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FEM or SPH ?

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Abstract. This paper brings, in front of the reader, some aspects regarding the using of those numerical methods, perhaps most used, for analysis of the fluids and structures. Next to the FEM (Finite Element Method), new numerical methods appeared, among these, the methods named meshfree methods are nowadays most used. The SPH (Smoothed Particle Hydrodynamics) method belongs to this category of meshfree method, being the most used in different fields like astrophysical phenomena, fluid dynamics, structure dynamics and others. The paper put face to face some results obtained by FEM and SPH, so the reader can alone to appreciate which method is better in a given problem or other. For to facilitate analysis and to understand the results, the fundamentals of SPH method are presented. In contrast to FEM, the SPH method is less known and less used in Romania. This finding underlies the emergence of this article. The answer to the title question depends on every one and it is influenced by many factors. Finally, the author suggests an answer by a correction of the title question: FEM and SPH or FEM with SPH.

Keywords. FEM, SPH, smoothing length, kernel function, particle

1. Introduction

For the numerical simulation of the simplest up to the most complicated problems, the used numerical methods can be categorised in three types. So, the first category, the oldest one, is represented by the grid-based methods; the second category is represented by the meshfree methods and the third category is represented by the meshfree particle methods. More details are presented in [5].
Referring to the grid-based methods, three approaching ways exist, for describing of the physical phenomena. These ways are based on Lagrangian description, Eulerian description and ALE (Arbitrary Lagrangian-Eulerian) description.

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The basic concepts of these three approaching ways are synthetically presented in the Figure 1.
The Lagrangian description is usually represented by the FEM, and the Eulerian description is usually represented by the finite difference method (FDM).
As ALE description is concerned, this is a newer formulation which put together the advantages of those two ways: Lagrangian and Eulerian. So, the main difficulties coming from large deformations can be avoided. Such formulations (ALE) are usually used in modelling of the explosion, of the impact problems etc.
A less used formulation is that named coupled Eulerian Lagrangian (CEL). In this description, those two basic formulations are used in separate regions of the same domain of the problem.
The differences between ALE and CEL, specially consist in used technique, for the numerical implementation of Lagrangian and Eulerian formulation.

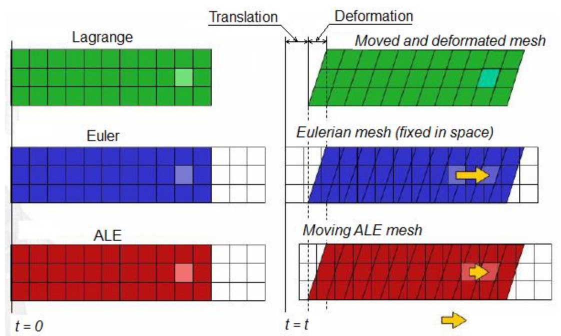


Fig. 1. The state of the mesh and of the analyzed domain by approaching ways

These aspects presented above, are only a few improvements of the grid-based methods. Such methods [1], [14], have already a long history comparatively with meshfree methods or meshfree particle methods, having already many improvements. Jugging by its basic theoretical fundamentals [5], [11], the SPH method could belong to both the meshfree methods and the meshfree particle methods. The history of SPH method started in 1977, with the papers of Lucy, Gingold and Monaghan.

2. Some Examples, Results and Discusions

Some problems and their results obtained by the FEM and SPH methods are presented below. These examples could make it easier to understand the basic theoretical fundamentals of the SPH method.
The first example, referring to the bullet-plate impact, is based on the geometrical model shown in the Figure 2. Such a problem can be modelled by finite element (Figure 3-a) or by free particles (Figure 3-b). The model presented in the Figure 3-b shows a combined use of those two methods: in the same problem a part is modelled by free particles (plate) and an other part (bullet), by finite elements.

The models presented in the Figure 3 are full 3D models; such problems can be also modeled by simplified 3D model (because of symmetry) or by a 2D axis-simmetric model.

As we can see in the Figure 4, the error between the maximum values of the impact direction displacements, obtained by FEM and by SPH, is 1.11% .

