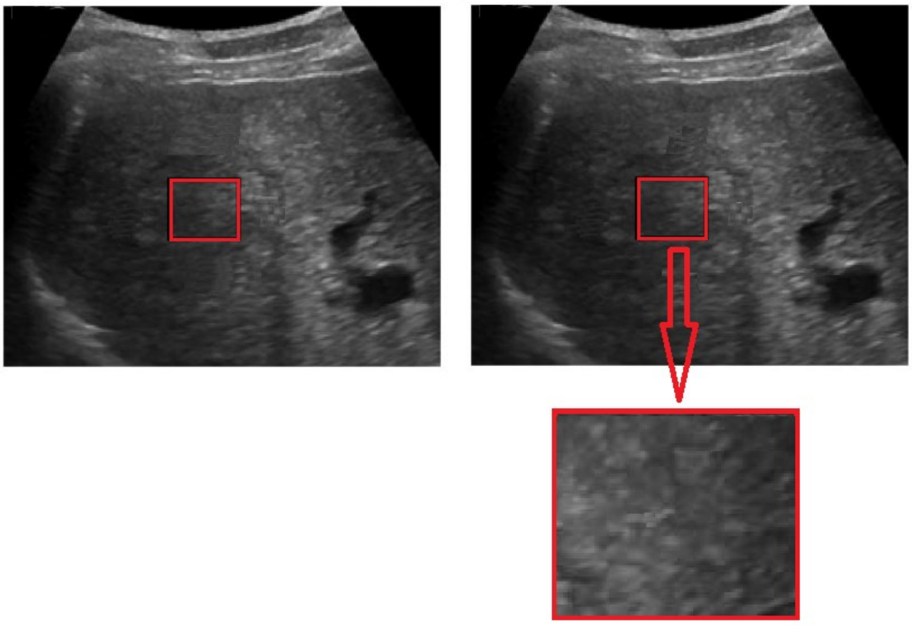

Fig. 5. Ultrasound image of a cirrhosis on which hepatocellular carcinoma before and after sonification.

For the next exercise, we consider the work of Salameh [30] which studies the detection of nonalcoholic steatohepatitis in the fatty rat livers by magnetic resonance (MR). MR is a technique which uses radiology and strong magnetic fields to obtain pictures of the organs. The Salameh study is useful in the early detection of fibrosis in the at livers.

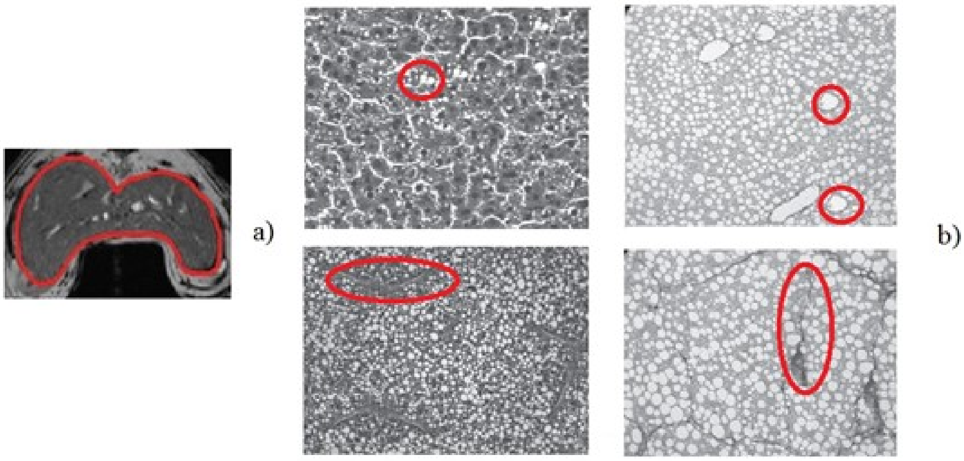

Fig. 6. a) The MR image of a liver rat;
b) All initially hidden details were found by sonification technique.