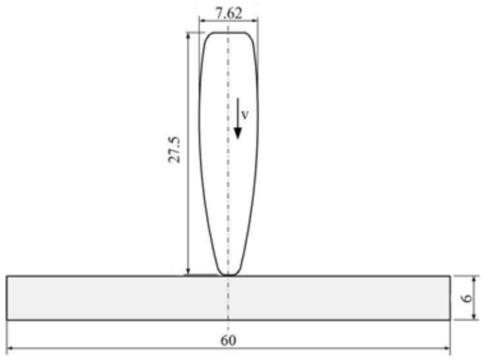
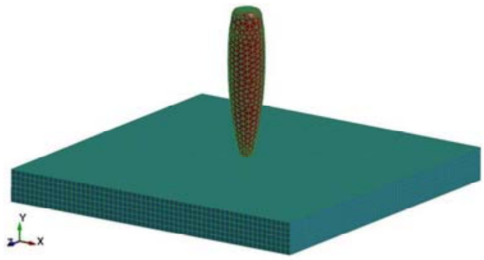
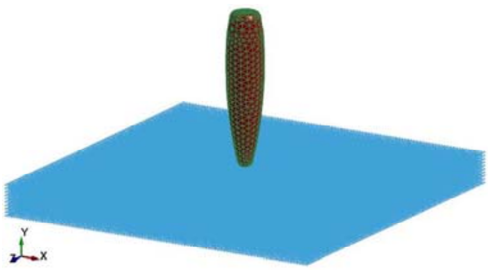


Fig. 2. Geometrical model of an impact bullet-plate

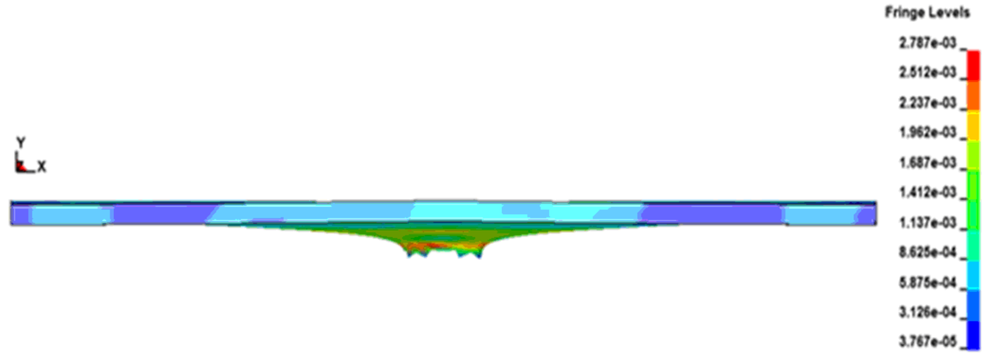


a)



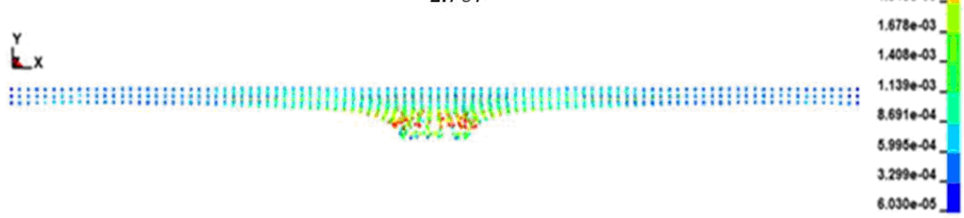
b)

Fig. 3. Numerical analysis models (a-by FEM and b-by SPH)



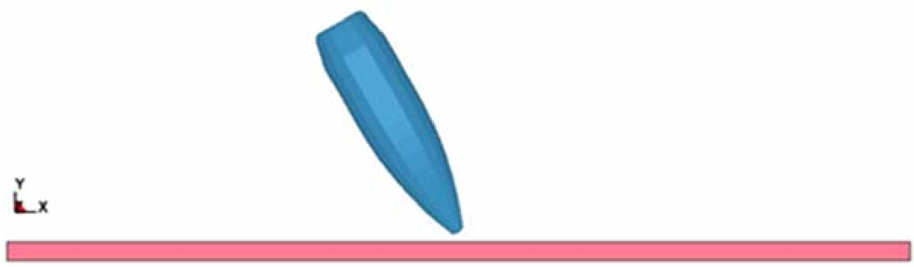
a)

$$Er. = \frac{2.787 - 2.756}{2.787} 100 = 1.11\%$$

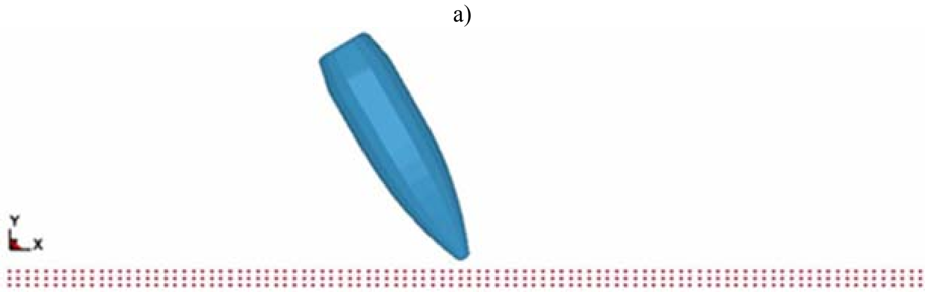


b)

Fig. 4. The deformed state of the plate and its displacements field



a)



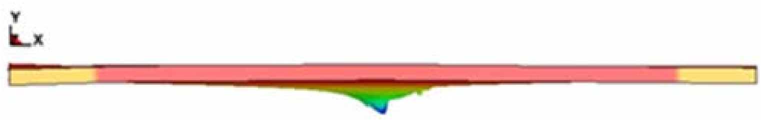
b)

Fig. 5. Oblique impact (300) bullet-plate, in the same modeling

A very good concordance between results exist in the oblique impact case too. The maximum displacement occurs in Y direction (negative displacement). The error between FEM and SPH is 1.27%. The Figure 7 shows the time variation of the bullet velocity, resulting from FE (7-a) and SPH (7-b) modelling.

As the residual velocity of the bullet is concerned, the error between the values obtained by FEM, respectively by SPH, is only of 0.91%, for Y bullet velocity. Next to the curve allure, which is a correct one for oblique impact, we can also see that at the same time moment, the bullet velocity has a constant value, meaning that the plate is perforated.

In the Figures 8 and 9, an impact problem is presented, but for the case when the target is a composite one.

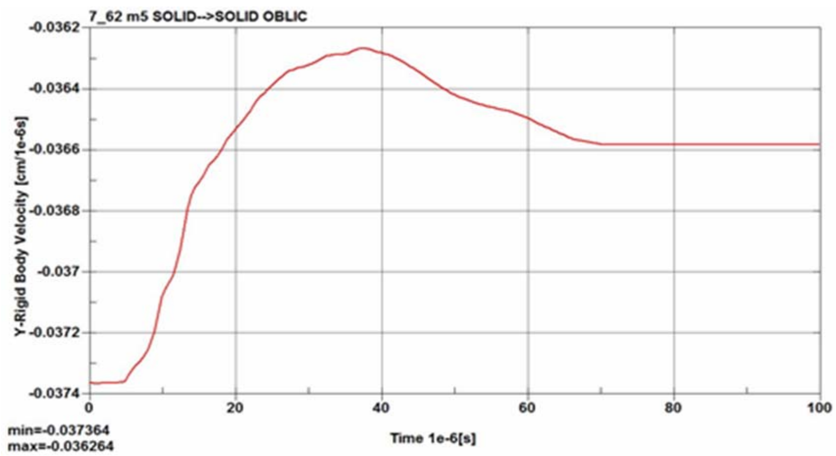


a)

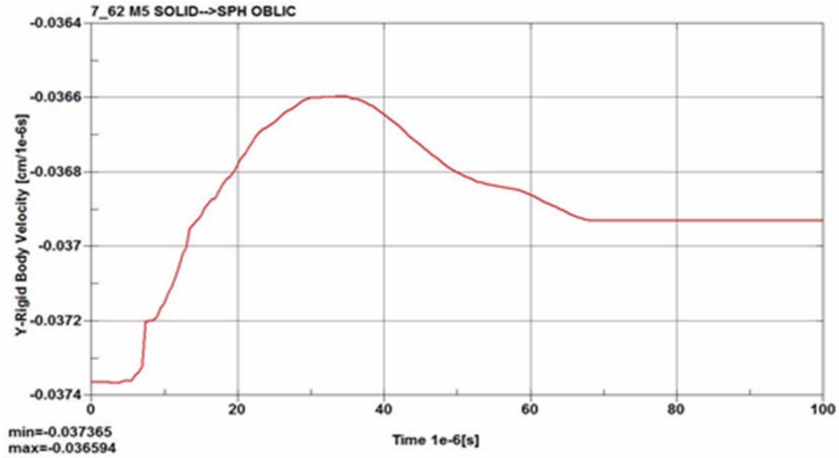
$$Er. = \frac{-1.632 + 1.691}{-4.632} = -1.27\%$$

b)

Fig. 6. Deformed plate with Y-displacement field, for oblique impact

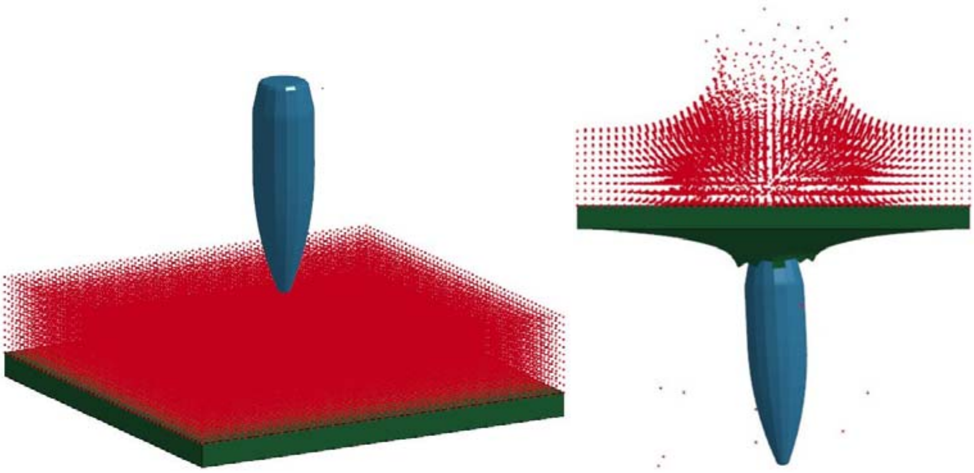


a)