The sonification operator S given by (17) is exercised on fictive MR images of the fatty rat livers. Fig. 6a shows the MR image of a liver rat with severe loss of architecture, disturbances of the hepatocytes and strong hepatocellular.
To verify the truthfulness of the sonification results, we intentionally hide some details in these images (shown in red Fig. 6b). The application of the inverse sonification operator was successful in the sense that all initially hidden details were found by the operator.
The last application considers the case of a tumor (pink color) located near the portal tree of the vascular territory (Fig. 7a) [31, 32].
The vascular territory (1) and the vessel branches in the vicinity of tumor (2) are shown in Fig. 7b. We obtain through sonification three images shown in Fig. 8 for the frontal, caudal and cranial views. Besides new details on the tumor reported to surrounding areas, the shape and size of the tumor is better visualized.

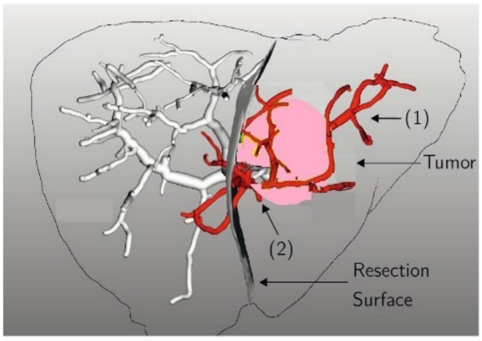

Fig. 7. a) Location of the tumor;
b) Vascular territory (1) and the vessel branches in the vicinity of tumor.

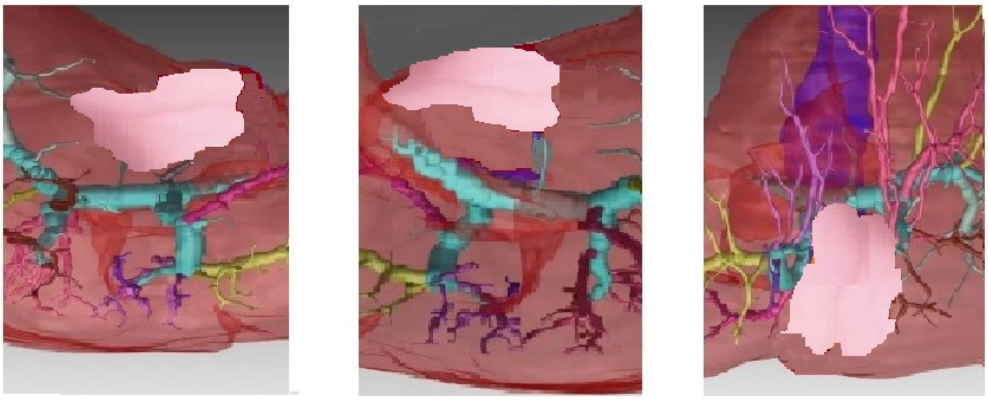

Fig. 8. The new images in the vicinity of the tumor after sonification.

Details about other applications in liver surgery can be found in [33-37].

6. Conclusions

Medical imaging tends to become more accurate in sensitivity and clarity for predicting of diseases and efficiency of surgical operations. The sonification operator proposed in this paper is converting the digital data field into sounds, using a set of basic functions defined as series of cnoidal vibrations. By inverting the sound into image, the results ***highlight hidden details seen by the sound and not seen by the eyes.*** The application of the inverse sonification operator was successful in the sense that all initially hidden details were rediscovered by the operator. In contrast to the direct problem of sonification that convert the data points into audio samples, the inverse problem is turning back the sound representation into images in order to obtain clarity, good contrast and low noise. The inverse sonification technique is exercited on ***fictive ultrasound images of human and rat*** livers with some regions of medical interest. The new sonification operator has the following advantages: 1) does not distinguish between the data time and the sonification time; 2) reflects indirectly the influence of the propagation of sound though abrupt changes in the tissue structure due to Burger equation; 3) discovers of new hard-to-find details in original medical images; 3) provides a better convergence.

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