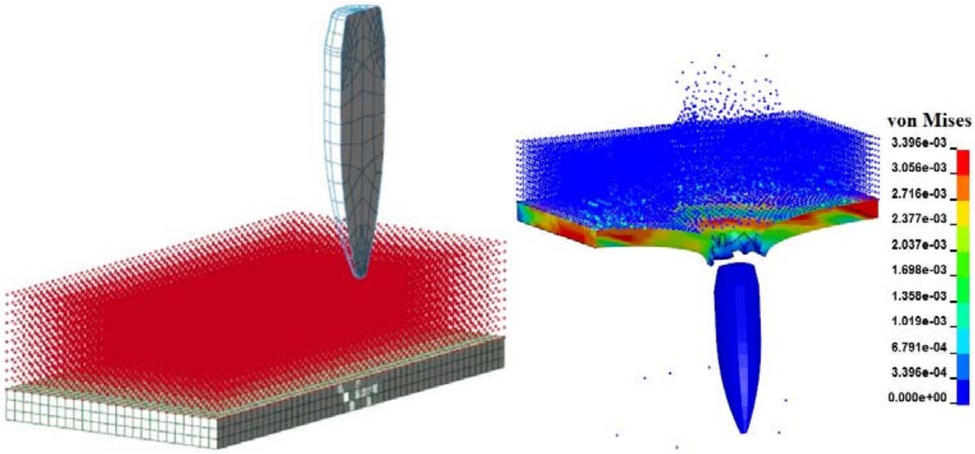


b)

Fig. 7. Time variation of the Y-velocity of the bullet



a) a 3D model



b) a half 3D model

Fig. 8 The impact bullet-composite plate with two layers

The first layer is modelled by SPH and the second layer and the bullet are modelled by finite elements. In these Figures we can watch a good working together between finite elements and free particles.

The Figure 8-b, put in evidence how the finite elements and free particles work together. In the Figure 9, variation of the bullet velocity in time is presented. The allure of the curve, the continuity without any jump, with only the changed slope, can be noticed in the Figure 9, as it is known from technical literature [2], [9].

The Figure 10 presents the water collapse in a closed space, in the presence of an obstacle, at different time moments. In the same figure, the velocity field along X-direction (flow direction) is also presented. Figure 10 shows the great possibilities for fluid dynamics simulation, including the fluid free surface evolution.

As the using of SPH method in fluid dynamics is concerned, it is known that the SPH method is fully validated and it is the best, without meaning that the using of the FEM is abandoned.

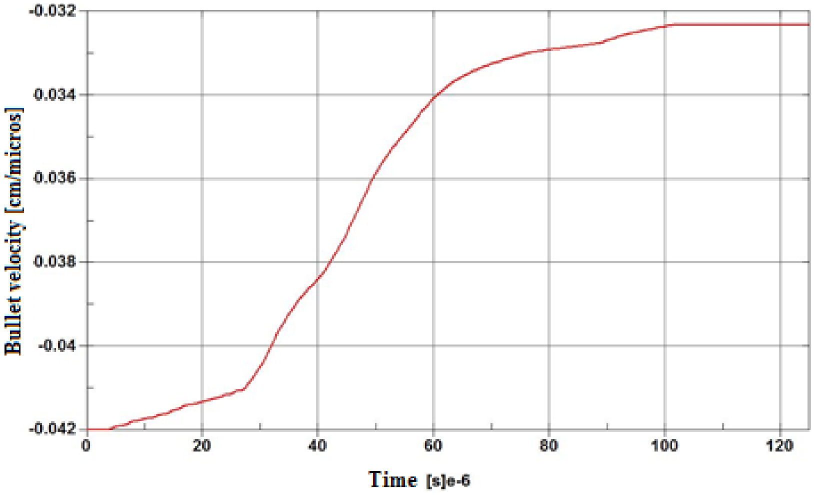


Fig. 9. Bullet velocity variation during composite plate penetration

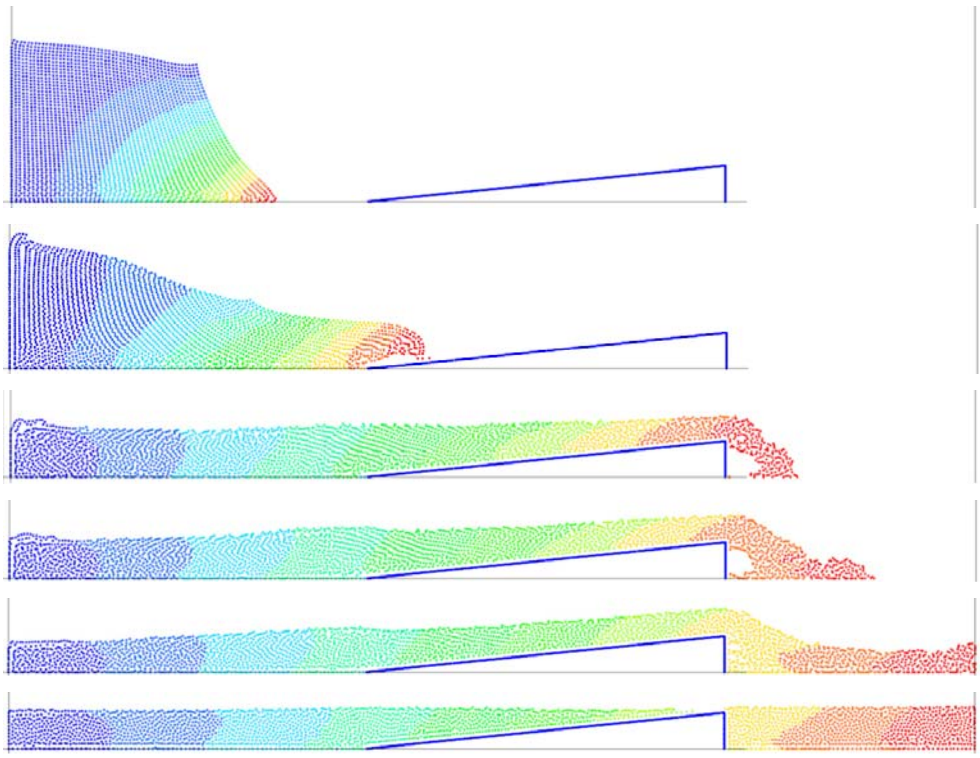
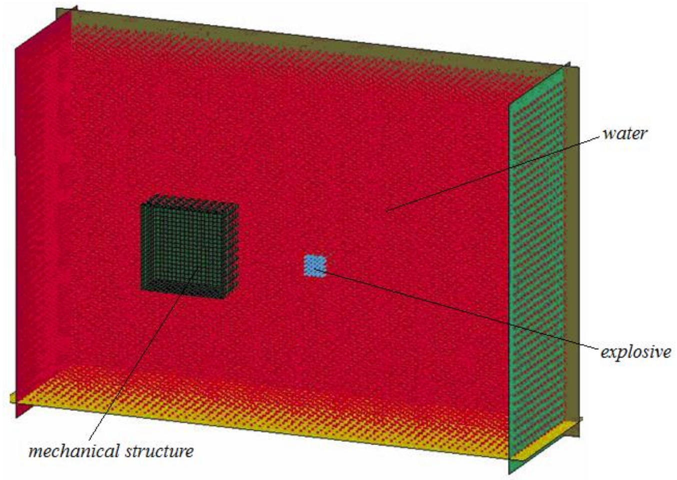
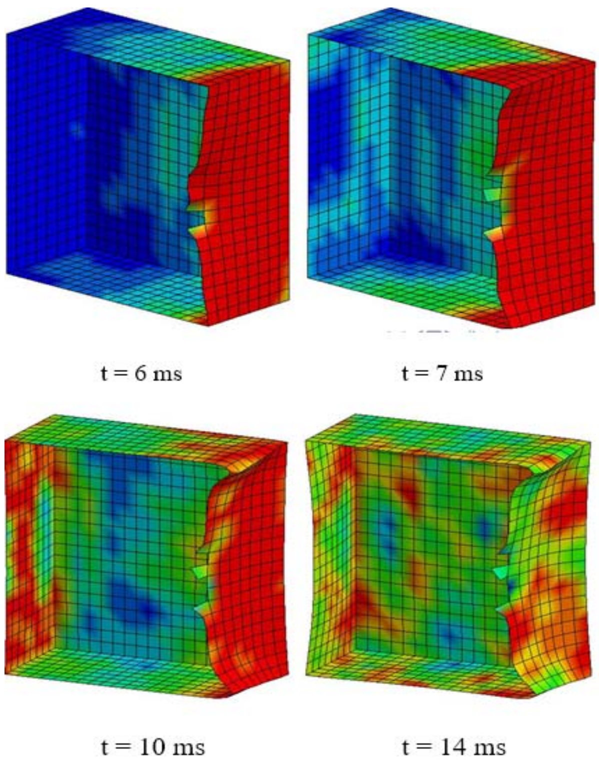


Fig. 10 Water collapse in a limited space, over an obstacle

In the Figure 11, an underwater explosion is modelled using free particle for water and for explosive and finite elements (shell) for an immersed mechanical structure modelling.



a)



b)

Fig. 11. The state of the structure with von Mises stress field, after an underwater explosion

The underwater explosion is simulated by a 3D model, in the case of a water space closed by perfect rigid walls, having only water surface free one. The model presented in the above figure is a half of 3D because of symmetry.

All the above figures, where SPH using is presented show an uniform initial distribution of the particles. This aspect is a very important one and it is a characteristic of SPH models.

As we can see, the interaction between fluid and structure is implicit solved. After the numerical solving, both results are available, referring to structure and in the same time referring to the water and explosive. It is a very important advantage of SPH using. The large deformations appearing as a result of an explosion are avoided using SPH method. The interaction fluid-structure (free particles-mechanical structure) is modelled by special procedures regarding the contact.

3. Theoretical Fundamentals of the SPH Method

From the point of view regarding the description types, SPH method is a Lagrangian type. The interested or analysed domain (a structure, a fluid, generally a continuum) is represented in its initial state by a set of particles without any physical connection between them, but each particle being characterised by all necessary parameters; e.g.: density, mass, velocity, acceleration, forces, position and others.

Finally, at the end of the numerical analysis, also at any time step, the number of the particles will be the same, but their positions and many parameters, defined or not at the beginning, will be changed for all the particles, or for a part of them.

All the physical laws (e.g.: conservation of mass, momentum and energy) are available for the considered system, even represented by this set of such particles.

The theory of SPH method starts from the following mathematical identity, representing an integral representation of a function $F(x)$:

$$F(x) = \int_{\Omega} F(x')\delta(x - x')dx' \tag{1}$$

where $F(x)$ is a function, representing any particle parameter, for the particle defined by the position vector x, Ω is the domain of analysis, $F(x')$ has the same signification of $F(x)$, but for a particle definited by the position vector x'. The notation dx' is referring to the elementary domain, and $\delta(x - x')$ is the Dirac delta function given by:

$$\delta(x - x') = \begin{cases} 1 & \text{pentru} \quad x = x' \\ 0 & \quad\quad\quad\; x \neq x' \end{cases} \tag{2}$$

Relation (1), representing an integral formulation of the function $F(x)$, is an exact definition. In SPH method, Dirac function is replaced with a smoothing, or kernel function $W(x - x', h)$ with h finite dimension of its (support) domain.

If we would be referring to a support domain for Dirac function, then we have to notice that its domain is only one particle defined by the position vector x.

Using a smoothing function, having a domain with h dimension, in which more particles exist, relation (1) represents an approximation of the function $F(x)$, denoted $\langle F(x) \rangle$ and it will be written:

$$\langle F(x)\rangle = \int_{\Omega} F(x')W(x-x',h)dx' \quad (3)$$

For the 1D space, the smoothing function and its domain are presented in the Figure 12 (inside of computational domain with SPH).

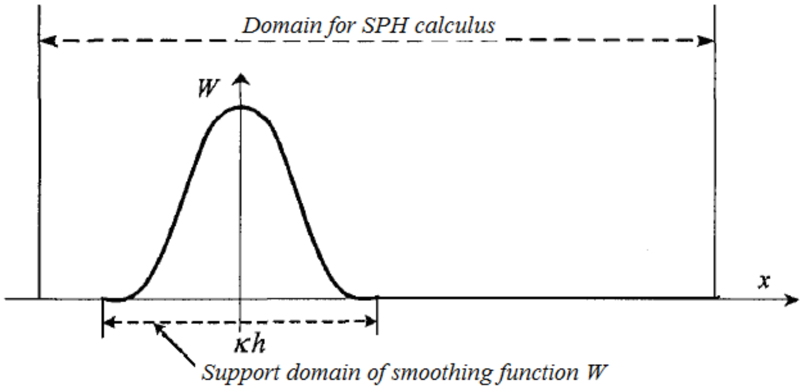
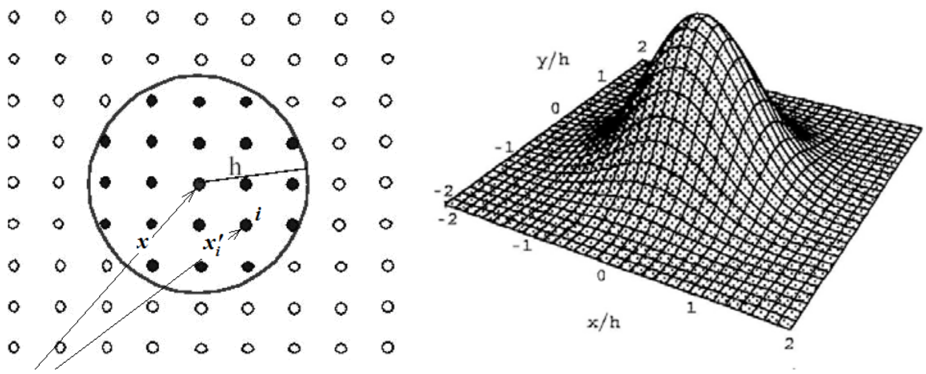


Fig. 12. Calculus domain and support domain for the space of 1D

For a 2D dimension space, the Figure 13-a shows the meaning of parameters x, x' and h. Geometrical shape of the smoothing function over its 2D domain can be watched in the Figure 13-b.



a) b)

Fig. 13. Smoothing function (a-parameters, b-geometrical shape) in the space of 2D dimension

In the space of 3D, the support domain of the smoothing function W, usually, is represented by a sphere with radius of h, or for generality, kh.

The smoothing function has to have some properties. A first requirement is that the smoothing function to be an even one. This is necessary for avoiding the negative values and for kipping the symmetry when this exists. The discussion about this

subject would have been longer, but it is not an aim of this paper. More information can be found in [5] and [8].
The smoothing function W has also to fulfil some conditions. The most important conditions are presented below. So:

- normalization condition, which is expressed by the relation (4);

$$\int_{\Omega} W(x-x',h)dr' = 1 \tag{4}$$

- Delta function property, when dimension of the support domain go to zero;

$$\lim_{h->0} W(x-x',h) = \delta(x-x') \tag{5}$$

- compact condition,

$$W(x-x',h) = 0 \quad \text{when} \quad |x-x'| > kh \tag{6}$$

where k is a constant, by which a non-zero domain is defined, especially when support domain is different of computational domain;
There are some other conditions which can not be expressed by relations, but which are, or can be (depending on the problem), very important for calculus result. Among these conditions, it should be mentioned the require of *monotonically decreasing* with the increase of the distance away from the particle of interest. Then, the smoothing function should be *sufficiently smooth*. This property has a great influence upon obtaining of a better approximation accuracy.
In many professional programs, which implemented SPH method, the most used smoothing function is the cubic B-spline function. This was originally used by Monaghan and Lattanzio, having the following mathematical form:

$$W(s) = \frac{\alpha}{h^n} \begin{cases} \frac{2}{3} - s^2 + \frac{1}{2}s^3 & 0 \le s \le 1 \\ \frac{1}{6}(2-s)^3 & 1 < s < 2 \\ 0 & s \ge 2 \end{cases} \tag{7}$$

where $s = \frac{r}{h} = \frac{|x-x'|}{h}$, r being the distance between two particles, α is a constant which can takes the values 1, $\frac{15}{7\pi}$ or $\frac{3}{2\pi}$, depending on the space dimension n (1 for 1D, 2 for 2D and 3 for 3D).
This function and its first two derivatives are presented in the Figure 14.
In time, many smoothing functions were tried, in order to improve the results coming from SPH utilisation. The Figure 15 shows some smoothing functions.

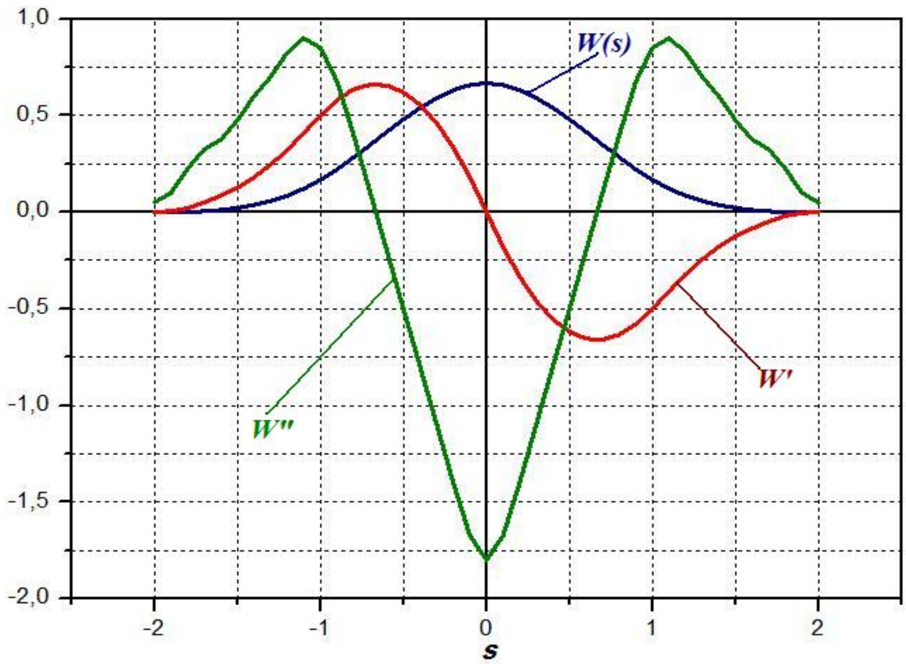


Fig. 14. The cubic B-spline smoothing function

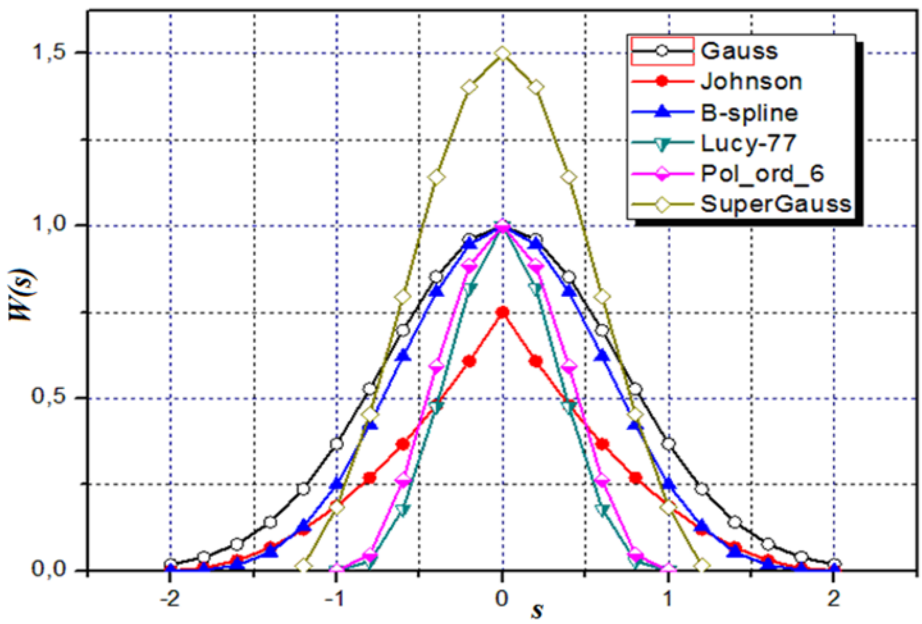


Fig. 15. Allures of different smoothing functions

For computational calculus, the integral forms of the function $F(x)$ are changed into summation form. The elementary domain dx', generally an elementary volume, is replaced, so:

$$m_j = \Delta V_j \rho_j \tag{8}$$

where j index is referring to the particle j. ΔV and ρ are elementary volume and respectively density – assigned parameters to the particle (j). From (8), we can write:

$$dx_j^{'} \approx \Delta V_j = \frac{m_j}{\rho_j} \tag{9}$$

The relation (3) becomes:

$$\langle F(x) \rangle = \sum_{j=1}^{N} F(x_j) W(x - x_j, h) \frac{m_j}{\rho_j} \tag{10}$$

where N is the particle number in support domain.

This form of the function $F(x)$ is used for any parameters attributed to a particle and it is the mathematical form which is used for getting of its derivatives (first and second order). Of course, starting from these mathematical expressions, all the corresponding laws to the investigated problem are formulated. About all these aspects, the technical literature [4], [5], [8] present in detail the mathematical ways and the results.

4. Conclusions

Although examples, presented in this paper, shows a full compatibility between FEM and SPH method, a firm answer, unequivocally one, to the title question can not be formulated. Rather the question is attractively and not correctly formulated. The answer has to be FEM and SPH or FEM with SPH.

These final considerations are based on several observations arising from those presented even in this paper, directly or implicitly.

It is certain that we have a new numerical method (SPH) for solving of many difficult and kind problems of engineering and other areas of science. The SPH method can solve (like FEM) many problems governed by partial differential equations (PDE). Some of these problems are solved better by SPH than by FEM, but other of them can be correctly solved only with SPH method or only by FEM.

SPH method has some especial advantages comparatively with numerical methods based on grids. Here are some of them: it is the best way for numerical simulation of the fluid free surface, it is the best for analysis of the fluid-structure interaction, it has applications in high energy phenomena such as explosion, underwater explosion, high velocity impact, and penetrations, and it is suitable for problems where the object under consideration is not a continuum.

By its Lagrangian nature and formulations, made at the particle level, the SPH method offers some advantages in most complicated problems.

So, some especially procedures like the contact or the fracture can be easily used or can be abandonated (unnecessary used), for solving such problems.
Finally, the answer of the title question depends on each researcher, user or any person interested in knowing and using of SPH method. Unfortunately, in Romania, unlike other countries, the SPH method has a few users and researchers. I hope their number to be greater and greater with the progress in the growth of consistency and stability of the solution, especially in applied mechanics.

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